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THE

Theory of Navigation

DEMONSTRATED:

AND

Its RUDIMENTS Clearly and Plainly Proved,

From the *First* and *most Simple*

Principles *of the* Mathematicks,

WITH

All the *Rules* and *Tables* useful in the Practice

To which is added,

A *New Table* of Difference of LONGITUDE to every Point, Half Point and Quarter Point of the Compass; very useful for the Ready and Expeditious Working of *Traverses* upon the Principles of *Mercators Sailing*, whereby the Difference of *Longitude* answering to every Single Course and Distance may be had by Inspection.

The Third Edition, with Alterations and Additions.

By *JAMES HODGSON*, F.R.S. and Master of the *Royal Mathematical-School* in *Christ-Hospital.*

LONDON:

Printed for *William Mount* and *Thomas Page*, in *Postern-Row*, on *Tower-Hill.* 1738.

TO

The Right Hon. Sir Charles Wager,
Sir Thomas Lyttelton,
Sir Thomas Frankland,
Thomas Clutterbuck, *Esq;*
Lord Harry Powlett,
John Campbell, *Esq;*
Lord Vere Beauclerk,

LORDS COMMISSIONERS for executing the Office of *Lord High Admiral* of *Great Britain*, &c. *Ireland*, &c. and of all His MAJESTY'S Plantations, *&c.*

This TREATISE is, with the Greateſt Submiſſion, moſt humbly Dedicated by

Your LORDSHIPS

Moſt Obedient and

Moſt Devoted Servant,

James Hodgſon.

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THE

PREFACE.

THIS *Book, at its firſt Appearance, met with a kind Reception from the Skillful and Ingenious Part of Mankind; and was afterwards Re-printed at* Dublin, *wherein the Editor took the Liberty to make ſome trifling Additions, as done by myſelf, though without either my Knowledge then, or Approbation afterwards.*

His Royal Highneſs George Prince of Denmark, *who was at that time* Lord High Admiral, *was pleaſed to honour it with His Patronage: And the principal Motive which induced me to write it, was for the Uſe of the Gentlemen Voluntiers on Board the* Royal Navy; *for whoſe Benefit and Improvement, as well as to encourage and promote the* Nautical Science, *ſo conducive to the Wealth and Grandeur of this Moſt Potent Iſland, Her late Majeſty Queen* Anne *had appointed Teachers on Board the ſeveral Ships of War.*

As the Books at that Time moſt in Vogue, conſiſted of a Parcel of Rules put together, without ſo much as an Attempt to ſhew the Reaſon of any one individual Thing; I thought I could not do a more acceptable Piece of Service, than to digeſt the whole into a Natural Order, and ſhew how every thing might be deduced from the Firſt *and* moſt Simple *Principles of the* Mathematicks.

How

How beneficial a Treatise of this Kind, wherein every Thing is naturally inferred from the Depths of Geometry, *may be to young Beginners, is well known to the judicious World; for by giving them right Notions at first, and letting them see the Reason of every Step that is taken, they will retain what they have thus learned better, and be enabled to make much greater Advances in* Geography, Navigation, *and* Astronomy, *than otherwise they would do.*

My System *of the* Mathematicks, *published some few Years since, was formed after the Model of* this Book; *but as there are several Things contained therein, which are not absolutely necessary for every* Navigator *to understand; and as the Price of* that *did not suit every ones Pocket, I have been prevailed upon to revise* this, *once more, and send it Abroad into the World.*

In this Edition, I have increased the Number of Geometrical Theorems, *taken out of the* First Six Books *of* Euclide's Elements, *from whence I have selected such only as I think most useful, and necessary for the intended Purpose; and though there cannot be so strict a Connection between them, as there is between the* Propositions *from whence they are extracted, yet in the Order they are placed, the Conclusions drawn from them are over and above sufficient to answer the present Design.*

In the Trigonometrical Part, *I have added an easy Method of making* Logarithms, *and illustrated it with various Examples; and though to some Persons this Addition may seem needless, because the Tables are already made to great Exactness, yet it must be confessed, that no Person can understand either* Plain *or* Spherical Trigonometry, *the very Basis of* Navigation, Astronomy, *and several other useful Branches*

of

of the Mathematicks, *without being well acquainted with the Nature and Conſtruction of the* Artificial Sines, Tangents *and* Secants, *as well as the* Natural; *and the beſt Way, no doubt, for young Beginners to attain this End, is by exerciſing themſelves at firſt in making a few of theſe incomparable Numbers.*

In the Nautical Part, *I have added ſome uſeful Things, and taken the Liberty to leave out entirely* Middle Latitude Sailing, *and ſuch* Problems *in* Mercator's Sailing, *wherein the* Departure *made a Part of the* Data; *and in the room thereof have put in* New Tables, *by which the* Difference of Longitude *may be found from the* Courſe *and* Diſtance, *in a Manner as eaſy as the* Departure *is in the* Common Tables, *which I hope will be a ſufficient Inducement for Perſons hereafter not to make Uſe of approximating Methods, when they may come at a* true Solution *with the ſame, if not leſs Trouble; and, indeed, if* Plain Sailing *itſelf was laid aſide, I cannot ſee any ill Conſequence would attend it: For to take a Perſon who is entirely ignorant, and inſtruct him in Principles which he muſt afterwards be ſhewn to be* erroneous, *is doing a great Piece of Injuſtice, beſides inſtilling into him* falſe Conceptions *of Things which he will not eaſily get rid of.*

In converſing with Sir Iſaac Newton, *after I was choſen Maſter of the* Royal Mathematical School, *about inſtructing Youth in* Navigation, *in order to form a* Scheme of Learning, *He hinted this very Affair; but at the ſame Time I could not help mentioning, That ſo prevalent is the Power of Cuſtom, ſo tenacious are our* Nautical *Gentlemen of the Methods they have made Uſe of, and ſo afraid of taking a little Pains extraordinary, though of the utmoſt Conſequence, that they*

will not give the leaſt Attention to any Thing that ſeems to run counter, though it carry with it the greateſt Force of Reaſon.

True it is, that before the Invention of Mercator's Sailing, *Mariners were obliged to make Uſe of the beſt Helps they could meet with; but after a Method has been diſcovered above One Hundred Years (which will never fail them, let their Voyages be never ſo long, or never ſo far to the* Northward *or* Southward, *and which, if it had been generally made Uſe of, would undoubtedly have ſaved the Lives, as well as Fortunes, of great Numbers of Perſons) it is ſurpriſing to find even at this time of Day, that the much greater Part of the* Engliſh Sailors *will not be perſuaded out of their* old Way; *and I know of no better Method to prevent its Continuance, but by Teaching the* True Sailing *only.*

In the Aſtronomical Part, *I have left out ſeveral* Problems, *and introduced Others in their Room, which I think more uſeful; and at the ſame Time I have inſerted* New Tables *of the* Sun's Longitude, Right Aſcenſion *and* Declination, *for the Years* 1737, 1738, 1739, *and* 1740, *in the room of the* Old Ones.

The Catalogue *of* Fixed Stars, *and the* Table *of* Latitudes *and* Longitudes *of* Places *have been enlarged, the Former is adapted to the preſent Time, and the Latter is deduced from the* lateſt *and* beſt Obſervations: *This, together with ſome uſeful Matters interſperſed throughout the whole Work, will, I hope, render this Impreſſion as compleat as the* Nature of the Subject *and the* Bulk *will admit of.*

THE

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N. B. *The* Sun's Right Ascension *being calculated to Hours and Minutes only, will serve very well for* 20 *Years, which is the Reason its* Variation *is not set down.*

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Postscript.

AS the *Table* of *Meridional Parts*, as well as the *Table* of *Difference of Longitude*, are computed to Minutes only, when any *Fractional Part* arises in the Course of the Solution, a proper Allowance must be made for it after the Manner following; and to this End, Let us suppose a Ship in the Latitude of 46° 30′ N. to Sail NE. $\frac{1}{4}$ E. 59 Miles, and let it be required to find how much she has altered her Longitude.

Entering the *Table*, *Page* 217, under 4 $\frac{1}{4}$ Points, and against 59 in the *1st Column*, I find 39.62 Miles for the *Difference of Latitude*; this added to 46° 30′, the Latitude came from, will give 47° 09′ .62 for the Latitude the Ship is in.

Entering again the *Table*, in *Page* 182, under 47° and against 9 Minutes, the next less whole Minute to the Latitude found, I find 3215.9 Miles; and against 10 Minutes in the same Column, the next greatest Minute to the Latitude, I find 3217.4 Miles; then to find the Proportional Part answering to .62, say, As 100 is to 15, the Difference between 3215.9 and 3217.4 Meridional Miles; so is .62, the Excess of the Latitude, above 47° 09′ to 9 Tenths, which being added to 3215.9, the Meridional Parts answering to 47° 09′, will give 3216.8, the Meridional Parts answering to 47° 09′ .62, the Latitude the Ship is in; from which taking away 3158.9, the Meridional Parts answering to 46° 30′, the Remainder 57.9 Miles, will give the Merid. Difference of Latitude.

Entering the *Table* again in *Page* 217, under 4 $\frac{1}{4}$ Points, againſt 57, (in the *1ſt Column*) the next leſs whole Number to 57.9, I find in the Column of *Difference of Longitude* 62.89 Miles, and againſt 58, the next greater whole Number above 57.9, I find 64.00 Miles; Then I ſay, As 100 is to 1.11, the Difference between 62.89 and 64.00; ſo is .9, the Exceſs of the Meridional Difference of Latitude 57 Miles, to .999, or rather 1.00, (for when the Figure to be rejected is more than 5, as in this *Caſe* it is 9, the Unit's Place muſt be increaſed by 1) which added to 62.89, the Miles of Difference of Longitude anſwering to 57 Miles, will give 63.89 Miles, for the Difference of Longitude made.

As the *Denominators* of all *Decimal Fractions* are 10, 100, *&c.* ſo if the Difference between the two Tabular Numbers be multiplied by the Fractional Exceſs, and there be cut off from the Product one Figure for Tenths, two Figures for Hundreds, *&c.* the remaining Figures will give the Proportional Part in all Caſes whatſoever. And indeed the Thing itſelf is ſo very ſimple and eaſy, that when a Perſon has a little accuſtomed himſelf to it, he will be able, by Inſpection, to pronounce the Proportional Part without the Trouble of an Operation.

After the ſame Manner may the *Difference of Longitude* for any Number of *Courſes* be found, and conſequently the *whole Difference of Longitude* made to the one Hundredth Part of a Mile.

The Advantage ariſing from keeping *Journals* after this Manner, and to this Exactneſs, is of more Conſequence than at firſt appears; for if the *Courſe* and *Diſtance* be true, the *Difference of Longitude* thus found, is as near as can be determined by any *Cœleſtial Obſervation:* Whence it follows, that the truer the *Courſes* and *Diſtances* are, the more certain will the Mariner be at all Times of the Place he is in; and conſequently, whoever can hit upon a Method, whereby the *True Courſe* a Ship has ſailed, and the *Diſtance* ſhe has run, may be exactly determined, may juſtly be ſaid to have found out the *Longitude*, and is legally intitled to the Reward for the Diſcovery.

As there happened to be a vacant Space at the End of the Contents, *I hope this farther Explanation of the* Uſe of the Tables *will not be unacceptable to the Learner.*

The two Series *in* Page 40 *ſhould ſtand thus:*

Suppoſe 0 . 1 . 2 . 3 . 4 . 5 . 6 , *&c.* to be a, *&c.*

And 1 . 10 . 100 . 1000 . 10000 . 100000 . 1000000, *&c.* to be a, *&c.*

As it is very difficult in Works of this Kind to avoid Typographical Errors, *ſo if any have crept in, I am ſatisfied they are very few, and ſuch as may be corrected at firſt Sight.*

GEOMETRY.

PART I.

Definitions.

THE Subject of *Geometry*, is *Magnitude* or *Quantity continued*; its Business to discover the Nature and Properties of those Quantities about which it is employed, in order to find out the Proportion they have to each other, that so from the Knowledge of one, we may be led to the Knowledge of the other according to Quantity.

Magnitudes are of three Kinds, *viz. Lines*, *Superficies* and *Solids*.

2. A *Solid* is that which hath *Length*, *Breadth* and *Depth*, or *Thickness*; its Limits or Boundaries, are called *Superficies*; so that,

3. A *Superficies* hath *Length* and *Breadth*, without *Thickness*; the Limits of which are *Lines*.

4. And if it lies equally betwixt its Lines, 'tis called a *Plain Superficies*.

5. A *Line* then hath only *Length*, and is bounded by *Points*; so that,

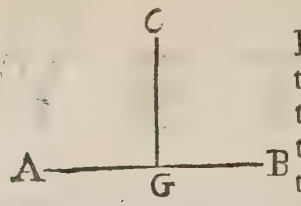
6. A *Point Mathematical* is incapable of being divided, and therefore hath no Parts.

7. A *Right* or *Streight Line* is that which lies equally betwixt, or is the nearest Tract between its Points; as AB.

A ——————— B

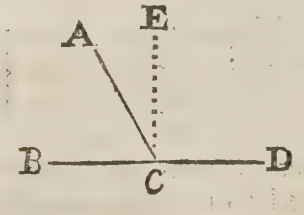
8. A *Plain Angle*, is the Inclination of two Lines, the one to the other; the one touching the other in the same Plain, yet not lying in the same streight Line.

9. And if the Lines which contain the Angle be Right-lines, it is called a *Right-lined Angle.*



10. When a Right Line CG, ſtanding upon a Right Line AB, makes the Angles on either ſide thereof CGA, CGB, equal one to the other, then both thoſe Angles are *Right Angles*; and the Right Line CG, which ſtandeth on the other, is termed a *Perpendicular* to that (AB) whereon it ſtandeth.

Note; *When ſeveral Angles meet at the ſame Point as at (G) each particular Angle is deſcribed by three Letters, whereof the middle Letter ſheweth the Angular Point, and the other two Letters the Lines that make that Angle; as the Angle which the Right Lines CG, AG make at G, is called CGA, or AGC.*



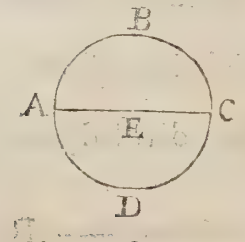
11. An *Obtuſe Angle*, is that which is greater than a Right Angle; as ACD.

12. An *Acute Angle*, is that which is leſs than a Right Angle; as ACB.

13. A *Limit*, or *Term*, is the End of any Thing.

14. A *Figure*, is that which is contained under one or more Terms.

15. A *Circle*, is a plain Figure, contained under one Line, which is called a *Circumference*; unto which all Lines drawn from one Point within the Figure, and falling upon the Circumference thereof, are equal the one to other.



16. And that Point is called the *Center* of the Circle.

17. A *Diameter* of a Circle, is a Right-Line drawn through the Center thereof, and ending at the Circumference on either ſide, dividing the Circle into two equal Parts.

18. A *Semi-circle*, is a Figure which is contained under the Diameter, and under that part of the Circumference which is cut off by the Diameter.

In the Circle EABCD, E is the *Center*, AC the *Diameter*, ABC the *Semi-circle.*

N. B.

N. B. *The equal Diviſions of the Circumference of every Circle into* 360 *Parts, are called* Degrees; *the equal Diviſion of each of theſe again into* 60 *Parts more, are called* Minutes; *and the equal Diviſion of each of theſe into* 60 *Parts more, are called* Seconds, *&c. So that the Arch of the Semi-circle contains* 180 Degrees, *and the Quadrant is equal to* 90 Degrees.

19. *Right Lined Figures* are ſuch as are contained under Right-Lines.

20. *Three Sided* or *Trilateral Figures*, are ſuch as are contained under three Right Lines.

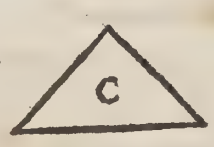
21. *Four Sided* or *Quadrilateral Figures*, are ſuch as are contained under four Right Lines.

22. *Many Sided Figures*, or *Polygons*, are ſuch as are contained under more Right Lines than four.

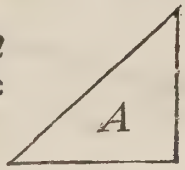
23. Of *Trilateral Figures*, that is, an *Equilateral Triangle*, which hath three equal Sides; as the Triangle A.

24. *Iſoſceles*, is a Triangle which hath only two Sides equal; as the Triangle B.

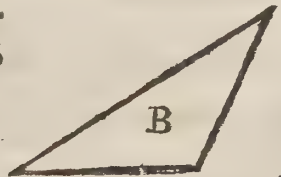
25. *Scalenum*, is a *Triangle* whoſe three Sides are all unequal; as C.

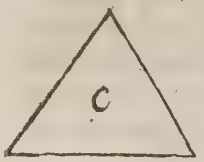


26. Of theſe Trilateral Figures, *A Right-angled Triangle* is that which has one Right Angle; as the Triangle A.



27. An *Amblygonium*, or *Obtuſe-angled Triangle*, is that which has one Angle Obtuſe; as B.

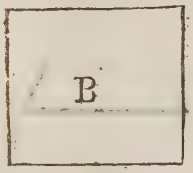




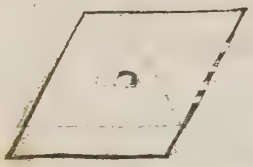
28. An *Oxygonium*, or *Acute-angled Triangle*, is that which hath three Acute Angles; as C.

An *Equiangular*, or *Equal-Angled Figure*, is that whereof all the Angles are equal: Two Figures are *Equiangular*, if the ſeveral Angles of the one Figure be equal to the ſeveral Angles of the other. The ſame is to be underſtood of *Equilateral Figures*.

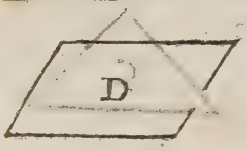
29. Of *Quadrilateral*, or *Four-Sided Figures*, a *Square* is that whoſe Sides are equal and Angles right; as A.



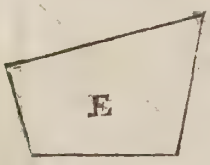
30. A Figure on one part longer, or a *Long Square*, or *Oblong*, is that which hath Right Angles, but not equal Sides; as B.



31. A *Rhombus*, or *Diamond Figure*, is that which hath four equal Sides, but is not Right Angled; as C.



32. *Rhomboides*, or *Diamond-like Figure*, is that whoſe oppoſite Sides and oppoſite Angles are equal; but has neither Equal nor Right Angles; as D.



33. All other Quadrilateral Figures beſides theſe are called *Trapezia*, or *Tables*; as E.

34. *Parallel*, or *Equi-diſtant Right Lines*, are ſuch, which being in the ſame Superficies, if infinitely produced, would never meet; as A and B.

35. A

35. A *Parallelogram*, is a *Quadrilateral Figure*, whoſe oppoſite Sides are parallel or equi-diſtant; as the Figures A. B. C. D.

A *Problem*, is when ſomething is propoſed to be done or effected.

A *Theorem*, is when ſomething is propoſed to be demonſtrated.

A *Corollary*, is a Conſectary, or ſome conſequent Truth gain'd from a preceeding Demonſtration.

A *Lemma*, is the Demonſtration of ſomething Premiſed, whereby the Proof of the thing in Hand becomes the ſhorter.

Poſtulates, or Petitions.

1. From any Point to any Point, to draw a Right Line.
2. To produce a Right Line finite, ſtreight forth continually.
3. Upon any Center, and at any Diſtance, to deſcribe a Circle.

The Explications of the Signs made uſe of.

When this Sign + is placed between two Quantities, it ſignifies that the latter is to be added to the former.

When this Sign −− is placed between two Quantities, it ſignifies that the latter is to be taken from the former.

When this Sign ×, it ſignifies the two Quantities are to be multiplied together.

When this Sign = is placed between two Quantities it ſignifies that they are equal.

Four Quantities thus expreſs'd a : b : : c : d, are thus to be read, as a is to b, ſo is c to d.

AXIOMS.

1. Things equal to the ſame thing, are alſo equal one to the other.

As A=B=C. Therefore A=C. Or therefore all, A,B,C, are equal one to the other.

Note, *When ſeveral Quantities are joined one to the other continually with this Mark=, the firſt Quantity is by vertue of this Axiom, equal to the laſt, and every one to every one. In which Caſe we often abſtain from citing the Axiom, for brevity ſake; although the force of the Conſequence depends thereon.*

2. If

2. If to equal things you add the ſame, or equal things, the wholes ſhall be equal. *And ſince Multiplication is nothing but an Addition of the Thing, a certain number of Times to its ſelf, it is equally true and equally intelligible, that, if equal things are multiplied by the ſame, or equal things, the Products or Reſults will be equal.*

3. If from equal things you take away the ſame, or equal things, the things remaining will be equal. *And ſince Diviſion is nothing elſe but the taking away the Diviſor, a certain number of times from the Dividend, it is equally true, and equally intelligible, that if equal things are divided by the ſame or equal things, the Quotients or Reſults will be equal.*

4. If to *unequal* things you add *equal* things, the Wholes will be *unequal.*

5. If from *unequal* things, you take away *equal* things, the Remainders will be *unequal.*

6. Things which are *double* to the ſame thing, or to equal things, are *equal* one to the other. Underſtand the ſame of *Triple, Quadruple,* &c.

7. Things which are *half* of one and the ſame thing, or of things equal, are *equal* the one to the other. Conceive the ſame of *Subtriple, Subquadruple,* &c.

8. Things which agree together, are equal one to the other.

The Converſe of this Axiom is true in Right Lines and Angles, but not in Figures, unleſs they be alike.

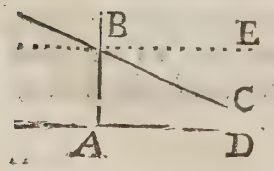
Moreover, Magnitudes are ſaid to agree, when the parts of the one being apply'd to the parts of the other, they fill up an equal, or the ſame place.

9. Every *whole* is *greater* than its *parts.*

10. Two Right Lines cannot have one and the ſame Segment, (or part) common to them both.

11. Two Lines meeting in the ſame Point, if they be both produced, they ſhall neceſſarily cut one the other in that Point.

12. All *Right-Angles* are equal one to the other.



13. If a Line BA, falling on two Right Lines AD, CB, make the internal Angles on the ſame Side BAD, ABC, leſs than two Right Angles; thoſe two Right Lines produced ſhall meet on that Side where the Angles are leſs than two Right Angles.

14. Two

14. Two Right Lines do not contain a Space.

15. If to *equal things*, you add things *unequal*, the Exceſs of the Wholes ſhall be *equal* to the Exceſs of the *Additions*.

16. If to *unequal things equal* be added, the Exceſs of the Wholes ſhall be *equal* to the Exceſs of thoſe which were at firſt.

17. If from *equal things*, *unequal things* be taken away, the Exceſs of the *Remainders* ſhall be equal to the Exceſs of the things taken away.

18. If from things *unequal*, things *equal* be taken away, the Exceſs of the *Remainders* ſhall be equal to the Exceſs of the Wholes.

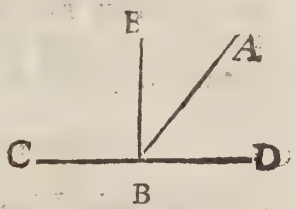
19. Every Whole is equal to all its parts taken together.

20. If one Whole be *double* to another, and that which is taken away from the *Firſt*, be *double* to that which is taken away from the *Second*, the Remainder of the *Firſt* ſhall be *double* to the Remainder of the *Second*.

Propoſition I. Theorem I.

When a Right Line AB, ſtanding upon a Right Line CD, maketh Angles ABC, ABD, it maketh either Right Angles, or two Angles equal to two Right.

Dem. If the Angles ABC, ABD, be equal, then they make two Right Angles *(by Def. the 10th)* If unequal, ſuppoſe BE, a Perpendicular to the Line CD, at the Point B.



Becauſe the Angle ABC=to a Right Angle+ABE *(by Ax. the 19th)* and the Angle ABD=to a Right Angle—ABE (*by Ax. the 3d.*) therefore ſhall the Angles ABC+ABD=to two Right Angles+ABE—ABE (*by Ax. the 2d.*)=to two Right Angles, which was to be demonſtrated.

Corollaries.

1. Hence if one Angle ABD, be Right, the other Angle ABC is alſo Right; if one Angle be Acute the other is Obtuſe, and ſo on the contrary.

2. If more Right Lines than one ſtand upon the ſame Right Line at the ſame Point, all the Angles ſhall be equal to two Right Angles.

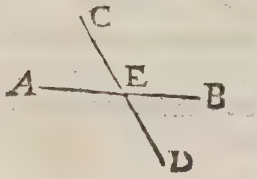
3. Two

3. Two Right Lines cutting each other, make Angles equal to four Right Angles.

4. All the Angles made about one Point, make four Right Angles, as appears by *Cor.* 2*d.*

Propoſition II. Theorem II.

If two Right Lines AB, CD, cut thro' one another, then are the two Angles which are oppoſite, *viz.* CEB, AED, equal one to the other.



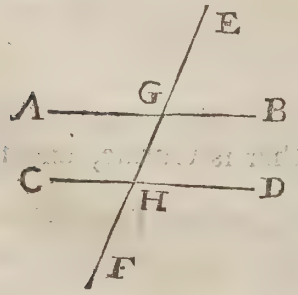
Dem. For the Angles AEC+CEB = to two Right Angles *(by Theor. the 1ſt.)* =AEC+AED, taking away from each the Common Angle AEC, and there will remain the Angle CEB=AED *(by Ax. the 3d)* Which was to be demonſtrated. After the ſame manner, it may be proved that the Angles AEC and DEB are equal one to the other.

Schol. 1.

If to any Right Line AB, and in it a Point E, two Right Lines being drawn EC, ED, not taken on the ſame ſide, make the Vertical or Oppoſite Angles AEC and DEB equal; theſe Right Lines CE, DE do meet directly and make one ſtreight Line.

Propoſition III. Theorem III.

If a Right Line EF, fall upon two parallel Lines AB, CD, it will both make the alternate Angles DHG, AGH, equal to each other, and the outward Angle BGE, equal to the inward and oppoſite Angle, on the ſame Side DHG, as alſo the inward Angles on the ſame Side AGH, CHG, equal to two Right Angles.



Dem. It is evident that AGH+CHG= two Right Angles; otherwiſe AB, CD, would not be parallel *(by the 13th Ax.)* which is contrary to the Hypotheſis. But moreover, the Angles DHG+CHG=two Right Angles *(by the 1ſt Theorem.)* Therefore, is the Angle DHG=Angle AGH *(by the 1ſt and 2d Ax.)* =Angle BGE *(by the 2d Theorem.)* W.W.D.

1. Hence

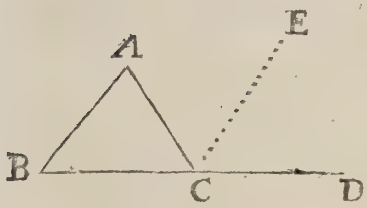
1. Hence, if a Right Line EF falling upon two Right Lines AB,CD, makes the alternate Angles DHG,AGH, equal the one to the other, then are the Right Lines parallel, (*which is the 27th of the 1ſt.*)

2. If a Right Line EF falling upon two Right Lines AB, CD, makes the outward Angle AGE of the one Line equal to CHG, the inward and oppoſite Angle of the other on the ſame Side, or makes the inward Angles on the ſame Side AGH, CHG, equal to two Right Angles, then are the Right Lines AB, CD parallel, (*which is the 28th of the 1ſt.*)

3. Hence it follows, that every *Parallelogram* having one Angle Right, the reſt are alſo Right.

4. Hence Right Lines parallel to one and the ſame Right Line, are alſo parallel the one to the other, (*which is the 30th of the 1ſt.*)

Propoſition IV. Theorem IV.



Of any Triangle ABC, one Side BC being produced, the outward Angle ACD ſhall be equal to the two inward and oppoſite Angles A and B; and the three inward Angles of the Triangle A, B, and ACB, ſhall be equal to two Right Angles.

Dem. Let CE be parallel to BA, Then is the Angle A=ACE (*by the 3d Prop.*) and the Angle B=ECD (*by the ſame.*) Therefore Angle A + Angle B = ACE + ECD (*by the 2d Ax.*) =ACD (*by the 13th Ax.*) W. W. D.

Again, ACD + ACB = two Right Angles (*by the 1ſt Prop.*) Therefore, Angle A + Angle B + Angle ACB=two Right Angles (*by the 1ſt Ax.*) W. W. D.

Corol.

1. The three Angles of any Triangle taken together, are equal to the three Angles of any other Triangle taken together; whence it follows,

2. That if in one Triangle, two Angles (taken ſeverally or together) be equal to two Angles of another Triangle (taken ſeverally or together) then is the remaining Angle of the one equal to the remaining Angle of the other. In like manner, if two Triangles have one Angle of the one equal to one Angle of the other, then is the Sum of the remaining Angles of the one Triangle equal to the Sum of the remaining Angles of the other Triangle.

3. If one Angle in a Triangle be Right, the other two are equal to a Right Angle; likewiſe, that Angle in a Triangle which is equal to the other two, is itſelf a Right Angle.

4. When in an Iſoſceles Triangle, the Angle made by the equal Sides is Right, the other two upon the Baſe are each of them half a Right Angle.

5. An Angle of an Equilateral Triangle makes two thirds of a Right Angle; for one third of two Right Angles is equal to two thirds of one.

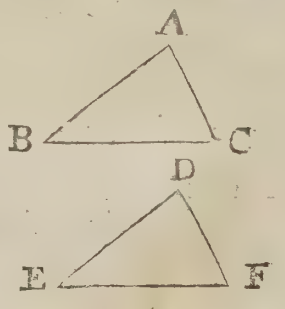
6. The Angles of a Right-lined Figure do together make twice as many Right Angles, bating four, as there are Sides of the Figure, as will be manifeſt, if, from any Point within, Lines be drawn to every Angle.

7. All the outward Angles of any Right-lined Figure, taken together, make up four Right Angles.

8. All Right-lined Figures, of whatſoever Species, have the Sum of their outward Angles equal.

Propoſition V. Theorem V.

If two Triangles BAC, EDF, have two Sides of the one, BA, AC, equal to two Sides of the other, ED, DF, each to its correſpondent Side (that is BA=ED, and AC=DF) and have the Angle A equal to the Angle D, contained under the equal Right Lines, they ſhall have the Baſe BC equal to Baſe EF, and the Triangle BAC equal to the Triangle EDF; and the remaining Angles, B, C, ſhall be equal to the remaining Angles E, F, each to each; under which the equal Sides are ſubtended.



Demon.

Demonſtration. If the Point D be applied to the Point A, and the Right Line DE placed upon the Right Line AB, the Point E ſhall fall upon B; becauſe DE=AB (*by Hyp*) alſo the Right Line DF ſhall fall upon AC, becauſe the Angle A=D (*by Hyp.*) Moreover, the Point F ſhall fall upon the Point C; becauſe AC=DF (*by Hyp.*) therefore the Right Lines EF, BC, ſhall agree (*by the* 14*th Ax.*) becauſe they have the ſame Terms, and conſequently are equal, (*by the* 8*th Ax.*) W. W. D. Whence,

1. If two Triangles, ABC, DEF, have two Sides, AB, AC, equal to two Sides, DE, DF, each to each, and the Baſe BC equal to the Baſe EF, then the Angles contained under the equal Right Lines ſhall be equal.

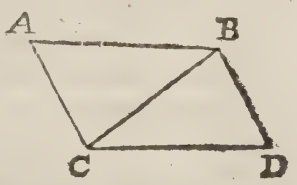
2. Triangles mutually Equilateral are alſo mutually Equiangular.

3. Triangles mutually Equilateral are equal one to the other: Wherefore,

4. The Diagonal of every *Parallelogram* divides the *Parallelogram* into two Triangles equal to each other; and therefore each of the Triangles is half the *Parallelogram*. Wherefore,

5. A *Parallelogram* of the ſame Baſe and Height with a Triangle, ſhall be double the Triangle.

6. Hence, from the 3*d Prop.* it follows, that if two equal and parallel Lines AB, CD, be joined together with two other Right Lines AC, BD, then are thoſe Right Lines alſo equal and parallel (*which is the* 33*d of the* 1*ſt.*)



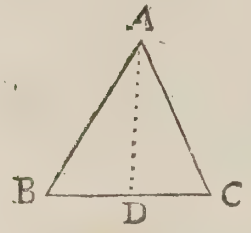
7. Alſo, Hence and from the 3*d Prop.* it follows, that in *Parallelograms*, as ABDC, the oppoſite Sides AB, CD, and AC, BD, are equal to each other, and the oppoſite Angles A, D, and ABD, ACD are alſo equal; and the Diameter BC biſects the ſame, (*which is the* 34*th of the* 1*ſt.*)

8. Hence we are taught how from a Point given C, to draw a Right Line CD, parallel to a Right Line given AB.

Conſt. Take in the Line AB any Point E, from the Centers E and C, at any Diſtance draw two equal Circles EF, CD, from the Center F, by the Space of EC, draw a Circle

FD, which ſhall cut the former Circle CD, in the Point D, then ſhall the Line drawn CD be parallel to AB; For CEFD is a *Parallelogram:* Therefore, *&c.*

Propoſition VI. Theorem VI.



If two Sides, AB and AC, of a Triangle BAC, be equal to each other, then ſhall the Angles ACB and ABC, oppoſite to thoſe equal Sides, be alſo equal.

For let D be the middle Point of the Line BC, and draw the Line AD, then are the Triangles ABD and ACD mutually Equilateral, and conſequently mutually Equiangular; whence the Angle B will be equal to the Angle C.

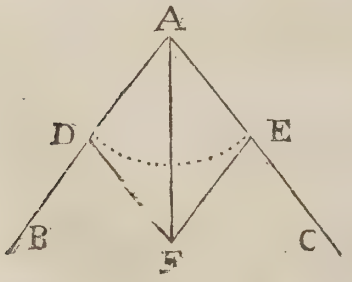
Corol.

1. If in a Triangle BAC, there be two Angles equal to each other, then ſhall the Sides ſubtending thoſe equal Angles be alſo equal.

2. If the Baſe of an Equilateral Triangle be biſected by a Line, that Line ſhall alſo biſect the Angle oppoſite to the Baſe, and ſhall be itſelf a Perpendicular to the Baſe.

3. Alſo, Hence it follows, that every Equilateral Triangle is alſo Equiangular; and every Equiangular Triangle is alſo Equilateral.

4. Hence we are taught how to biſect or divide into two equal Parts a given Right-lined Angle BAC.



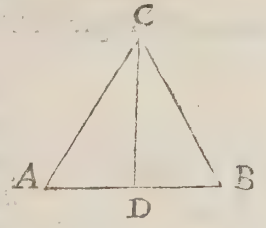
Conſtr. On A, as a Center, with any Diſtance deſcribe a Circle, cutting AB and AC, in D and E, (*by the 3d Poſt.*) Again; with the ſame, or any other Diſtance, ſetting one foot of the Compaſſes in the Points D and E, deſcribe two ſmall Arches, cutting each other in the Point F, and draw the Line AF, which will biſect the given Angle DAE.

5. For

For the Triangles, ADF and AEF, are mutually Equilateral by Conſtruction, and conſequently the Angle BAF=CAF

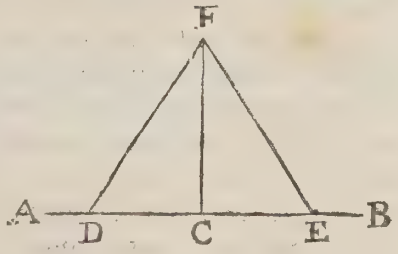
5. Hence we are taught how to biſect a given Right Line.

On the Centers A and B, at the Diſtance of AC or BC, deſcribe two Arches cutting each other in the Point C, and draw the Lines AC and BC; then biſect the Angle C (*by the former Cor.*) with the Line CD, and the Thing is done: For AC=BC (*by Conſtr.*) the Side CD common, and the Angle ACD = BCD (*by Conſtr.*) therefore AD=DB.



6. Hence we are taught how from a given Point C, in a Right Line given AB, to erect a Perpendicular, or Line CF, at Right Angles.

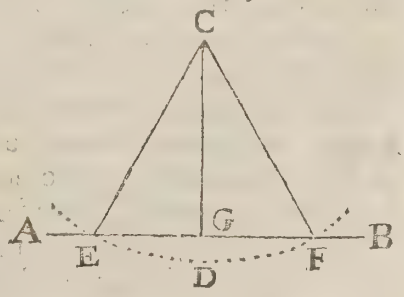
Conſtr. Take CD equal to CE, from the Centers D and E, at the Diſtance of DF or EF, deſcribe two Arches cutting each other in the Point F, and draw the Line FC, and it will be the Perpendicular required.



Dem. Having drawn the Lines DF and EF, it is evident, the Triangles DCF and ECF are mutually Equilateral; conſequently the Angles DCF and ECF will be equal: Wherefore CF is Perpendicular to AB, (*by the 10th Dif.*)

7. Hence again, we are taught, how, upon an Infinite Right Line given, AB, from a Point given C, that is not in it, to let fall a Perpendicular.

Conſtr. From the Center C, deſcribe a Circle, cutting the Right Line given AB, in the Points E and F, then biſect EF in G, and draw the Line CG, which will be the Perpendicular required.

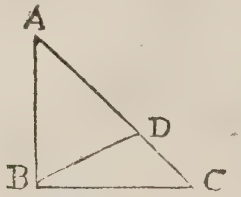


Dem. Let the Lines CE and CF be drawn, then are the Triangles EGC and FGC, mutually Equilateral; therefore the Angles EGC and FGC are equal, and conſequently Right; wherefore GC is a Perpendicular to AB.

Propo-

Propoſition VII. Theorem VII.

The greateſt Side AC, of every Triangle ABC, ſubtends the greateſt Angle ABC



Dem. Let AD be equal to AB, and join the Points B, D, by the Line BD, the Angles ADB and ABD are equal, (*by Theor. 6th.*) but the Angle ADB is greater than the Angle ACB, (*by the 4th Theo.*) therefore the Angle ABD is greater than the Angle ACB, and conſequently the whole Angle ABC is greater than ACB.

1. Hence in every Triangle ABC, under the greateſt Angle B, is ſubtended the greateſt Side CA.

2. Hence likewiſe, of every Triangle, BDC, two Sides BD, DC, any way taken, are greater than the Side BC that remains.

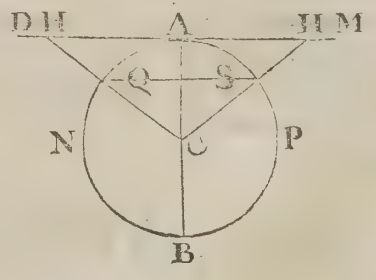
For, produce CD till DA=DB, and draw BA, then will the Angle DAB be equal to the Angle DBA, (*by the 6th Theor.*) but the Angle ABC is greater than ABD, (*by the 9th Ax.*) therefore the Angle ABC is greater than the Angle BAC; therefore DC+DB (=AC) is greater than BC.

Propoſition VIII. Theorem VIII.

A Line DM, drawn from the extreme Point A, of the Diameter AB, of a Circle CANBP, perpendicular to the ſaid Diameter, ſhall fall without the Circle.

Conſtr. From the Center C to any Point H, in the Right Line DM, draw the Right Line CH.

The Side CH ſubtending the Right Angle, is greater than the Side CA, which is oppoſite to the acute Angle (*by Theor. the 7th.*) Therefore, whereas CA=CQ reaches to the Circumference, CH ſhall reach farther, and ſo the Point H, and for the ſame Reaſon any other Point of the Line DM, is placed without the Circle.



Corol.

Corol.

1. Hence a Right Line drawn from the Extremity of the Diameter of a Circle, and at Right Angles to it, is a Tangent to the ſaid Circle

2. If any Right Line touch a Circle, and from the Center to the Point of Contact a Right Line be drawn, that Line ſhall be perpendicular to the Tangent.

3. Hence, and (*from Theor. the 6th.*) it follows, that if from the Point Q a Right Line be drawn parallel to the Tangent DM, it ſhall be biſected by the Diameter BA.

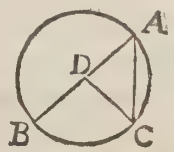
4. Wherefore, the Segments of the Circumference out of the Subtence and Diameter are equal.

5. Hence all Lines drawn within the Circle, parallel to the Tangent, are biſected by the Diameter paſſing through the Point of Contact

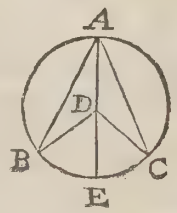
6. Wherefore, if a Line be drawn any wiſe within the Circle, and through the middle of it, and at Right Angles to it another Line be drawn, that Line ſhall paſs through the Center.

Propoſition IX. Theorem IX.

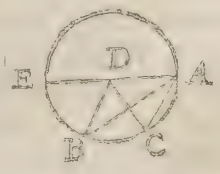
In a Circle DABC, the Angle BDC at the Center, is double the Angle BAC at the Circumference; when the ſame Arch of the Circle, BC, is the Baſe of the Angles.



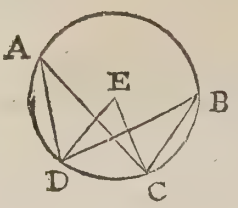
If one of the Lines paſs through the Center, as in the firſt Caſe, then the Angle BDC=DAC +DCA, (*by the 4th Theor.*) but the Angle DAC =DCA, (*by Theor. the 6th.*) therefore the Angle BDC = 2 DAC.



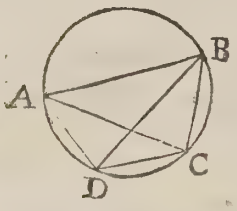
If neither paſs through the Center, as in the 2*d* and 3*d* Caſes, draw the Diameter ADE, then the outward Angle BDE = DAB + DBA (*by the 4th Theor.*) = 2 DAB (alſo the Angle EDC = 2 DAC, by the ſame; therefore, in the 2*d* Caſe, the whole Angle BDC = 2 BAC, (*by the 2d Ax.*) and in the 3*d* Caſe, the remaining Angle BDC = 2 BAC (*by the 2oth Axiom.*)



1. Hence

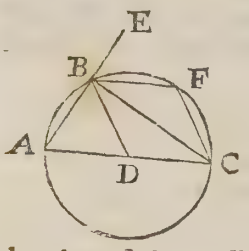


1. Hence and *(from the 7th Ax.)* it follows, that in a Circle EADCB, the Angles DAC and DBC, which are in the ſame Segment, are equal; or which ſtand upon equal Parts of the Circumference, in equal Circles, are equal.



2. That the Angles ADC, ABC, of a Quadrilateral Figure ABCD, inſcribed in a Circle, which are oppoſite one to the other, are equal to two Right Angles.

For, the Angle ABC + (BCA =) BDA + (BAC =) BDC equal two Right Angles *(by Theor. the 6th)* = AEC + ADC.



3. In a Circle, the Angle ABC, which is in the Semicircle, is a Right Angle; but the Angle which is in the greater Segment BAC, is leſs than a Right Angle; and the Angle which is in the leſſer Segment BFC, is greater than a Right Angle; moreover, the Angle of the greater Segment is greater than a Right Angle, and the Angle of the leſſer Segment is leſs than a Right Angle.

Draw from the Center D, DB; becauſe AD, DB, DC, are equal, *(by the 15th Def.)* therefore is the Angle A = Angle DBA, *(by Theor. the 6th.)* and the Angle DCB = Angle DBC, by the ſame. Therefore the Angle ABC = Angle A + Angle ACB *(by the 2d Ax.)* = a Right Angle *(by the 3d Cor. of Theor. the 4th.)* = EBC, *(by the 4th Prop.)*

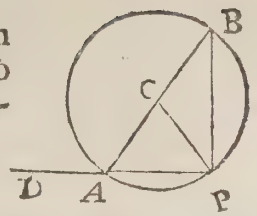
Again, becauſe Angle ABC is Right, the Angle BAC is Acute, and becauſe Angle BAC + BFC = two Right, therefore the Angle BFC is an obtuſe Angle.

Laſtly, The Angle contained under the Right Line CB, and the Arch BAC, is greater than the Right Angle ABC; but the Angle made by the Right Line CB, and the Periphery of the leſſer Segment BFC, is leſs than the Right Angle EBC, *(by the 9th Ax.)*

4. Whence, in a Right-angled Triangle, if the Hypothenuſe AC, be biſected in D, a Circle drawn from the Center D, through the Point A, ſhall alſo paſs through the Point B, which is of Uſe in the Conſtruction of Quadratick Equations.

Hence

5. Hence we are taught, how from a given Point P, at the End of a Right Line DP, to erect a Perpendicular PB, or Line at Right Angles.



Conſtr. Placing one Foot of the Compaſſes in P, the Point given, with any Diſtance CP, placing the other Foot in C, deſcribe the Circle BPA (*by the 3d Poſt.*) and through A and C draw the Right Line AC, and produce it till it cut the Circumference in B, and draw BP, and the Thing is done.

For becauſe ACB is a Diameter, the Angle APB is a Right Angle, and BP a Perpendicular, (*by the 10th Def.*)

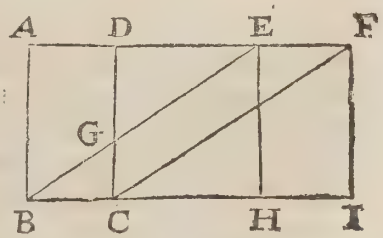
6. Hence we are taught, how from a Point given B, over the end of a Right Line DP, to let fall a Perpendicular BP.

For from the given Point B, to any Point A in the Right Line DP, draw the Line BA, and biſect AB in the Point C, on C as a Center, with the Diſtance CA or CB, deſcribe the Circle CBPA, cutting the Line DP in P, and draw BP, and the Thing is done.

For, becauſe C is the Center of the Circle CAPB, and AB a Diameter, therefore is the Angle APB a Right Angle, and BP a Perpendicular, (*by the 10th Def.*)

Propoſition X. Theorem X.

Parallelograms BCDA, BCFE, which ſtand upon the ſame Baſe BC, and between the ſame Parallels AF, BC, are equal one to the other.



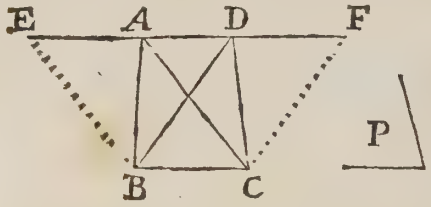
For AD=BC=EF (*by the 7th Cor. of the 5th.*) add DE common to both; then is AE=DF (*by the 2d Ax.*) alſo AB=DC (*by the 7th Cor. of the 5th.*) and the Angle A=Angle D (*by the 3d.*) therefore is the Triangle BAE=CDF; take away from each the Triangle GDE common to both Triangles, and there will remain the *Trapezium* BADG=CGEF (*by the 3d Axiom.*) Add to each *Trapezium*

 the

the Triangle BGC common to both, then is the *Parallelogram* BADC=BEFC (*by the 2d Axiom.*)

1. Hence, and from the *1ſt Ax.* it follows, that *Parallelograms*, as ABCD and EFIH, ſtanding upon equal Baſes BC and IH, and betwixt the ſame Parallels AF, BI, are equal to each other.

2. Hence we are taught, how to find the Area of any *Parallelogram*; for ſince the Area of a Right-angled *Parallelogram* is found by the Drawing or Multiplication of any two contiguous Sides, one into another, the Area of any other *Parallelogram* will be found by the Multiplication of the Baſe into the Perpendicular Height.



3. Hence, and from the *7th Ax.* it follows, that Triangles BCA, BCD, ſtanding upon the ſame Baſe BC, and between the ſame Parallels BC, EF, are equal one to the other; for they are the Halves of the equal *Parallelograms* BEAC and BDFC.

4. Hence, and from the *1ſt Ax.* it follows, that Triangles, as EBA, DCF, ſtanding upon equal Baſes EA, DF, and between the ſame Parallels EF, BC, are equal one to the other.

5. From the *3d Cor.* and *6th Ax.* it follows, that if a *Parallelogram* EBCA (*ſee the Figure to the 3d Cor.*) have the ſame Baſe BC, with the Triangle BDC, and between the ſame Parallels BC, EF, then is the *Parallelogram* EBCA, double to the Triangle BDC.

For the Triangle BDC = BCA = half the *Parallelogram* EBCA.

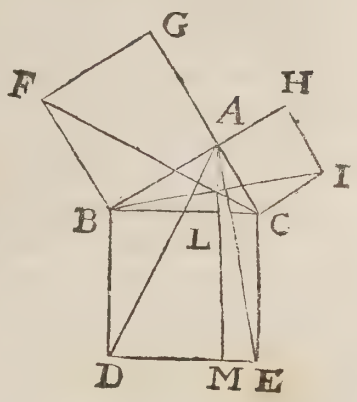
6. Hence we are taught, how to find the Area of any Triangle; for ſince every Triangle is half the *Parallelogram*, having the ſame Baſe and Altitude, the Area thereof will be found by the Multiplication of the Baſe into half the Perpendicular Height, or of half the Baſe into the whole Height; or of half the Product of the Baſe into the Height.

7. Hence we are taught, how to find the Area of any Right-lined Figure; for if it be reduced into Triangles, by drawing Diagonals from Angle to Angle, the Sum of the Area's of all the Triangles, ſhall be the Area of the Figure (*by the 19th Axiom.*)

Propo-

Propoſition XI. Theorem XI.

In Right-angled Triangles, as BAC, the Square BE, which is made of the Side BC, that ſubtends the Right Angle BAC, is equal to both the Squares BG,CH, which are made of the Sides AB, AC, containing the Right Angle.



Conſtr. Join AE and AD (*by the 1ſt Poſt.*) and draw AM parallel to CE, (*by the 8th Cor. of the 5th.*)

Dem. Becauſe the Angle DBC=FBA (*by the 12th Ax.*) and the Angle ABC common to both, then is the Angle ABD=FBC (*by the 2d Ax.*) moreover, AB = BF (*by the 29th Def.*) and BD = BC (*by the ſame*) therefore is the Triangle ABD=Triangle FBC (*by the 5th Prop.*) but the *Parallelogram* BM = twice the Triangle ABD (*by 5th Cor. of the 10th.*) and the Square BG=twice the Triangle FBC (*by the ſame.*) For, GAC is one Right Line, (*by Hyp.*) therefore *Parallelogram* BM = Square BG (*by the 6th Axiom.*) By the ſame way of Argument is the *Parallelogram* CM=Square CH; therefore is the Square BE = *Parallelogram* BM + *Parallelogram* CM (*by the 19th Ax.*) = Square BG + CH (*by the 1ſt Axiom.*)

Corol.

1. Hence we are taught how to find the Difference of two Squares, or the Sum of any given Number of Squares.

2. Hence, and by the Help of the Square Root, we are taught how from any two Sides being given, the Third may eaſily be found.

For, Suppoſing the two Sides containing the Right Angle were given, to find the Side ſubtending the Right: In this Caſe, to the Square of one of the Sides, add the Square of the other, the Square Root of their Sum will be the Side required.

Again, Suppoſe the longeſt Side, or Side ſubtending the Right, were given, and one of the containing Sides, to find the other:

In this Caſe, from the Square of the longeſt Side, ſubtract the Square of the other Side; the Square Root out of the Remainder will be the remaining Side; as is abundantly manifeſt from what went before.

Def. Four Numbers, Lines or Quantities, are ſaid to be Proportional, when the Firſt as oft contains, or is contained in the Second, as the Third contains, or is contained in the Fourth.

Corol. Wherefore, if of four Quantities, the Quotient ariſing from the Diviſion of any two Quantities, one by the other, which ſhews how oft one is contain'd in the other, be equal to the Quotient ariſing from the Diviſion of any two other Quantities, one by the other, (provided the Quantities are all of the ſame Kind) then thoſe four Quantities are Proportional.

Propoſition XII. Theorem XII.

If four Numbers, Lines, or Quantities, be proportional, the Product or Rectangle of the Extreams ſhall be equal to the Product or Rectangle of the Means; *that is*, If $a : b :: c : d$, I ſay, $a \times d = b \times c$.

For, becauſe the Quantities are Proportional, $\frac{a}{b} = \frac{c}{d}$ (*by the preced. Def.*) multiplying therefore each Quantity by $\frac{b\,d}{}$ and we ſhall have $\frac{bda}{b} = \frac{bdc}{d}$ (*by Ax. the 2d.*) that is, $a \times d = b \times c$, which was to be demonſtrated.

Again, If of four Quantities a, b, c, d, the Product or Rectangle $a \times d$ of the Extreams be equal to the Product or Rectangle $b \times c$ of the Means, then are the four Quantities proportional.

For, becauſe (*by Hyp.*) $a \times d = b \times c$, if each Quantity be divided by $b \times d$, we ſhall have $\frac{a \times d}{b \times d} = \frac{b \times c}{b \times d}$ (*by Axiom the 3d*) that is, $\frac{a}{b} = \frac{c}{d}$, conſequently $a : b :: c : d$. (*by the Cor. of the preced. Def.*)

1. Whence it follows, that Quantities that are proportional to the ſame Quantities are the ſame betwixt themſelves.

Hence

Hence we are taught the various Ways of Reaſoning with Proportional Quantities; that is,

If the Quantities a, b, c, d, are Proportional, they ſhall alſo be Proportional, when

Alternated, *that is*, —— $a : c :: b : d$
Inverted —— —— —— —— $b : a :: d : c$
Compounded —— —— —— $a+b : b :: c+d : d$
Divided —— —— —— —— $a-b : b :: c-d : d$
Converted —— —— —— —— $a : a-b :: c, c-d$
Mixed —— —— —— —— —— $a+b : a-b :: c+d : c-d$

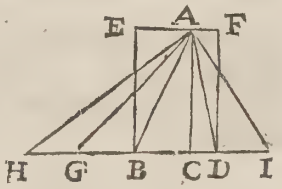
As is manifeſt, if one multiply the Extreams of any Rank together and the Means, for the Products in each will come out equal.

Hence the Reaſon of the common *Rule of Three* will evidently appear.

Let the fourth Quantity be repreſented by d, becauſe (*by Hyp.*) $a : b :: c : d$, therefore $a \times d = b \times c$, and dividing each Quantity by a, you will have $d = \frac{b \times c}{a}$ (*by the 3d Axiom*) that is, to the Product of the Second, multiplied by the Third, and divided by the Firſt.

Propoſition XIII. Theorem XIII.

Triangles, as ABC, ACD, and *Parallelograms*, as BCAE, CDFA, which have the ſame Height, are in Proportion one to the other, as their Baſes BC, CD, are.



Take as many as you pleaſe, BG, GH = BC, and alſo DI = CD, and join AG, AH, AI, (*by the 1ſt Poſt.*)

Becauſe the Triangles ACB, ABG, AGH, are equal (*by the 3d Cor. of the 10th.*) and the Triangle ACD = ADI, by the ſame; therefore the Triangle ACH, as oft contains the Triangle ACB, as the Baſe HC contains the Baſe BC; and therefore (*by the Def. of the Prop.*) the Triangle ACH : Tri. ACB :: HC : BC.

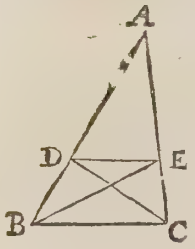
Again,

Again, the Triangle CAI, as oft contains the Triangle CAD, as the Baſe CI contains the Baſe CD; and therefore *(by the Def. of Prop.)* the Tri. ACI : Tri. : ACD : : CI : CD; now comparing theſe two Ranks of Proportionals together, it will follow, that the Tri. ABC : Tri. ACD : : BC : CD.

Again, becauſe *(by the 5th Cor. of the 10th)* the *Parallelograms* CE, CF, are double of the Triangles ABC, ACD; therefore, *Pgr.* CE : *Pgr.* CF : : BC : CD.

1. Hence, Triangles and *Parallelograms*, whoſe Baſes are equal, are in the ſame Proportion as their Altitudes.

Propoſition XIV. Theorem XIV.



If to one Side BC, of a Triangle ABC, be drawn a parallel Right Line DE, the ſame ſhall cut the Sides of the Triangle proportionally. (AD : BD : : AE : EC) and if the Sides of the Triangle be proportionally cut, (AD : BD : : AE : EC) then a Right Line DE, connecting the Sections D, E, ſhall be parallel to the remaining Side of the Triangle BC.

1. *Hyp.* Becauſe the Triangle DEB=DEC *(by the 3d of the 10th)* therefore ſhall the Triangle ADE : DBE : : ADE : ECD *(by the Def. of Prop.)* but the Triangle ADE : DBE : : AD : DB *(by the 14th Prop.)* and the Triangle ADE : DEC : : AE : EC, therefore *(by the 1ſt of the 12th)* AD : DB : : AE : EC.

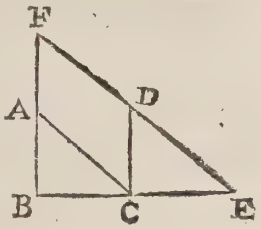
2. *Hyp.* Becauſe AD : DB : : AE : EC, therefore *(by the 13th)* Triangles ADE : DBE : : ADE : ECD; therefore is the Triangle DBE = ECD, and therefore DE and BC are parallel *(by the 3d of the 10th.)*

Scholium.

If there be drawn many Parallels to one Side of any Triangle, then all the Segments of the Sides ſhall be proportional; as may eaſily be deduced from the preceding Propoſition.

Propo-

Propoſition XV. Theorem XV.



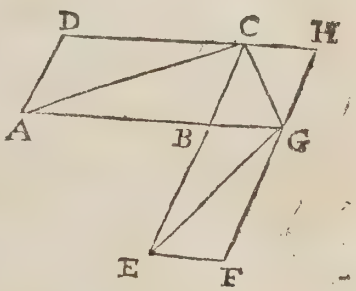
Of *Equiangular* or Similar Triangles ABC, DCE, the Sides are proportional which are about the equal Angles B,DCE, (AB : BC : : DC : CE) and the Sides AB, DC, *&c.* which are ſubtended under the equal Angles ACB and E, *&c.* are Homologous, or of like Ratio.

Set the Side BC in a direct Line to the Side CE, and produce BA and ED till they meet.

Becauſe the Angle B=ECD (*by Hyp.*) therefore BF,CD are parallel (*by the 2d of the 3d*) alſo, becauſe Angle BCA=CED, therefore are CA, EF, parallel (*by the ſame,*) therefore the Figure CAFD is a *Parallelogram:* Therefore AF=CD, AC=FD, (*by the 7th of the 5th.*) whence it is evident (*by the 14th Prop.*) AB : (AF=) CD : : BC : CE; therefore (*by Alternation*) AB : BC : : CD : CE; alſo, BC : CE : : (FD=) AC : DE; therefore (*by Alternation*) BC : AC : : CE : ED; wherefore alſo (*by Equality*) AB : AC : : CD : DE.

1. Hence, AB : DC : : BC : CE : : AC : DE.

Propoſition XVI. Theorem XVI.



Equal *Parallelograms*, as BD,BF, having one Angle ABC, equal to one EBG, have the Sides which are about the equal Angles reciprocally proportional; (AB : BG : : EB : BC) and thoſe *Parallelograms* BD, BF, which have one Angle AB, equal to one EBG, and the Sides which are about the equal Angles reciprocally proportional, are equal.

For let the Sides AB, BG, about the equal Angles make one Right Line; wherefore EB : BC ſhall do the ſame (*by the Scholium of Theor. the 2d.*) Let FG,DC, be produced till they meet.

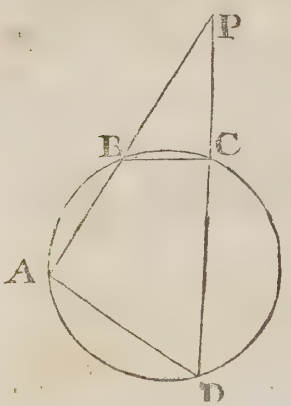
1. *Hyp.* AB : BG : : *Pgr.* BD : *Pgr.* BH (*by the 13th.*) : : *Pgr.* BF : *Pgr.* BH (*by the Cor. of the Def.*) : : BE : BC; therefore, *&c.*

2. *Hyp.*

2. *Hyp.* ℘*gr.* BD : ℘*gr.* BH :: AB : BG, (*by the* 13*th.*) BE, BC (*by Hyp.*) ℘*gr.* BF : ℘*gr.* BH ; therefore, the ℘*gr.* BD = BF (*by the Cor. of Def.* ℘*rop.*).

Corol.

1. Hence *Parallelograms* having one Angle equal, and the Sides about that Angle reciprocally proportional, are equal.



2. Hence it follows, that if from any Point P, taken without the Circle, two Right Lines, as PA and PD be drawn, cutting the Circle in the Points B and C, that the Rectangle comprehended under the whole Lines PA and PD, and the external Parts, PB and PC, will be equal between themselves.

For draw the Lines BC and AD, because the Angles ABC and ADC are equal to two Right Lines (*by the 2d Cor. of the 5th Theor.*) and the Angles ABC and PBC are equal to two Right Lines, (*by the 1st Theor.*) therefore the Angles PBC and ADC are equal (*by the 5th Axiom*) but the Angle BPC is common to both Triangles ; wherefore the Triangles PBC and PAD are similar, and it will be (*by the preceding Theor.*) PA : PD : : PC : PB.

And consequently PA : × PB = PD × PC, (*by Theor. the* 12*th.*)

SECT.

SECT. II.

OF

TRIGONOMETRY;

OR,

The Doctrine of Triangles.

THE principal Buſineſs of *Trigonometry*, is to teach us how to compute the Length of the Sides and Quantities of the Angles of Triangles from proper *Data*; and to this End it has been ſhewn in the former Section.

1. That a *Plain Triangle*, the Subject of *Plain Trigonometry*, is a Figure contained under three Right Lines lying in the ſame plain Superficies, any two Sides of which, taken together, are greater than the Third that remains.

2. That the greateſt Angle of every Triangle is oppoſite to the greateſt Side, and, *vice verſa*, that the greateſt Side ſubtends the greateſt Angle.

3. That the Sum of the Angles of every Plain Triangle is equal to a Semicircle, or 180° 00′: Wherefore,

4. If two Angles of a Plain Triangle are known, the Third is alſo known; for being added together, and their Sum taken from 180° 00′, there will remain the Third.

5. If one Angle of a Plain Triangle be obtuſe, each of the other two ſhall be acute.

6. If one Angle of a Plain Triangle be Right, the other two ſhall be equal to one Right Angle: Wherefore,

7. In a Right-angled Plain Triangle, if one of the acute Angles be given, the other is alſo known, being what the given Angle wants of a Quadrant, or 90° 00′; and this Defect is called its Complement.

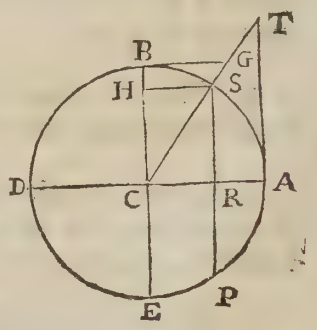
As every Triangle conſiſts of ſix Parts, *namely*, Three Sides and Three Angles, ſo from any Three of theſe being known (or given,) except the three Angles of a Plain Triangle, (for in that Caſe the Proportion of the Sides one to another can only be diſcovered;) the other Three may be directly found: And how this may be done ſhall be ſhewn in its proper Place.

And inaſmuch as the Angles of Plain Triangles, as alſo the Angles and Sides of Spherical Triangles, are meaſured and eſtimated by Arches of Circles; and as the exact Ratio of the Circumference of a Circle to its Diameter cannot be expreſs'd by our preſent way of Notation, neither can any Proportion between Arches of Circles and Right Lines be aſcertain'd, Mathematicians have been neceſſitated to ſubſtitute certain Right Lines in the room of the Arches or Angles; the Nature and Conſtruction of which comes next to be ſhewn.

Lines uſually applied to the Circle, are Chords, Sines, Tangents, and Secants.

Definitions.

1. The Chord, or Subtence of an Arch, is a Right Line drawn within the Circle, and dividing the Circumference into two Parts, and is a Chord to them both; thus SP is the Chord of the Arch SAP, as alſo of the Arch SDP.



A Sine is either Right or Verſed.

2. The Right Sine of an Arch is a Right Line drawn from one End or Termination of the Arch perpendicular to the Radius, drawn

drawn through the other Termination of the Arch; or it is half the Chord of twice that Arch: Thus SR is the Right Sine of the Arch SA, and of the Arch SD, the Sum of which Arches make a Semicircle.

3. The Verſed Sine is that Part of the Diameter which is contained between the Right Sine and the Arch; for which Reaſon RA is the Verſed Sine of the Arch SA, and RD the Verſed Sine of the Arch SD.

4. The Tangent of an Arch is a Right Line drawn perpendicularly from one End of the Diameter, and limited by a Right Line drawn from the Center through the other Termination of the Arch; thus AT is the Tangent of the Arch SA, and of the Arch SD.

5. And the Right Line drawn from the Center to the Extremity of the Tangent is called the *Secant* of the ſame Arch; wherefore CT is the Secant of the Arch SA, and of the Arch SD.

Corol.

1. From *Definition the 1ſt*, it follows, that the Chord of any Arch, and its Supplement to a Circle, is repreſented by the ſame Line: Wherefore, if the Chord of any Arch, leſs than a Semicircle, be given, the Chord of another Arch, as much exceeding a Semicircle as the former is defective, is alſo given.

2. From *Definitions the 2d, 4th and 5th*, it follows, that any Arch, and its Complement to a Semicircle, have one Right Line, Tangent and Secant common to them both: Wherefore, the Right Sine, Tangent and Secant of any Arch, leſs than a Quadrant, is the ſame with the Right Sine, Tangent and Secant of an Arch, as much exceeding a Quadrant as the former is defective.

3. Alſo, from *Definition the 3d*, it follows, that the Verſed Sine of an Arch greater than a Quadrant, is greater than the Radius; but the Verſed Sine of an Arch leſs than a Quadrant is leſs than the Radius; and that the Sum and Difference of the Radius and Co-ſine of any Arch ſhall be the Verſed Sines of that Arch, and its Supplement to a Semicircle:

$$\text{For } DC \pm CR = \left\{\begin{matrix} DR \\ RA \end{matrix}\right\} \text{ the Verſed Sine of } \left\{\begin{matrix} DS \\ AS \end{matrix}\right\}$$

4. Again, Hence (and from *Theor. the 11th)* it follows, that the Square of the Radius leſſened by the Square of the Right Sine, is equal to the Square of the Co-ſine.

For $CS^q - SR^q = CR^q$.

Alſo, by comparing the ſimilar Triangles CRS and CBG one with another, it appears,

5. That the Co-ſine of an Arch is to the Right Sine, as the Radius is to the Tangent of the ſame Arch, and the contrary; for,

CR : RS : : CA : AT.

6. That the Co-ſine of an Arch is to the Radius, as the Radius to the Secant of the ſame Arch, and the contrary; for,

Corol. RC : SC : : AC : CT.

Hence the Radius is a mean Proportional between the Co-ſine and the Secant of the ſame Arch; wherefore the Co-ſines and Secants of Arches are reciprocally proportional.

7. That the Right Sine of an Arch is to the Radius, as the Tangent of the ſame Arch is to its Secant, and the contrary; for,

RS : SC : : AT : TC.

8. That the Tangent of an Arch is to the Radius, as the Radius is to the Co-tangent of the ſame Arch, and the contrary; for,

TA : AC : : CB : BG.

Corol. Hence the Radius is a Mean Proportional between the Tangent and Co-tangent of an Arch, wherefore the Tangents of Arches and their Co-tangents are reciprocally proportional.

9. That the Tangent of an Arch is to the Secant of the ſame Arch, as the Radius is to the Secant of its Complement, and the contrary; for

AT : TC :: CB : CG.

10. By viewing the Right-angled Triangle CSR, it appears, that if the longeſt Side or Hypothenuſe CS be made the Radius of a Circle, the other Sides or Legs SR and CR will be the Sines of their oppoſite Angles: But if either of the Sides containing the Right Angle be made the Radius, as in the Triangle CAT, the other Leg will be the Tangent of its oppoſite Angle, and the Hypothenuſe the Secant of the ſame Angle.

This Property, by reaſon of its excellent and frequent Uſe, is called *The firſt Axiom of Plain Trigonometry*; and the Reader is to take Notice, that whenever we mention the *Firſt Axiom* we mean this Property.

But

But how to determine the Lengths of the ſeveral Sines, Tangents and Secants belonging to the ſeveral Arches, anſwering to every Degree and Minute of the Quadrant in Parts of the Radius firſt given, is the Conſtruction or making of the Tables of Natural Sines, Tangents and Secants, which ſhall be ſhewn both Geometrically and Arithmetically, or by Calculation.

SECT. III.

Containing the Geometrical Conſtruction of the Natural Sines, Tangents *and* Secants.

1. DEſcribe a Semicircle *c a b d*, and divide it into two equal Parts by the Radius *b c*.

2. Divide the Arch of the Quadrant *a b* into nine equal Parts, in the Points 10, 20, 30, *&c.* then will each Diviſion anſwer to 10 Degrees; and if you imagine Lines drawn from *a* to the ſeveral Points 10, 20, 30, *&c.* in the Arch *a b*, they will be the Chords of their reſpective Arches (*by the 1ſt Def. of the 2d Sect.*) and being transferred to the Chord Line *a b*, will form a Line of Chords; and if each Diviſion of the Quadrant be again ſub-divided into ten equal Parts more, and the Diſtances of theſe ſeveral Diviſions from the Point *a*, be transferred to the Line *a b*, you will by this means have a Line of Chords to every ſingle Degree.

3. For the Line of Right Sines from the Points of the ſeveral Diviſions 10, 20, 30, *&c.* of the Quadrant *b a*, draw Lines parallel to *b c*, 'till they cut the Radius *c a*, ſo ſhall *c a* be divided into a Line of Sines; and if, as we ſuppoſed before, the Arch *b a* be divided into 90 equal Parts, and from the ſeveral Points of the Arch *a b*, Lines parallel to the former be drawn, the Line *c a* will be divided into a Line of Sines anſwering to every ſingle Degree of the Quadrant (*by the 2d Def. of the 2d Sect.*) which Diviſions muſt be numbred from *c* towards *a*.

4. If the ſame Line of Right Sines be numbred from *a* towards *c*, you will have a Line of Verſed Sines (*by the 3d Def. of the 2d Sect.*) which may be continued to 180 Degrees, if the ſame Diviſions be transferred on the other Side of the Center *c*.

5. From

Fig. 1.

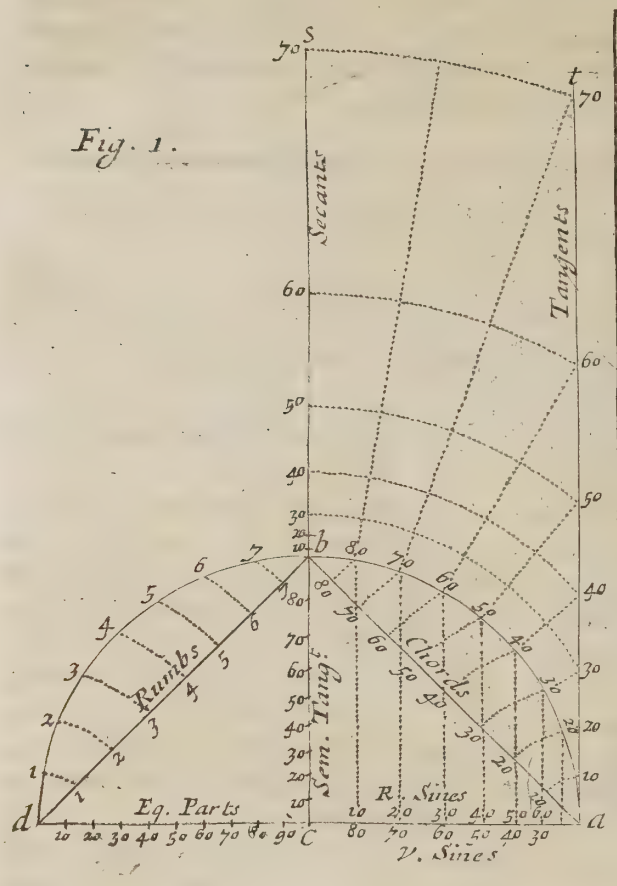
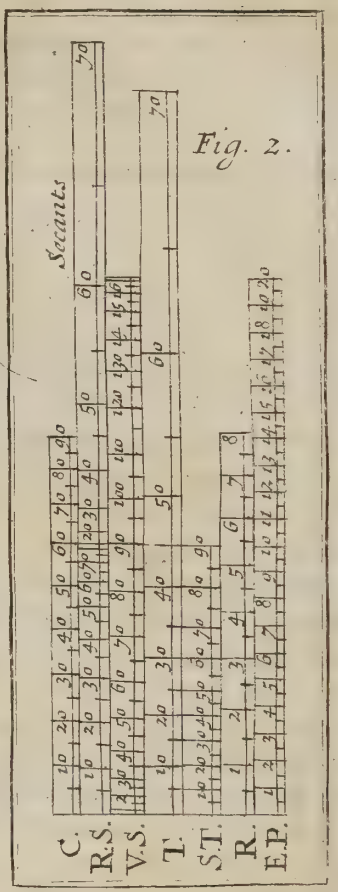


Fig. 2.



5. From *a*, the extream Point of the Diameter, raiſe the Perpendicular *a t*, and from the Center *c*, through the ſeveral Diviſions of the Quadrant *a b*, draw Lines 'till they cut the Tangent *a t*; ſo will the Line *a t* become a Line of Tangents (*by Def. the 4th of Sect. the 2d.*) which muſt be numbred from *a* towards *t*.

6. If inſtead of laying the Ruler from *c* the Center, you lay it from *d* to the ſeveral Diviſions of the Quadrant *a b*, and obſerve where it cuts the Line *c b*, and there make Marks, the Line *c b* will become a Line of Semi-tangents (*by Theor. the 9th, Def. the 4th of Sect. the 2d.*)

7. And

7. And becauſe Lines drawn from the Center through the ſeveral Diviſions of the Quadrant *a b*, to form the Tangents, are the Secants of the reſpective Arches (*by the 5th Def. of the 2d Sect.*) if their Lengths be transferred to the Line *c b*, continued to *s*, the Line *c s* will be the Line of Secants, which muſt be numbred from *c* towards *s*.

After the ſame manner may the Chords, Sines, Tangents and Secants of every Minute of the Quadrant be conſtructed, if the Circle will admit of it, or of any particular Arch.

Now if the ſeveral Diviſions of the Line *a b* (*in Fig.* 1.) be transferred to the Line *C* (*in Fig.* 2.) alſo the Diviſions of the Tangent Line *a t* (*in Fig.* 1.) to the Line T (*in Fig.* 2.) In like manner the Diviſions of the other Lines of Right and Verſed Sines and Secants *in Fig.* 1. *to Fig.* 2. you will have the Inſtrument formed, called the *Plain Scale*. And ſince we are ſpeaking of this Inſtrument, it may not be amiſs to ſhew the Conſtruction of another Line uſed in Sailing, call'd *Rhombs*, which is as follows:

Divide the Quadrant *d b* (in *Fig.* 1.) into eight equal Parts in the Points 1, 2, 3, *&c.* and ſetting one Foot of the Compaſſes in the Point *d*, transfer the ſeveral Diſtances *d* 1, *d* 2, *d* 3, *&c.* from the Arch *d b* to the Line *d b*, ſo will you have formed a Line of Rhombs, each of which will anſwer to 11° 15′, which Diviſions may be transferred to the Line R (in *Fig.* 2.)

And laſtly, if the Radius *c d* be divided into 100, *&c.* equal Parts, and the ſeveral Diviſions of this Line be transferr'd to the Line EP, (in *Fig.* 2.) the Inſtrument will be compleat.

Now if the Numbers anſwering to the ſeveral Lengths of the Sines, Tangents and Secants correſponding to the ſeveral Arches of the Quadrant in equal Parts of the Radius be ſet down in a Table, each in its proper Column, you will by this means have a triangular Cannon of Numbers, by which the ſeveral Caſes of *Trigonometry* may be eaſily done: But theſe being not accurate enough for *Arithmetical Calculations*, it is therefore abſolutely neceſſary that the Lengths of thoſe Right Lines (eſpecially ſuch as are uſed in the Calculation of Triangles) be determined to greater Exactneſs; and how this may be done will be ſhewn in the next Section.

SECT.

SECT. IV.

Containing Methods for Computing the Tables of Natural Sines, Tangents *and* Secants.

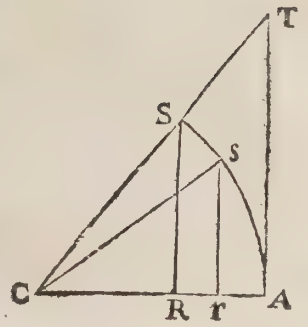
IT has been ſhewn and demonſtrated, in the 67*th*, 68*th*, and 81*ſt* Pages of my *Syſtem of the Mathematicks*, Vol. I. If the Diameter of any Circle be conſider'd as an Unit, the Circumference of that Circle will conſiſt of 3,1415.9265 + ſuch Parts; and conſequently, if the Radius of the ſame Circle be ſuppos'd to be divided into 10000.0000 equal Parts, its Periphery will contain 6.28318530 + ſuch Parts, whence the Length of any Part, or Portion of the Circumference will be readily found by the common Rule of Proportion.

For, as 360, the Length of the Circumference numbred in Degrees, is to 6.28318530 +, the Length of the ſame Circumference number'd in ſuch Parts, as the Radius is 1.00000000, or the whole Diameter 2.00000000, ſo is the Length of any Arch of the Circumference number'd in Degrees and Minutes, to the Length of the ſame Arch of the Circumference, in ſuch Parts as the Radius is 1.00000000, *&c.*

Wherefore, if the Circumference 6.28318530 + be divided by 21600, the Minutes in the Circumference of a Circle, the Quotient is 2908 : 88, will be the Length of the Arch of one Minute in the ſame Parts, or the Sine of the ſame Arch, ſeeing there is no Difference between them (in the largeſt Tables generally uſed) the Square of which being ſubtracted from the Square of the Radius, will leave the Square of the Co-ſine of one Minute, whoſe Root 9.999999.57 will be the Sine of 89° 59′, or the Sine of 90° 01′.

And that in very ſmall Arches, the Sines and Tangents of the ſame Arches are very nearly in a Ratio of Equality, may be demonſtrated after this Manner:

Let



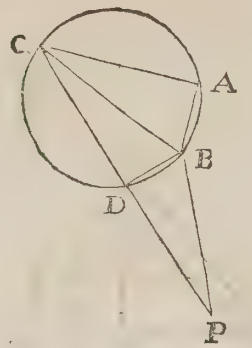
Let SR be the Sine and AT the Tangent of any Arch, ſuppoſe AS: Becauſe the Triangles CRS and CAT are ſimilar, it will be as CR, CA, RS, AT; but the nearer the Point S approaches to the Point A, or the ſmaller the Arch SA, the leſſer will be the Verſed Sine, till at laſt, when the Point S nearly coincides with the Point A, the Verſed Sine RA, will be infinitely ſmall, with reſpect to the Arch AS; CR will become nearly equal to CA, and conſequently SR, the Right Sine, will become nearly equal to AT, the Tangent.

Again, Since the Arch AS is leſs than the Tangent AT, but greater than the Right Sine SR; and inaſmuch as the Sines and Tangents of infinitely ſmall Arches are nearly equal, the Arch itſelf will be nearly equal to the Sine or Tangent, and conſequently, in intenitely ſmall Arches, the Arches will be to each other very near, as their correſpondent Sines and Tangents.

And how far this Law obtains, will be manifeſt to any one who will but give himſelf the Trouble to inſpect the *Tables of Natural Sines and Tangents*, where he will find, that the Sines and Tangents of the firſt 10 Minutes of the Quadrant, to a Radius of 1.0000000, do not only increaſe in a Ratio of Equality with their correſpondent Arches, but are expreſs'd by the ſame Numbers.

The Sine and Co-ſine of one Minute being thus obtained, the Sines of all the intermediate Minutes between 1, and 89° 59′, may be found by one general Rule; namely, As the Radius to the double Sine of 89° 59′, ſo is the Sine of 1 Minute to the Sine of 2 Minutes; and ſo is the Sine of 2 Minutes to the Sum of the Sines of 1 Minute and 3 Minutes; from whence taking away the Sine of 1 Minute, there will remain the Sine of 3 Minutes; and ſo is the Sine of 3 Minutes laſt found, to the Sum of the Sines of 2 Minutes and 4 Minutes; from whence taking the Sine of 2 Minutes already found, there will remain the Sine of 4 Minutes; and by proceeding after the ſame manner may the Sine of every Minute of the Quadrant be found.

And that the Reaſon of this Rule may the more evidently appear,



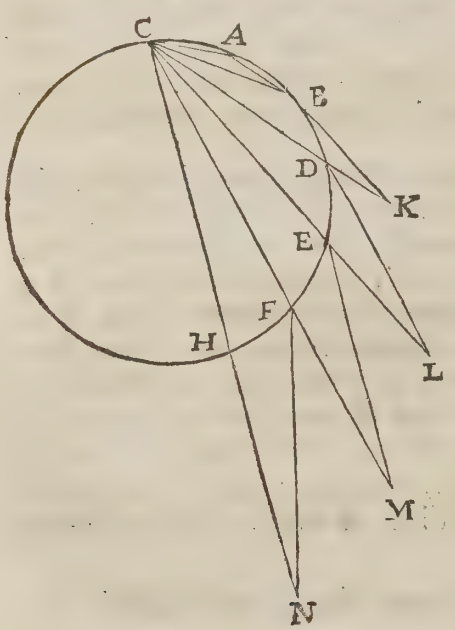
In the adjacent Circle CABD, if any Arch, as AD, be biſected in the Point B, and from any other Point C, in the Periphery of the ſame Circle, taken without the Arch AD, the Chords or Subtences, as CA, CB and CD, be drawn to the ſeveral Points A, B, and D, CD being produced without the Circle, till BP be equal to CB: I ſay, the external Part DP of the Chord CD, produced to P, will be equal to the Chord CA.

Conſtruction.

Let the Lines AB and BD be drawn.

Demonſtration.

Becauſe the Angle ACB=BCP (*by the 1ſt Cor. of the 9th Prop.*)= CPB, (*by Conſtr. and the 6th Prop.*) and the Angle PDB=CAB (*by the 2d and 3d Cor. of the 9th Prop.*) and the Side CB=BP, (*by Con.*) therefore CA=DP, that is, DP, the external Part of the Chord CP, is equal to the Chord CA; which was to be demonſtrated.



Whence it follows,

If from any Point C, in the Circumference of a Circle, any Number of equal Parts be ſet off from C to A, from A to B, from B to D, *&c.* and the ſeveral Chords or Subtences, CA, CB, CD, *&c.* be drawn, CD being produced till BK be equal to CB, and CE being produced till DL be equal to CD, *&c.* I ſay, it follows, that CK will be equal to CA + CD, CL = CE + CB, *&c.* For,

Becauſe the Triangles CBK, CDL, CEM, *&c.* are ſimilar, it will be,

As

As CB : (CK =) CA + CD :: CD : (CL =) CB + CE ; or,

As CB : (CK =) CA + CD :: CF : (CN =) CE + CH, *&c.*

That is, in a Rank of Equidiffering Arches, the Chord of any Arch in that Rank, is to the Sum of the Chords of any two Arches, equally remote from it on each Side, as the Chord of any other Arch in the ſame Rank, is to the Sum of the Chords of two other Arches next to it, on each Side equally remote, or having the ſame common Difference, whether the Progreſſion be continued or interrupted.

And ſince the Chords are the double Sines of half the Arches they ſubtend, and that the Halves are in the ſame Proportion as the Wholes, it follows, That in a Rank of Equidiffering Arches, the Sine of any Arch in that Rank will be to the Sum of the Sines of any two Arches, equally remote from it on each Side, as the Sine of any other Arch in the ſame Rank, is to the Sum of the Sines of two Arches next to it, on each Side equally remote, or having the ſame common Difference, whether the Progreſſion be continued or diſcontinued.

Now the Sine of 90° 00′, being the middle Sine between the Sines of 89° 59′ and 90° 01′, each of theſe being equally diſtant from the Radius, it follows, That as the Sine of 90° 00′ is to the Sum of the Sines of 90° 01′ and 89° 59′, ſo is the Sine of any other Arch in the Quadrant, to the Sum of the Sines of the two neighbouring Arches, removed at the Diſtance of 1 Minute from the given Arch: And ſince the Sines of 90° 01′ and 89° 59′, are repreſented by the ſame Line, it follows, That as the Radius is to twice the Sine of 89° 59′, ſo is the Sine of any other Arch (ſuppoſe 1 Minute) to the Sum of the Sines of 0′ and 2′, or to the Sine of 2 Minutes; and ſo is the Sine of 2 Minutes, to the Sum of the Sines of 1 Minute and 3 Minutes; from which Sum ſubtracting the Sine of 1 Minute, there will remain the Sine of 3 Minutes; and ſo is the Sine of 3 Minutes to the Sum of the Sines of 3 Minutes, 4 Minutes, *&c.*

The Right Sine of 90° 00′, in the Common Tables, is put equal to 1.0000000, whence the Sine of 89° 59′ has been found to be 9999999,5; wherefore it will be, As 1.0000000, is to 1.9999999, ſo is the Sine of any Arch: Suppoſe 1 Minute equal to 0002909,

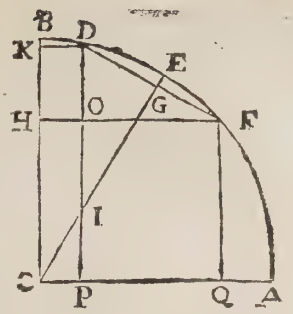
to 0005818, the Sine of 2 Minutes; and ſo is 0005818 the Sine of 2 Minutes, to 0011636 the Sum of the Sines of 1 Minute and 3 Minutes; from whence taking away 0002909, the Sine of 1 Minute, there will remain 0008927, the Sine of 3 Minutes, *&c.*

And inaſmuch as in this Proportion, the firſt Term being Unity, with a Number of Cyphers annex'd to it, the Trouble of Diviſion is wholly avoided; and ſince the ſecond Term conſtantly remains the ſame, if a Table of the ſeveral Multiples of it by the 9 Digits, be formed in a regular Order, the whole Canon may be formed by *Addition* and *Subtraction* only.

And it is farther obſervable, that as the ſecond Term is very nearly double of the Firſt, the Fourth, according to the Rules of Proportion, muſt be nearly double of the Third; whence the Sum of the Sines of two Arches, whoſe Difference is 2 Minutes, will be nearly double to the Sine of the middle Arch; wherefore the Difference between the Middle and each Extream will be nearly the ſame, but decreaſing as you aſcend; that is, the Difference between the greater Extream and the Mean, will be leſs than the Difference between the Mean and the leſſer Extream; but is ſo very ſmall, in a Table to 7 Places in Decimals, ſuch as the common Tables are, and does decreaſe ſo regularly, that an Equation Table may be made with much Eaſe; by the Help of which, the whole Table of Sines may be readily made, by a continual Addition of the Sine of 1 Minute, leſſen'd accordling as the Table ſhall direct, which is ſo wonderful eaſy, as nothing can be more: For what the Ancients were forced to do by Continual Multiplications, Diviſions, and Extracting of Roots, may be here done by the Simple Addition of a very ſmall Number, which will always be leſs than the Sine of 1 Minute, and conſtantly decreaſing, till at laſt it vaniſhes.

The Sines to 30 Degrees being obtain'd, the Sines to 60 Degrees may be had, by multiplying the Sines of thoſe already found, by the Square Root of 3, equal to 1.73205807 +, (that is, if the Sine of any Arch leſs than 30 Deg.) Suppoſe 10 Degrees be added to the Sine of its Defect to 30 Deg. namely, the Sine of 20 Deg and this Sum be multiplied by the Square Root of 3, or 1.732050807+, you will have the Sine of an Arch (namely 50 Degrees) as much exceeding 30 Degrees, as the Arch (of 10 Degrees) firſt propoſed wanted, or was leſs than 30; which may be proved after the Manner following.

Let



Let BE be an Arch of 30 Deg. BD any Arch leſs than BE, then will ED be the Arch of Defect; make EF = ED, and BF will be an Arch as much exceeding BE, the Arch of 30 Degrees, as BD wanted of 30 Degrees.

Now becauſe the Angles FOD and IGD are Right Angles, and the Angles IDG or FDO, common to the Triangles IGD and FOD; therefore the remaining Angle OFD, of the Triangle FOD, will be equal to the Angle DIG, of the Triangle DGI, (*by Prop. the 5th.*) equal to the Angle BCE (*by Prop. the 3d*) equal to 30 Deg. (*by Hyp.*) and DO equal to DG (*by the 5th Prop.*) $=\frac{1}{2}$DF: But becauſe (*by the 11th Prop.*) $FO^q = FD^q - DO^q$; therefore $FO^q = 4\,DG^q - DG^q = 3\,DG^q$; therefore $FO = DG^q \times \sqrt{3}$: But becauſe DK = HO, therefore FH = FO + KD = KD + DG × $\sqrt{3}$, which was to be demonſtrated.

Again, Becauſe the Radius is to twice the Co-ſine of an Arch as the Sine of any other Arch is to the Sum of the Sines of two Arches, equally remote on each Side, if the Sine of 30 Degrees, which is equal to half the Radius, be taken for one of the extream Arches, of which the Radius, or Sine of 90° 00′, is the Mean, the double of its Co-ſine, or the ſecond Term, will be equal to the Radius, or firſt Term; whence, according to the Laws of Proportional Quantities, the fourth Term muſt be equal to the Third: That is, the Sine of any Arch (ſuppoſe 80 Degrees) is equal to the Sum of the Sines of 20 Degrees, and 140 Degrees, (or of two Arches as far diſtant from the Sine of 80 Degrees, in the progreſſional Order) as the Sine of 30 Degrees, or Co-ſine of 60 Degrees, is removed or diſtant from the Radius, or Sine of 90° 00′. And becauſe the Sine of 140 Degrees, and its Supplement 40, are repreſented by the ſame Line, and expreſs'd by the ſame Number, therefore the Sine of 80 Degrees is equal to the Sum of the Sines of 40 Degrees and 20 Degrees.

In like manner, The Sine of 70 Degrees is equal to the Sum of the Sines of 10 Degrees, and (of 130 Degrees, or of its Supplement) 50 Degrees, each being diſtant 60 Degrees from the Sine of 70 Deg. that is, the Sine of any Arch, leſs than 60 Deg. being added to the Sine of its Defect, the Sum will give the Sine of an Arch, as much exceeding 60 Degrees, as the Arch firſt taken wanted

wanted of 60 Degrees: That is, If to the Sine of 35 Degrees be added the Sine of 25 Degrees, its Defect to 60 Degrees, the Sum of these two Sines, will give the Sine of 85 Degrees, or of an Arch, which exceeds 60 Degrees by 25 Degrees, which is just as much as the Arch of 35 Degrees, first given, wanted of 60 Degrees: Whence it appears, that the Sines of the first 60 Degrees being found, the rest to 90 Degrees may be obtained by the Addition of those already found. Or,

The Sines of the last 30 Degrees, *viz.* from 60 Degrees to 90 Degrees, might have been found directly from the *general Theorem*, by saying, As the Radius to twice the Sine of 89° 59′, so is the Sine of 89° 59′ to the Sum of the Sines of 90° 00′ and 89° 58′; and so is the Sine of 89° 58′ to the Sum of the Sines of 89° 59′ and 89° 57′; from the former of which, if you take the Sine of 90° 00′, and from the latter the Sine 89° 59′, you will have the Sines of 89° 58′ and 89° 57′, *&c.* and proceeding in the same manner, the remaining Sines of the Quadrant, in a descending Order, might be found, if required.

From this Property of the Sines only, several useful Rules might be drawn, that would contribute very much to an expeditious and easy Computation of the Tables of Natural Sines; but as they are already done to a very great Exactness, and sufficiently large for any Uses to which they may be applied, what has been said on this *Section*, is sufficient to form an adequate Notion in the Reader's Mind, of the Nature of these Numbers, and to instruct him how to form or investigate them, if his Curiosity shall lead him so far.

The Sines of all the Arches in the Quadrant being found, the correspondent Tangents and Secants will be easily had.

For, because (*by the 5th Cor. of the 2d Sect.*) the Co-sine of an Arch is to the Right Sine, as the Radius to the Tangent of the same Arch, if the Radius, multiplied into the Right Sine (which is done by the Addition of a certain Number of Cyphers to the Right Sine) be divided by the Co-sine, the Quotient will give the proper Tangent.

And again, Because (*by the 8th Cor.*) the Radius is a Mean Proportional, between the Tangent and Co-tangent of an Arch, if the Square of the Radius (which is an Unit with a certain Number of Cyphers annex'd; the Radius being supposed to be 1.0000000, *&c.*) be divided by the Tangent of an Arch, the Quotient will give the Co-tangent of the same Arch: Whence it follows, that the

that the Tangents being made to half of the Quadrant, the reſt may be found by the Help of thoſe already known.

And again, Becauſe (*by the 6th Cor. of the 2d Section*) the Co-ſine of an Arch is to the Radius, as the Radius is to the Secant of the ſame Arch, if the Square of the Radius (equal to 1.000000, *&c.*) be divided by the Co-ſine of any Arch, the Quotient will be the Secant of the ſame Arch, whence the whole Table of Natural Sines, Tangents and Secants, may be readily found.

How the Natural Sine, Tangent and Secant of any Arch in the Quadrant may be found, independently from the Length of the Arch itſelf, in equal Parts of the Radius, to much greater Degrees of Exactneſs, than this Method can pretend to, by a direct Method, has been ſhewn and demonſtrated at large, in the *Second Section of the Second Part of the Firſt Volume* of my *Syſtem of the Mathematicks*; to which I refer the more curious and inquiſitive Reader, who deſires a farther Knowledge of theſe Things, than *this Section* can afford him. But as every Perſon has not had an Opportunity of attaining to a ſufficient Fund of Mathematical Knowledge, to underſtand theſe more abſtruſe Methods, and yet is deſirous of knowing how theſe uſeful Tables are formed; I ſay, 'tis for the Sake of ſuch Perſons, that I have made Uſe of this plain and eaſy demonſtrative Method, not doubting but it will give them entire Satisfaction in their Inquiries into the Nature and Conſtruction of theſe very uſeful Numbers.

By the Help of the Tables thus made, and a Line of equal Parts of a convenient Length, may the Lines upon the *Plain Scale* be more readily formed, and with greater Accuracy, than by the former Method of Conſtruction, as will be manifeſt to any one that will attempt the Performance.

Having ſufficiently explain'd the Nature of Sines, Tangents and Secants, and ſhewed their Conſtruction from their Nature conſidered, I ſhall, in the next *Section*, proceed to enquire into the Nature of *Logarithms*, and ſhew the manner how they may be computed, as well as the way how the Artificial Sines, Tangents and Secants, may be inveſtigated; and from thence, how the Lines upon the *Gunter's Scale* are formed; an Inſtrument of very great Uſe for the ready and expeditious Solution of all ſuch Problems, wherein the Aſſiſtance of Logarithms, Sines, Tangents, *&c.* are concerned.

SECT.

SECT. V.

Containing Methods for Computing the Tables of Logarithms, Artificial Sines, Tangents *and* Secants.

LOGARITHMS were first invented by *John Neper*, Baron of *Marcheſton* in *Scotland*, in the Year 1614, and afterwards reduced into a better Form, and publiſhed at *London* in the Year 1624, in the ſame Manner as they are uſed at this Day.

Logarithms may be conſidered as (and were uſually defined to be) a Series of Numbers in *Arithmetical Progreſſion*, fitted or adapted to a Series of Numbers in a *Geometrical Progreſſion*: For Example; Suppoſe 0, 1, 2, 3, 4, 5, 6, *&c.* to be a Series of Numbers in *Arithmetical Progreſſion*, and 1, 10, 100, 1000, 10000, 100000, 1000000, *&c.* to be a Series of Numbers in a *Geometrical Progreſſion*.

It is evident, that as the Numbers in the firſt Rank are formed by a continual Addition of Unity, and have therefore one common Difference, ſo the Numbers in the ſecond Rank are formed by a continual Multiplication by 10, and have therefore one common Ratio.

As the Sum of the two Extreams 3 and 5 of any three Terms in the upper Rank, ſuppoſe 3, 4 and 5, is double of the Mean, or middle Term, for $2 \times 4 = 8$; ſo the Product ariſing from the Multiplication of the two extream Terms, 1000 and 100000 of any three Terms in the loweſt Rank. Suppoſe 1000, 10000, and 100000, whoſe Product is 100000000, will be equal to the Square of the Mean, or middle Term; for $10000 \times 10000 = 100000000$.

Again; As half the Sum of any two Extream Terms of any three Terms in the upper Rank, will produce the Mean, or middle Term; for half 8, the Sum of 3 and 5, is 4, the Mean, or middle of the three Terms 3, 4 and 5, ſo the Square Root of Product ariſing from the Multiplication of any two extream Terms of any three Terms in the lower Rank, will produce the Mean, or middle Term, in the ſame Rank; for 10000, the Square Root of 100000000,

100000000, arising from the Multiplication of 1000 by 100000, is the Mean, or middle Term of the three Terms, 1000, 10000, and 100000.

Again; As 8, the double of the Mean, or middle Term 4, of any three Terms; suppose 3, 4 and 5, in the upper Rank, lessen'd by one of the Extreams, 3 will give 5, the other Extream; so 100000000, the Square of 10000, the Mean, or middle Term of any three Terms; suppose 1000, 10000, and 100000, in the lower Rank, divided by 1000, one of the Extreams, will give in the Quotient 100000, the other Extream.

Again, As 7, the Sum of the Extreams, 6 and 1 of any four proportional Numbers in the upper Rank is equal to the Sum of any two Means equally removed from them; for $2 + 5 = 3 + 4 = 7$, so 10000000, the Product of the two Extreams 10 and 1000000 of any four proportional Numbers in the lower Rank, is equal to the Product of any two mean Terms equally removed from them; for $100 \times 100000 = 1000 \times 10000 = 10000000$.

Again, As the Difference between the Sum of any two mean Terms, and one of the Extreams of any four Terms, equally remote from each other in the first Rank or Series, will be equal to the other Extream in the same Series; so the Product arising from the Multiplication of any two mean Terms of four Terms equally remote from each other in the lower Rank or Series, being divided by one Extream, will produce the other Extream in the same Rank or Series, as far distant from one of the mean Terms in the progressional Order, as the extream Term taken was distant from the other Mean; for, as 7, the Sum of 2 and 5, or 3 and 4, considered as mean Terms in the progressional Order of the upper Rank or Series, lessened by 1, considered, as an extream Term will produce 6, the other Extream in the same Rank; so 10000000, the Product arising from the Multiplication of 100 by 100000, or of 1000 by 10000, considered as mean Terms in the lower Rank or Series, being divided by 10, considered as an Extream, will give in the Quotient 1000000 the other Extream in the same Series, as far distant from 100000, considered as a Means, as 10 considered as an Extream was distant from 100.

Again, As the Sum of 2 and 3, the third and fourth Terms in the upper Rank or Series, will produce 5, the sixth Term in the same Series; so the Product answering from the Multiplication of 100 by 1000, the third and fourth Terms in the lower Rank or

Series, will produce 100000, the corresponding ſixth Term in the ſame Series.

Again, as the Difference between 6 and 4, the ſeventh and fifth Terms in the upper Rank produces 2, the third Term in the ſame Rank; ſo the Quotient ariſing from the Diviſion 1000000, the ſeventh Term in the lower Rank, by 10000, the fifth Term in the ſame Rank, will produce 100, the correſponding third Term in the ſame Rank.

Whence it is manifeſt, if the Numbers in the upper Rank be conſidered as the Logarithms of the Numbers in the lower Rank, in a Progreſſional Order, each to its correſpondent Number; that is, if 1 be conſidered as the Logarithm of 10, 2 the Logarithm of 100, 3 the Logarithm of 1000, *&c.* that the Addition and Subtraction of the Terms in the upper Series, will produce Terms in the ſame Series that ſhall correſpond with Terms in the lower Series, ariſing from the Multiplication and Diviſion of the correſponding Terms in the lower Series amongſt themſelves.

For as 2, the third Term in the upper Series, the ſuppoſed Logarithm of 100 in the lower Series being added to 3, the fourth Term in the upper Series, the ſuppoſed Logarithm of 1000 in the lower Series produces 5, the ſixth Term in the upper Series, the ſuppoſed Logarithm of 100000 in the lower Series; ſo the third Term 100 in the lower Series, of which 2 is the Logarithm, being multiplied by 1000, the fourth Term in the ſame Series, of which 3 is the Logarithm, will produce 100000 the ſixth Term in the ſame Series, of which 5 is the Logarithm.

Again, as 1, the ſecond Term in the upper Series, the ſuppoſed Logarithm of 10 in the lower Series, being taken from 4 the fifth Term in the upper Series, the ſuppoſed Logarithm of 10000 in the lower Series will leave 3, the fourth Term in the upper Series, the ſuppoſed Logarithm of 1000 in the lower Series; ſo 10000 the fifth Term in the lower Series, of which 4 the fifth Term in the upper Series is the ſuppoſed Logarithm, being divided by 10, the ſecond Term in the lower Series, of which 1 is the ſuppoſed Logarithm, will produce 1000, the fourth Term in the lower Series, of which the ſuppoſed Logarithm is 3.

Again, as 2, the third Term in the upper Rank, the ſuppoſed Logarithm of 100 in the lower Rank, being doubled, will produce the Number 4, the fifth Term in the firſt Rank, the ſuppoſed Logarithm of 10000, the fifth Term in the lower Rank; and being

ing tripled, will give 6, the ſeventh Term in the firſt Rank, the ſuppoſed Logarithm of 1000000, the ſeventh Term in the ſecond Rank, ſo the Number 100, the third Term in the loweſt Rank, of which the Logarithm is 2, being ſquared or multiplied by itſelf will produce 10000, the fifth Term in the ſame Series, of which the fifth Term, namely 4, is the ſuppoſed Logarithm; and being again multiplied by itſelf or cubed will produce 1000000, the ſeventh Term in the ſame Series, of which the ſeventh Term in the upper Series, namely 6, is the ſuppoſed Logarithm.

Again, As the one half of 4, the fifth Term in the upper Series, the ſuppoſed Logarithm of 10000, the fifth Term in the lower Series, will give 2, the third Term in the upper Series, the ſuppoſed Logarithm of 100, the third Term in the lower Series; and as one third of 6, the ſeventh Term in the upper Series, the ſuppoſed Logarithm of 1000000, the ſeventh Term in the lower Series, will likewiſe give 2, the third Term in the upper Series, the ſuppoſed Logarithm of 100, the third Term in the lower Series; ſo the Square Root of 10000, the fifth Term in the upper Series, namely 100, will be the third Term in the lower Series, the Logarithm of which is 2, the third Term in the upper Series; and the Cube Root of 1000000, the ſeventh Term in the lower Series, namely 100, will be the third Term in the lower Series, the ſuppoſed Logarithm of 2, the third Term in the upper Series.

Whence it is abundantly manifeſt, that the Addition and Subtraction of Logarithms anſwers to the Multiplication and Diviſion of the reſpective Numbers of which they are the Repreſentatives, that the Multiplication of theſe Logarithms anſwers to the Involution of Numbers or Raiſing of Powers, and the Diviſion of them to the Evolution of Powers or Extracting of Roots.

As a great Variety of Series in Arithmetical Progreſſion may be adapted to the ſame Series in Geometrical Progreſſion; and as a great Variety of Series in Geometrical Progreſſion may be adapted to the ſame Series in Arithmetical Progreſſion, ſo there may alſo almoſt infinite Variety in the Forms of Logarithms; but of all theſe Forms that is found to be the readieſt for Uſe, where the Logarithm of 10 is aſſumed to be 1 of 100, 2, &c. as I have here done.

The Nature of Logarithms being thus explained, and the Numbers in the upper Series being aſſumed, as Logarithms of the Numbers in the lower Series, each to its correſpondent Number, in a regular Order, that is, If 1 be the aſſumed Logarithm of 10, 2

the Logarithm of 100, 3 the Logarithm of 1000, *&c.* it remains now, that we ſhew how the Logarithms of the intermediate Numbers between 0 and 10, namely 1, 2, 3, *&c.* between 10 and 100, namely 11, 12, 13, *&c.* the intermediate Logarithms between 100 and 1000, namely 101, 102, 103, *&c.* may be found.

It has been ſhewn and demonſtrated, in the *25th Section of the First Vol. of my Syſtem of the Mathematicks*, *Page* 672, to which I refer the inquiſitive Reader; where he will find the Nature of Logarithms more fully inquired into, and the Manner of their Conſtruction ſhewn different Ways, from different Conſiderations of their Nature and Properties; I ſay, it has been ſhewed in that Page, that to find the Logarithm of any given Number, we muſt

To the given Number add 1 for a Denominator or Diviſor, and ſubtract 1 from the ſame Number for a Numerator or Dividend; then find the Equivalent Decimal Fraction thence reſulting, which Fraction being multiplied ſucceſſively by the Squares of the Fraction firſt found, if the ſeveral Products thus produced be divided by the ſeveral odd Numbers, namely 1, 3, 5, 7, 9, *&c.* the Sum of all theſe laſt Quotients will be the Natural Logarithm of the Number propoſed.

Let it therefore be required to find the Natural Logarithm of 2, the firſt prime Number to 7 Places in Decimals, which is as far as the common Tables uſually extend.

Becauſe the Number given is 2, if from 2 we take 1, the Remainder 1, will be the Numerator; and if to the ſame Number we add 1, the Sum 3 will be the Denominator; whence the reſulting Fraction will be $\frac{1}{3}$, and its equivalent Decimal Fraction 333,333,333; this therefore being ſucceſſively multiplied by $\frac{1}{9}$, the Square of $\frac{1}{3}$, the Fraction firſt reſulting, and the ſeveral Products being divided by the odd Numbers 1, 3, 5, 7, *&c.* the Sum of the laſt Quotients will be the Natural Logarithm of 2.

But becauſe to multiply by $\frac{1}{9}$ and divide by 9 produces the ſame Effect, if the Decimal Fraction 333,333,333 be divided ſucceſſively by 9, and the ſeveral Quotients thence reſulting be again divided by the odd Numbers 1, 3, 5, 7, *&c.* till the Terms vaniſh, the Sum of all the laſt Quotients will be the Natural Logarithm required to be found.

The Operation is as follows.

$\frac{1}{3}$ =	333.333.33 =	333.333.33 +
$\frac{1}{9}$ =	37.037.04 +	12.345.68 −
$\frac{1}{9}$ =	4.115.23 −	823.05 −
$\frac{1}{9}$ =	457.25 −	65.32 +
$\frac{1}{9}$ =	50.81 −	5.65
$\frac{1}{9}$ =	5.65 −	51
$\frac{1}{9}$ =	63	5
The Nat. Logarithm of 2 is		346.573.59

Hence 346. 573. 59, the Natural Logarithm of 2 being doubled, will give 693. 147. 18, for the Natural Logarithm of 4; and being trebled will give 103. 972. 077, for the Natural Logarithm of 8, *&c.*

Again, becauſe the Logarithm of 8, being added to the Logarithm of 1 $\frac{1}{4}$ will give the Logarithm of 10, if the Logarithm of 1 $\frac{1}{4}$ be inveſtigated, we ſhall thence readily find the Logarithm of 10: Now in order to find the Logarithm of 1 $\frac{1}{4}$, if to this Number be added 1, we ſhall have 2 $\frac{1}{4}$ for a Denominator; and if from the ſame Number we ſubtract 1, we ſhall have $\frac{1}{4}$ for a Numerator; ſo that the Fraction thence reſulting will be $\frac{1}{4}$ by $\frac{9}{4} = \frac{1}{9}$, and the equivalent Decimal Fraction will be 111.111.111, *&c.* Wherefore, if this be multiplied by $\frac{1}{81}$, equal to the Square of $\frac{1}{9}$, or divided ſucceſſively by 81, and the ſeveral Quotients thence reſultiug by 1, 3, 5, 7, *&c.* the Sum of the laſt Quotients will be the Natural Logarithm of 1 $\frac{1}{4}$.

The Operation is as follows.

$\frac{1}{9}$ =	111.111.111	111.111.11 +
$\frac{1}{81}$ =	1.371.74	457.25 −
$\frac{1}{81}$ =	16.94	3.39 −
$\frac{1}{81}$ =	21	3 −
The Nat. Logarithm of 1 $\frac{1}{4}$		111.571.78

This

This therefore being added to 1.039.720.77, the Natural Logarithm of 8 before found will give 1.151.292.55, for the Natural Logarithm of 10.

As our ſuppoſed or aſſumed Logarithms of 10 is Unity, in order to reduce all Natural Logarithms into our aſſumed Form, we muſt reaſon thus: As 1.151.292.55 the Natural Logarithm of 10, is to 1 our aſſumed Logarithm of 10, ſo is any other Natural Logarithm to the correſpondent Logarithm in the aſſumed Form: For ſince all Kinds of Logarithms are made up of ſimilar Quantities, Logarithms of any Form may be reduced into Logarithms of any other Form, by ſaying, As any one Logarithm in the given Form is to its correſpondent Logarithm in the New Form, ſo is any other Logarithm in the given Form to its correſpondent Logarithm in the New Form; the Reaſon of which has been ſufficiently ſhewed in the *25th Section of my Syſtem, Vol.* 1. already mention'd. Wherefore, if the Natural Logarithm of any one Number be divided by 1.151.292.55, or multiplied by the Quotient of 1.000.000, *&c.* divided by 1.151, *&c.* 865.588.96, which is called its *Reciprocal*, the Quotient in the firſt Caſe, and the Product in the latter Caſe, (as is manifeſt from the Laws of Decimal Arithmetick) will give the Logarithm in our ſuppoſed Form correſpondent to or belonging to the Number given.

Wherefore, if 346.573.59 the Natural Logarithm of 2 before found, be divided by 1.151.292.55, or multiplied by 868.588.96 the Quotient in the firſt Caſe, and the Product in the laſt Caſe will give 301.029.99, or 3010200, the Logarithm of 2 in our aſſumed Form, which is the *Brigg*'s Logarithm of 2, agreeing exactly with the Logarithm of 2 in the common Tables now in Uſe.

But if inſtead of taking the Decimal of $\frac{1}{3}$, as we did in finding the Natural Logarithm of 2, we take $\frac{1}{3}$ of the reciprocal Number (namely 868.588.96) for reducing the Natural Logarithm into our aſſumed Form, and divide it firſt ſucceſſively by 9, and each of thoſe Quotients again by 1, 3, 5, 7, *&c.* the Sum of all theſe laſt Quotients will be the Logarithm of 2 in our aſſumed Form.

The

The Operation is as follows.

$\frac{1}{3}$ of 868.588.96 is 289.529.65 : Wherefore

Put down	289.529.65 +	—— 289.529.65 +
Its $\frac{1}{9}$ is	32.169.96 +	——— 10.723.32
$\frac{1}{9}$	——3.574.44	——— 714.89 —
$\frac{1}{9}$	———397.16	———56.74 —
$\frac{1}{9}$	———44.13 —	———4.90 +
$\frac{1}{2}\frac{1}{9}$	———4.90 +	———45 —
$\frac{1}{9}$	———54 +	———4 —
The Assumed or *Brigg*'s Log. of 3		301.029.99

or 301.030.0, agreeing exactly with the Common Tables: For, in all Contractions of Places, when the Figure to be rejected is greater than 5, as here it is 9, the Figure next to it is always increased by Unity; and consequently the Number 301.029.99, will become by this Addition 30.10.300, the Reason of which is evident.

After the same Manner may the *Brigg*'s Logarithms of any other Prime Number be found.

Suppose it were required to find the Logarithm of the next Prime Number 3, because $1+3=4$ is the Denominator, and $3-1=2$ is the Numerator; therefore the Fraction thence resulting is $\frac{2}{4}=\frac{1}{2}$, whence if $\frac{1}{2}$ of 868.588.96 the Reciprocal be taken, and this last Number be divided successively by the Square of $\frac{1}{2}$ or $\frac{1}{4}$, and the succeeding Quotients by 1, 3, 5, 7, *&c.* we shall by summing up the last Quotients have the Tabular Logarithm of 3: But inasmuch as this Series converges very slow, the same Logarithm of 3 may be found much quicker, by finding the Difference between the Logarithm of 2 already known, and the Logarithm of 3 required.

And in this Case, the Numerator of the Fraction will be $3-2$ or 1, and the Denominator $3+2=5$, whence the Fraction thence resulting will be $\frac{1}{5}$: Wherefore, if $\frac{1}{5}$ of the Reciprocal 868.588.96 equal to 173.717.79 be successively divided by the Square of $\frac{1}{5}$ or $\frac{1}{25}$, and the several Quotients thence resulting by the odd Numbers 1, 3, 5, 7, *&c.* the Sum of the last Quotients will be the Difference of the Logarithms of 2 and 3.

The

The Operation is as follows.

	$\frac{1}{5}$ of 868.588.96+	
Is	173.717.79+	173.717.79+
Its $\frac{1}{25}$ is	6 948.71+	2.316.24—
$\frac{1}{25}$	277.95—	55.59—
$\frac{1}{25}$	11.12—	1.59
$\frac{1}{25}$	44+	5
Diff. between the Log. of 2 and 3		176.091.26

This therefore added to 301.029. 99 the Logarithm of 2 before found, will give 477.121.25, or 477.12.12; for the *Briggs* Logarithm of 3, agreeing exactly with the Tabular Logarithm.

Again, becauſe 2×2=4, therefore,

To find the Logarithm of 4.

If 301.0300, the Logarithm of 2, be added to itſelf, or multiplied by 2, the Sum or Product 602.0600 will be the Logarithm of 4.

Again, becauſe 2×5=10,

To find the Logarithm of 5.

From 1.000.000, the Logarithm of 10, take 301.0300, the Logarithm of 2, the Remainder 698. 9700 will be the Logarithm of 5.

And becauſe 2×3=6, therefore,

To find the Logarithm of 6.

To 477.1212, the Logarithm of 3, add 301. 0300, the Logarithm of 2, and the Sum 778. 151.25 will be the Logarithm of 6.

The Logarithm of 6 being known, the Log. of 7 will be eaſily had; for by adding 6 to 7, we ſhall have 13 for a Denominator, and by taking 6 from 7, we ſhall have 1 for a Numerator, whence $\frac{1}{13}$ is the reſulting Fraction, and conſequently, if $\frac{1}{13}$ of the Reciprocal be taken, and that Quotient be divided by the Square of $\frac{1}{13}$ or $\frac{1}{169}$, and the ſeveral Quotients by 1, 3, 5, 7, *&c.* the Sum of theſe laſt Quotients will give us the Logarithm of the Difference, as in the following Operation.

$\frac{1}{13}$ of

$\frac{1}{13}$ of ——	868 58.95	
is ——	66 814.54 ——	66.814.54
its $\frac{1}{169}$ —— ——	395 39 + ——	131.78 +
$\frac{1}{169}$ —— —— ——	2 34 —— ——	47
The Differ. of the Log. of 6 and 7 ——		66.946.79

Which being therefore added to the Logarithm of 6, will give 8450980⅘, for the Logarithm of 7.

Again, Becauſe 2×2×2=8, therefore, if 3010300, the Logarithm 2 be multiplied by 3, the Product 9030900 will be the Logarithm of 8.

And again, Becauſe 3×3=9, if 47712125 be multiplied by 2, the Product 95424250 will be the Logarithm of 9, whence the Logarithm of 2 has been found to be 3010300, the Logarithm of 3 to be 4771212, the Logarithm of 4 to be 6020600, the Logarithm of 5 to be 6989700; of 6 to be 7781512; of 7 to be 8450980; of 8 to be 9030900; of 9 to be 9542425, and the Logarithm of 10, as was at firſt aſſumed to be 1.000.0000; and proceeding after the ſame manner may the whole Tables of Logarithms (if need be) be formed.

The Tables of Logarithms being thus compleated, the Logarithmic or Artificial Sines, Tangents and Secants will be eaſily obtained.

For by entering the Logarithmic Table with the Natural Sine, Tangent and Secant of any Arch, you will find the Logarithmic Sine, Tangent or Secant of the ſame.

Suppoſe, for *Example*, it were required to find the Logarithmic or Artificial Sine of 30 Degrees, becauſe the Natural Sine of 30 Degrees is 5000000, if I enter the Logarithmic Table with the given Number 5000, *&c.* I ſhall find the correſpondent Logarithm to be 6989700, and becauſe the Logarithmic Sines, *&c.* are computed to a Radius of 1.0000000000, it will have 9 for its Index; whence the Logarithmic or Artificial Sine of 30 Degrees will be 9.6989700.

After the ſame Manner, by entering the Logarithmic Tables with the Numbers 5773503, *&c.* the Natural Tangent of 30 Degrees, we ſhall find its correſpondent Logarithm to be 7614394, to which prefixing 9 for the Index, for the above-mentioned Rea-

ſon, we ſhall have 9.7614394 for the Logarithmic Tangent of 30 Degrees; and laſtly, by entering the ſame Table with the Number 1.1547005, the Natural Secant of 30 Degrees, we ſhall have for its correſpondent Logarithm 064.694, which muſt have 10 for its Index, (becauſe the Natural Secant of 30 Degrees goes to the eleventh Place in Decimals, the ſame with the Radius, but expreſs'd by a greater Number) whence the Logarithmic Secant of 30 Degrees will be 10.0624694; and inaſmuch as the Tangent of 45 Degrees is equal to the Radius, it will have 10 for the Index, and all Logarithmic Tangents above that will have the ſame Index, till you come to the Tangent of 84° 18', when the Natural Tangent being increaſed by one Decimal Place, the Index will be increaſed by an Unit, and conſequently will become 11, *&c.* In like manner, as the Natural Secant is always greater than the Radius, the Index of the ſmalleſt or firſt Secant expreſſed in the Tables will have 10 for its Index, for the Reaſon already mentioned; and this Index will continue till you come to the Arch of 84° 16', when the Natural Secants will be increaſed by one Place in Decimals, whence the Index itſelf muſt be increaſed by Unity; and theſe, as well as the Natural Tangents, will continually increaſe till they become Infinite, and their Logarithmic Tangents and Secants not to be expreſſed by Numbers.

The Logarithmic Sines being obtained, the Logarithmic Tangents and Secants may from thence be inveſtigated; for, becauſe the Co-ſine is to the Right Sine, as the Radius to the Tangent of the ſame Arch: If from the Sum of the Radius, and the Right Sine of an Arch be taken, the Co-ſine of that Arch, the Remainder will be the Tangent; Thus, if from 19.6989700, the Sum of the Radius, and the Logarithmic Sine of 30 Degrees, which (becauſe the Logarithmic Radius is 10.0000, *&c.* is done by prefixing Unity to the Index of the Logarithmic Sine) be taken 9.9375306, the Logarithmic Co-ſine of 30 Degrees, or Sine of 60 Degrees, the Remainder 9.7614394 will be the Logarithmic Tangent; and becauſe the Tangent of an Arch is to the Radius, as the Radius is to the Co-tangent of the ſame Arch; if the Tangents are made to 45 Dgrees, the reſt of the Tangents in the Quadrant may be found from thoſe already had. For *Example*, If from twice the Radius, which is equal to 20.0000, *&c.* be taken 9.7614394, the Logarithmic Tangent of 30 Degrees, the Remainder 10.2385606 will be the Logarithmic Tangent of 60 Degrees. And again, becauſe

cauſe the Co-ſine of an Arch is to the Radius, as the Radius is to the Secant of the ſame Arch, if from twice the Radius equal to 20.0000, *&c.* be taken, For *Example*, 9.6989700, the Co-ſine of 60 Degrees, or Sine of 30 Degrees, the Remainder 10.3010300 will be the Logarithmic or Artificial Secant of 60 Degrees, *&c.*

The Knowledge of theſe Properties of the Sines, *&c.* are very neceſſary to be known by every one who has Occaſion to make Uſe of them, ſince they will not only teach him how to ſupply any Numbers that may be wanting, but enable him to examine and correct the Tables of any Errors that may have crept into them; for want of which Knowledge, People are ſometimes plunged into Difficulties, out of which they know not how to extricate themſelves.

How the Logarithmic Sine, Tangent and Secant of any Arch may be found independently from the Length of the Arch firſt given, in equal Parts of the Radius, without the Help of the Tables of Logarithms, Natural Sines, Tangents, *&c.* has been fully explained and exemplified, in the *25th Section of my Syſtem*, *Vol.* 1. *Page* 687, *&c.* where the ingenious Reader will find a much more ample Account of theſe Things.

As the *Gunter's Scale* is an Inſtrument of the moſt extenſive Uſe in all Branches of the *Mathematicks*, and very valuable for its eaſy and expeditious Solutions, it will not be amiſs to ſhew how the Lines upon it are formed, and more particularly in this Place, ſince they are no other than the Numbers, whoſe Conſtruction we have been ſhewing, laid down upon the Inſtrument by a Line of equal Parts. And,

1ſt. Having fixed upon a convenient length for your ſingle Line of Numbers, let a Line of the ſame Length be divided into 100, or 1000, or as many more Decimal Parts as it will admit of, and having Recourſe to the Tables of Logarithms, you will thence find, that the Log. of 90, for *Example*, rejecting the Index, will be 9542, *&c.* take therefore 95.42 Parts off the Scale of equal Parts, and ſet it off in the ſingle Line of Numbers from the Point of Commencement, and it will reach to 9, upon the ſame Line, which will be the 90th Diviſion upon the ſame Line: In like manner, againſt the Number 80, in the Tables of Logarithms, you will find (rejecting the Index) 90.30, ſetting off therefore, 90.30 taken from the Scale of equal Parts in the Line of Numbers from it, will reach

to the Figure 8 in the ſame Line, which is the Place of the 80th Diviſion.

And again, If the Log. of 70, which is 84.50, be taken off from the Line of equal Parts, and ſet off in the Line of Numbers it will give the 70th Diviſion; and proceeding after the ſame way, by taking the Logarithm of the ſeveral Numbers between 1 and 100 off from the Line of equal Parts, and applying them to the Line of Numbers from the Point of Commencement, you will have by this means the whole Line of Numbers formed; but if greater Exactneſs be required, and the Line will admit of more Subdiviſions, Suppoſe each of theſe were to be divided into 10 Parts more; then having Recourſe to the Tables of Logarithms, and taking out the Logarithm of 80.7, ſuppoſe 1.9068, and by rejecting the Index, it will be found to be 90.68: This Number therefore taken off from the Scale of equal Parts, and applied to the Line of Numbers, will give the correſpondent Diviſion: But if greater exactneſs were required, and the Length of the Line would admit of it; that is, if it were required to ſubdivide each of theſe into 10 Parts more, or which is the ſame thing, to find the Diviſion anſwering to 80.75; looking for 8075 in the Table of Logarithms, and rejecting the Index, you will find the Logarithm ſtanding againſt it to be 90.71; ſetting off therefore ſo many equal Parts upon the Line of Numbers from the Point of Commencement, and it will reach to the correſpondent Diviſion: And by the ſame manner of proceeding may each Diviſion be ſubdivided into 10, 100, 1000, *&c.* equal Parts, if the Breadth will admit of it. The ſingle Line being thus divided, what is called a double Line, which is upon all *Gunter*'s Scales, is no other than the ſame Diviſions repeated over again; and what is called a treble Line is the ſame Diviſions once more applied.

The Line of Numbers being thus formed, the Lines of Sines and Tangents are eaſily made; for having Recourſe to the Tables of Natural Sines and Tangents, take out the Numbers anſwering to the given Degree, *&c.* and right againſt that Number found upon the Line of Numbers will ſtand the correſpondent Degree, *&c.* in the Line of Sines or Tangents. For *Example*, The Natural Sine of 30 Degrees is 5000, *&c.* if I look for the Number 50 upon the Line of Numbers on the *Gunter*'s Scale, right under I ſhall find the Diviſion in the Line of Sines anſwering to 30 Degrees; and if I look for 577, the Natural Tangent of 30 Degrees,

in

in the Line of Numbers, I ſhall find right under it in the Tangent Line the Diviſion correſponding or ſtanding for 30 Degrees; wherefore, if the Diſtance from 5 to 10 in the Line of Numbers be ſet off in the Line of Sines backwards from the Sine of 90, and the Diſtance between 57.7 and 10 in the Line of Numbers be ſet off from the Tangent of 45 Degrees backwards, in the Line of Numbers, you will have in the firſt the Diviſion anſwering to 90 Degrees in the Line of Sines, and, in the ſecond Caſe the Diviſion anſwering to 30 Degrees in the Line of Tangents; and by repeating the ſame Operation may the whole Line of Sines and Tangents be formed.

The Diviſions upon the Lines, called the Sine R. and Tangent R, correſpond exactly with the Diviſions upon the Line of Sines and Tangents, that anſwer to the Number of Degrees and Minutes contained in each Point and quarter Point: For *Example*, Againſt 33° 45′ (the Number of Deg. and Min. in 3 Points) in the Line of Sines, you will ſee the Number 3 ſtand upon the Line called SR, as you will againſt 33.45 or 33 $\frac{3}{4}$, in the Line of Tangents, ſee the Number in the Tangent Rhumbs; wherefore, if the Diſtance between 33° 45′ and 90.00 in the Line of Sines be ſet off in the Line of Sine Rhumbs, from the Number 8 backwards, and the Diſtance between 33° 45′ and 45° 00′ in the Lines of Tangents be ſet off in the Line of Tangent Rhumbs, from the End where the Number 4 is marked backwards, you will have in the firſt Caſe the Diviſion anſwering to 3 Points in the Line of Sine Rhumbs, and in the ſecond Caſe the Diviſion anſwering to 3 Points in the Tangent Rhumb; and by this Method the whole Lines may be divided into Points, half Points, and quarter Points, if required.

The Lines of Sines and Tangents may be laid down without the Help of the Line of Numbers, by the Aſſiſtance of a Line of equal Parts only; for if 3 or 4 firſt Figures (according to the Largeneſs of the Scale of equal Parts) of the Artificial or Logarithmic Sine or Tangent, rejecting the Index or Characteriſtick, be taken off of the Scale of equal Parts, and laid down from the Point of Commencement of the Line of Sines or Tangents, it will give the proper Diviſions. Thus if 9408, *&c.* the Logarithmic Sine of 0° 03′ be taken off from the Line of equal Parts, and applied to the Line of Sines from the Beginning, it will give the Seat of the Diviſion anſwering to 0° 03′ in the ſame Line, *&c.* But if I was to find the Diviſion anſwering to 0° 30′, becauſe the Index is increaſed

increaſed by an Unit, (that being expreſs'd by the ſame Number, namely 9408, &c. with the Line 0° 30′ rejecting the Index) I ſay, the Index being thus increaſed, inſtead of taking off 9408, &c. of the equal Parts, you muſt take off 19408, &c. Parts, and by ſetting this Diſtance off in the Line of Sines from the Point of Commencement, it will give the correſpondent Diviſion. And again, inaſmuch as the Sine of 5° 00′, rejecting the Index is 9403, nearly the ſame with the Sine of 0° 03′ and 0° 30′, yet ſince its Index is increaſed by Unity (being in this Caſe 8, whereas that of 0° 30′ was 7, and that of 0° 036′) inſtead of 9403 we muſt take off 29403, and by ſetting off that from the Beginning of the Line of Sines we ſhall have the proper Diviſion. And again, As the Sine of 60° 46′ rejecting the Index is 9408, the ſame with the Sines of 0° 03′ and 0° 30′, yet ſince the Index of this Sine is increaſed by Unity, it being now 9, you muſt ſet off 39408, &c. equal Parts from the Point of Commencement, and you will have the proper Diviſion; and after the ſame manner may the whole Line be divided, the Reaſon of which is very evident.

But as the Index of the Sine of 1 Minute is 6 ſuch Parts, as the Index of the Sine of 90 Degrees is 10, the Point of Commencement of the Line of Sines will be diſtant from the Seat of the Sine of 90 Degrees, 4000 ſuch Parts, as a ſingle Line of Number anſwers to 1000; and conſequently, to lay down the whole Line of Sines by the Line of Numbers will require a quadruple Line of Numbers, or a Line of 4000 equal Parts of the ſame Length with the aforeſaid quadruple Line. Now as the Seat of the Sine of 0° 34′ 23″, whoſe Logarithm is 8.00000, &c. correſponds with the Number 1, the Commencement of the firſt Line of Numbers on the common *Gunter's* Scale, if from that Point you ſet off 2418, 5428, 7188 equal Parts, the Logarithmic Sines of 1, 2 and 3 Minutes, &c. rejecting the Index, you will have the Seats of the Sines of 1, 2, 3 Minutes, &c. on the Line of Sines. And again, becauſe the Seat of the Sine 5° 44′ 21″, whoſe Logarithm is 9.0000, correſponds with 1, the Point of Commencement of the ſecond Line of Numbers on the ſame Scale, if from this Point you ſet off 2396, 5340, 6989, &c. equal Parts, the Logarithmic Sines of 10, 20, 30, &c. Degrees, you will have the Places of the Sines of 10, 20, 30, &c. Degrees on the ſame Scale; and after the ſame manner you may find the Seats of the Sines of every Degree and Minute of the Quadrant, if the Scale will admit of it.

Again,

Again, By taking out the Artificial Tangent belonging to the ſeveral Arches of the Quadrant, rejecting the Indexes, and obſerving the ſame Laws as you are directed to do in forming the Line of Sines: You may by that Means form the Line of Tangents upon the Inſtrument.

The Line of Verſed Sines is readily formed from the Line of Sines; for by laying the Diſtance between the Sine of 10 Degrees, and the Sine of 90° 00′ twice from the Beginning of the Line of Verſed Sines, you will have the Verſed Sine of 160 Degrees, as will by ſetting off the Diſtance between the Sine of 15 and the Sine of 90° 00′ twice from the Beginning of the Line of Verſed Sines, give the Sine of 150: In like manner, the Diſtance between the Sine of 20 and the Sine of 90° 00′ ſet off twice from the ſame Point of Commencement, will give the Verſed Sine of 140; and the Diſtance between 30, and 90° 00′ upon the Line of Sines ſet off twice, will give the Verſed Sine of 120 Degrees. Again, the double of the Diſtance between the Sine of 45, and the Sine of 90° 00′ twice ſet off will give the Verſed Sine of 90° 00′; and the Diſtance between the Sine of 60 Degrees, and the Sine of 90 twice ſet off will give the Verſed Sine of 60 Degrees, *&c.* By which means the whole Lines of Verſed Sines may be divided into as many Parts as you pleaſe.

The Meridian Line upon the Scale may be formed after this Manner:

Having pitched upon a convenient Size for the Breadth of an equatorial Degree, reduce the Meridional Parts ſtanding againſt any Degree and Minute in the Tables of Meridional Parts into Degrees and Minutes, by dividing the Meridional Parts by 60, and the Degrees and Minutes anſwering to this Quotient taken off the Scale of equal Degrees, and ſet off from the Beginning of the Meridional Line, will give the proper Diviſion. For *Example*, If 31° 28′, the Degrees and Minutes in 1884, the Meridional Parts anſwering to 30 Degrees in the Table, be taken off the Scale of equal Parts or Degrees, and ſet off in the Meridian Line upon the Scale from the firſt Point, it will give the Point againſt which ſtands the Number 30 in the ſame Scale.

Again, If 43° 42′, the Degrees and Minutes in 2622, the Meridional Parts of 40 Degrees; or 57° 54′, the Degrees and Minutes in 3474, the Meridional Parts of 50 Degrees; or 75° 27′, the Degrees and Minutes in 4527, the Meridional Parts of 60 Degrees, be

be taken each off the Scale of equal Parts or Degrees, which is placed next below it on the *Gunter's* Scale, and ſet off in the Meridian Line of the Scale from the Point of Commencement, they will mark out the ſeveral Diviſions of 40 Degrees, 50 Degrees, and 60 Degrees, in the ſame Scale; and thus may the whole Meridian Line be formed.

The Inſtrument thus formed is uſeful in every Branch of the *Mathematicks* wherein Numbers are concerned, and is very valuable for its great Facility in its ſeveral Solutions, and in many Caſes gives Anſwers ſufficiently exact for the Purpoſes: For if you extend the Compaſſes from the firſt Number to the ſecond, the ſame Extent will reach from the third Number to the fourth; or if you extend from the firſt Number to the third, the ſame Extent will reach from the ſecond Number to the fourth, in all Kinds of Proportionals whatſoever.

Having cleared the Way hitherto, and opened a Path into the Knowledge of the ſucceeding Part, I ſhall now proceed on with my Deſign, not doubting but if what has been ſaid be but tolerably underſtood, the Reaſon of every thing that follows will be very obvious to the Learner.

SECT. VI.

THE Nature and Conſtruction of the Sines, Tangents and Secants Natural and Artificial, as well as of the Logarithms themſelves, having been fully inquired into and explained in the two preceding Sections, their Uſe and Application to the Solution of the ſeveral Caſes in *Plain Trigonometry*, will be very intelligible; eſpecially if it be premiſed, that the Tables of Sines, Tangents and Secants are Tables of Triangles ready calculated; that is, If the Baſe of a Right-angled Plain Triangle be conſidered as made up of 10000, *&c.* equal Parts, the Natural Tangent of the Degrees, *&c.* anſwering to the Quantity of the Angle at the Baſe, will be the Length of the Perpendicular; as will the Secant of the ſame Degrees, *&c.* be the Length of the Hypothenuſe of the ſame Triangle. In like manner, if the Hypothenuſe of a Right-angled Plain Triangle be conſider'd equal to the Radius of the Canon, or made up of 1000, *&c.* equal Parts, that the Natural

Sine

Sine of the Angle at the Baſe will be the Length of the Perpendicular, and the Natural Co-ſine of the ſame Angle will be the Length of the Baſe: So that no Triangle can be propoſed, but that a Triangle ſimilar to it may be found among the Tables of Sines, *&c.* all the Sides of which are known or given: Whence, and by the Help of the ſeveral *Trigonometrical Axioms*, all the Caſes that can poſſibly ariſe in the Doctrine of *Plain Triangles* will be readily anſwered, as will appear by the following Solutions.

Caſe the 1ſt.

The Angles and one of the Legs being given, to find the other Leg.

Or,

The Baſe and Angle at Baſe being given, to find the Perpendicular.

Example.

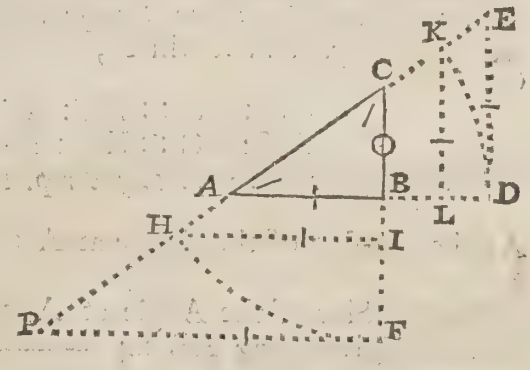
Given $\left\{\begin{array}{l} AB\ 64\ 00 \\ <A\ 36^{\circ}\ 52'\ 12'' \end{array}\right\}$ BC required.

1ſt *Geometrically.*

1. Make AB equal to 64 by a Line of equal Parts.

2. Erect the Perpendicular BC.

3. Make the Angle at A equal to 36° 52′ 12″, and produce the Line AC, till it meet the Perpendicular in C.



4. Then is the Triangle determined, and BC being meaſured by the Line of equal Parts will give its Length.

But if a more exact Anſwer be required in Numbers, produce AB to D, till AD be equal to the Radius of the Canon; then will DE be the Tangent of the Angle A, *by Ax. the 1ſt of Trigonometry*, and therefore given in the ſaid Tables; and becauſe the Triangles ABC and ADE are ſimilar, it will be *by Theor. the 15th.*

1ſt. AD : DE :: AB : BC. *i. e.*

As the Radius ——— 10.0000000

To the Tangent of Angle at A 36° 52′ 12″ —— 9.8750628
So is the given Leg AB 64 ——— 1.8061800

To the required Leg BC 48 ——— 1.6812428

Or,

Produce CB to F, till CF be equal to AD, then will FP be the Tangent of the Angle at C, (*by Axiom the 1st of Trig.*) and it will be (*by Theorem the 15th.*)

2d PF : FC : : AB : BC. *i. e.*

As the Co-tangent of the Angle at A 36° 52′ 12″ —— 10.1249372

To the Radius ——— 10.0000000
So is the given Leg AB 64 ——— 1.8061800

To the required Leg BC ——— 1.6812428

Or,

From the Points H and K, let fall the Perpendiculars HI and KL, which will be the Sines of their oppoſite Angles, (*by Ax. the 1ſt of Trig.*) and it will be,

3d { CH : CA : : HI : AB } *by Theor. the 15th.*
{ AK : AC : : KL : BC }

Wherefore, (*by Cor. 1. of Prop. 12.*) HI : AB : : KL : BC.

As the Co-ſine of the Angle at A 36° 52′ 12″ —— 9.9030895

To the Sine of the Angle at A 56° 52′ 12″ —— 9.7781523
So is the given Leg AB 64 ——— 1.8061806

To the required Leg BC 48 ——— 1.6812434

Caſe the 2d.

One of the Legs and the Angles being given, to find the Hypothenuſe.

Or,

The Baſe and Angle at Baſe being given, to find the Hypothenuſe.

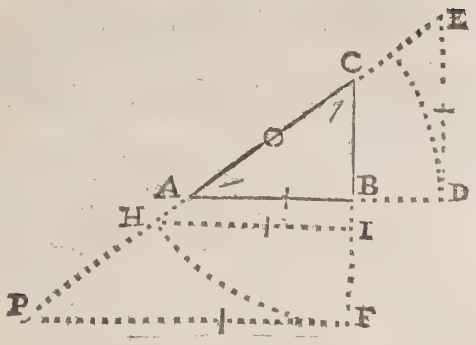
Example.

Example.

Given $\left\{\begin{array}{l} AB = 64 \\ \angle A\ 36^\circ\ 52'\ 12'' \end{array}\right\}$ AC required.

This Caſe is conſtructed like the former; and if AC be applied to a Line of equal Parts, its Length will be known.

And becauſe DE is the Tangent, AE the Secant of the Angle at A; alſo PF the Tangent, PC the Secant, and HI the Sine of the Angle at C, to the ſame Radius, *by Ax. the 1ſt of Trig.* it will be, *by Theor. the 15th.*



1ſt DA : AE :: BA : AC. *i. e.*

As the Radius	10.0000000
To the Secant of the Angle at 36° 52′ 12″	10.0969107
So is the given Leg AB 64	1.8061800
To the Hypothenuſe AC 80	1.9030907

Or,

2d. IH : HC :: BA : AC. *i. e.*

As the Sine of the Angle at C 53° 07′ 48″	9.9030895
To the Radius	10.0000000
So is the Leg AB 64	1.8061800
To the Hypothenuſe AC 80	1.9030905

Or,

3d. PF : PC :: AB : AC. *i. e.*

As the Tangent of the Angle at C 53° 07′ 48″	10.1249372
To the Secant of the ſame Angle 53° 07′ 48″	10.2218477
So is the given Leg 64	1.8061800
To the Hypothenuſe 80	1.9030905

Case the 3d.

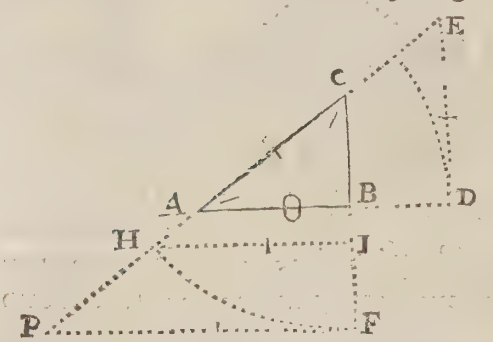
Given the Hypothenuse and Angles, to find one of the Legs.

Or,

The Hypothenuse and Angle at the Base being given, to find the Base.

Example.

Given $\left\{\begin{array}{l} AC = 80 \\ \angle A = 36^\circ\ 52'\ 12'' \end{array}\right\}$ AB required.



1st *Geometrically.*

1. Draw the Line AB at pleasure.

2. Make the Angle at A equal to 36° 52′ 12″, and continue the Line AC till it be equal to 80.

3. Let fall the Perpendicular CB, then is the Triangle compleat.

4. And AB the Leg required; and if measured by a Line of equal Parts, you will have its Length.

But to determine the Length by Calculation, let there be the same Preparation, as in the former Case, and it will be (*by Theor. the 15th.*)

1st. CH : HI :: CA : AB. *i. e.*

As the Radius	10.0000000
To the Sine of the Angle at C 53° 07′ 48″	9.9030895
So is the Hypothenuse 80	1.9030900
To the Leg required 64	1.8061795

2d. AE : AD :: AC : AB. *i. e.*

As the Secant of the Angle at A 36° 52′ 12″	10.0969107
To the Radius	10.0000000
So is the Hypothenuse 80	1.9030900
To the Leg sought 64	1.8061793

Or,

Or,

3d. CP : PF : : CA : AB. *i. e.*

As the Co-ſecant of the Angle at A 36° 52′ 12″ —— 10.2218477

To the Co-tangent of the ſame Angle 36° 52′ 12″ 10.1249372
So is the Hypothenuſe 80 —— —— —— —— —— 1.9030900

To the Leg ſought 64 —— —— —— —— —— —— 1.8061795

Caſe the 4th.

The Legs being given, to find the Angle.

Or,

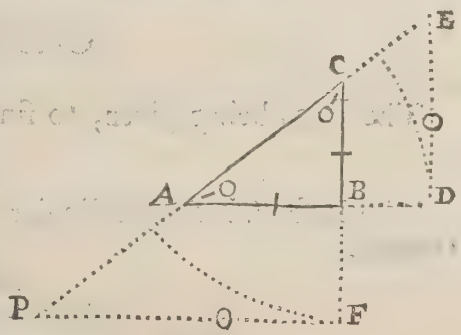
The Baſe and Perpendicular being given, to find the Angles.

Example.

Given $\left\{\begin{matrix} AB=64 \\ BC=48 \end{matrix}\right\}$ Angles at A and C requir'd.

1. Let AB be made equal to 64 of a Line of equal Parts.

2. On B, raiſe the Perpendicular BC, and lay off upon it 48 Parts of the ſame Scale from B to C.



3. Join the Points A and C by the Right Line AC, and the Triangle will be formed. And if the Angles at A and B be meaſur'd by a Line of Chords, their Quantities will by this means be known: But to find the ſame Angles by Calculation, produce AB to D, till AD be equal to the Radius of Canon; then will DE be the Tangent of the Angle at A, (*by Ax. the* 1*ſt of Sect. the* 2*d*) and it will be

AB : BC : : AD : DE, (*by Theor. the* 15*th*) *i. e.*

As the Leg AB 64 —— —— —— —— —— —— 1.8061800

To the Leg BC 48 —— —— —— —— —— —— 1.6812412
So is the Radius —— —— —— —— —— —— —— 10.0000000

To the Tangent of the Angle at A 36° 52′ 11″ —— 9.8750612

Whoſe

Whose Compliment to a Quadrant is the Angle at C = 53° 07′ 48″ ½, (*by the 3d Corol. of the 4th Theor.*)

Or,

If CF be made equal to AD, then will FP be the Tangent of Angle at C, (*by Ax. the 1st of Trig.*) and it will be,

BC : AB : : CF : FP. (*by Theor. the 15th.*)

As the given Leg BC 48	1.68124121
To the given Leg AB 64	1.8061800
So is the Radius	10.0000000
To the Tangent of the Angle at C 53° 07′ 48 ½	10.1249388

Which being subtracted from 90° 00′, leaves the Angle at A = 36° 52′ 11″ ½, as before, (*by the 3d Corol. of the 4th Theor.*)

Case the 5th.

The Legs being given, to find the Hypothenuse,

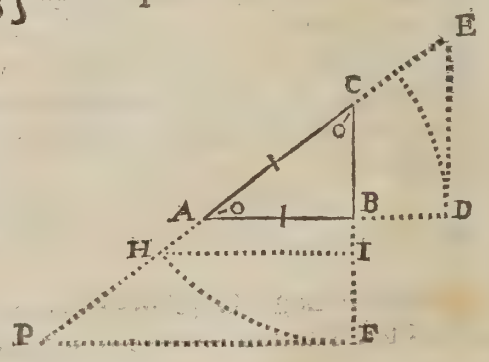
Or,

The Base and Perpendicular being given, to find the Hypothenuse.

Example.

Given $\left\{\begin{matrix} AB = 64 \\ BC = 48 \end{matrix}\right\}$ AC required.

The Geometrical Construction of this Case is, in all Respects, the same with the former: And if AC be measured by the same Scale, by which the Triangle was projected, its Length will be known.



And if an Answer be required by Numbers,

1*st.* Find the Angles by *Case the 4th.*

2*d.* Then the Hypothenuse by *Case the 2d.*

Or,

Or,

To the Square of one of the given Sides AB = 64 = —— 4096
Add the Square of the other Side BC = 48 = —— —— —— 2304

The Square Root of that Sum —— —— —— —— —— 6400

will be equal to the Side AC = 80.

Or,

From twice the Log. of the greater Side AB = 64 = 3.6123600
Subtract the Log. of the leſſer Side BC = 48 = 1.6812412

The abſolute Number anſwering to the Rem. is 85.3332 1.9311188

To this Number 85.3332, add the leſſer Side 48, and the Sum will be 133.3332.

To the Log. of this laſt Sum 133.3332 —— —— 2.1249379
Add the Log of the leſſer Side BC = 48 = —— —— 1.6812412

Half the Sum of theſe two Logarithms —— —— —— 3.8061791

will be the Log. of the Side required AC = 80 = —— 1.9030895

For put H for the Hypothenuſe, b for one of the Sides, and p for the other, then $HH = pp + bb = \frac{pp+bb}{b} \times b = \frac{pp}{b} + b \times b$,

whence $H = \frac{\sqrt{\frac{pp}{b}}}{b} + b \times b$.

Caſe the 6th.

The Hypothenuſe and one of the Legs being given, to find the Angles.

Or,

The Baſe and Hypothenuſe being given, to find the Angles.

Example.

Given $\left\{\begin{matrix} AC = 80 \\ AB = 64 \end{matrix}\right\}$ <at A and C required.

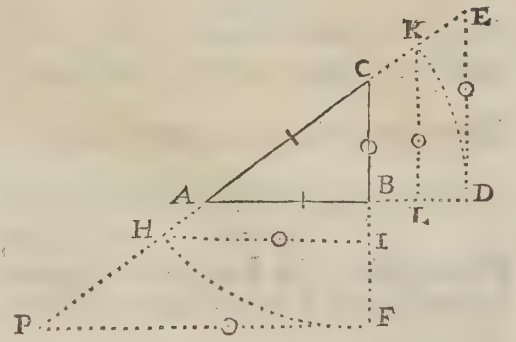
Geome-

Geometrically.

1. Let AB be made equal to 64, by a Line of equal Parts.

2. Erect the Perpendicular BC.

3. With the Diſtance of 80 between the Points of the Compaſſes, ſetting one Foot in A, with the other croſs the Perpendicular in C.



4 Draw the Line AC, then is the Triangle projected; and the Angles may be meaſured, as before, by a Line of Chords.

But in order to find the Angles Logarithmetically, make AD and CH equal to the Radius of the Canon; then will AE be the Secant of the Angle at A, and HI the Sine of the Angle at C, (*by Ax the 1ſt of Trig.*) and it will be,

1ſt. AB : AC : : AD : AE. (*by Theor. the 15th*) that is,

As the given Leg 64	1.8061800
To the Hypothenuſe 80	1.9030900
So is the Radius	10.0000000
To the Secant of the Angle at A 36° 52′ 11″ ½	10.0969100

Whoſe Defect to a Quadrant is the Angle at C=53° 07′ 48″ ½.

Or,

2d. AC : AB : : HC : HI. *i. e.*

As the Hypothenuſe 80	1.9030900
To the Leg given 64	1.8061800
So is the Radius	10.0000000
To the Sine of the Angle at C 53° 07′ 48″ ½	9.9030900

Which being ſubtracted from 90°, will give the Angle at A = 36° 52′ 11″ ½, as before, (*by Cor. the 3d of Prop.* 4.)

Caſe

Caſe the 7th.

The Hypothenuſe and one of the Legs of a Right-angled Plain Triangle being given, to find the other Leg.

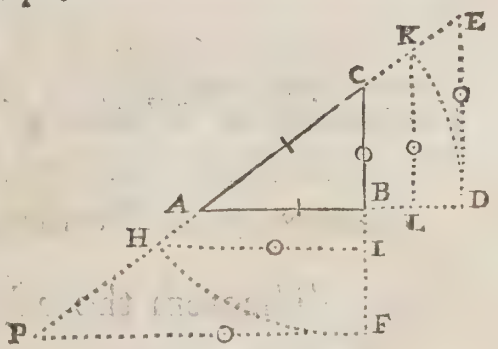
Or,

The Baſe and Hypothenuſe being given, to find the Perpendicular.

Example.

In the Triangle ABC, Right Angle at B,

Are given { AC=80 / AB=64 } req. BC.



The Geometrical Conſtruction of this Caſe, is the ſame with the fifth, and the Length of BC, may be meaſured by a Line of equal Parts.

This Caſe is likewiſe a Compound of the *6th* of the *1ſt*, and of the *3d*; for the Angles A and C being found by the *6th*, the Perpendicular BC may be found ſix different Ways, as in the laſt, *viz.* three Ways by the Help of the Angles, and the Baſe or Leg AB, as in the firſt Caſe; and three other Ways by the Help of the Angles and the Hypothenuſe, as in the third Caſe; all which I ſhall omit for Brevity-ſake, ſince the intelligent Reader may eaſily ſupply them himſelf.

The Perpendicular BC may likewiſe be found by *Prop. the 11th*, thus:

From the Square of the Hypothenuſe AC 80= — — —	6400
Take the Square of the Baſe AB 64= — — — —	4096

The Square Root of the Remainder will be 48, the Perpendicular.

Or,

To the Logarithm of the Sum of the Hypothenuſe and Baſe, or given Leg AB 144 — — — —	2.1583625
Add the Log. of their Diff. = 16 — — — —	1.2041200
Half the Sum of theſe two Log. will be the Log.	3.3624825
of the Perpendicular 48 — — — —	1.6812412

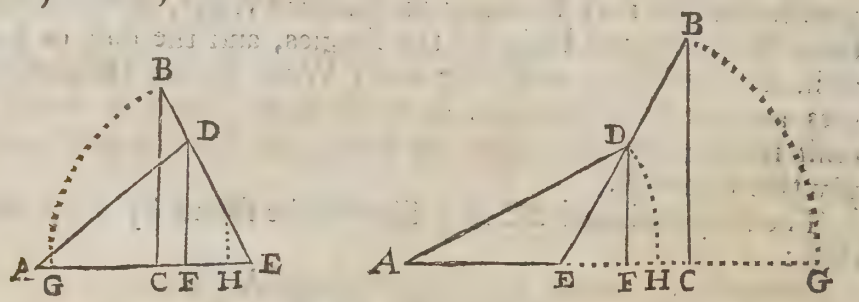
N. B. *Those Proportions in which the Radius is not concerned, are most easily work'd, by taking the Arithmetical Complement of the first or uppermost Number, which is what it wants of* 10.0000000, *for then the Sum of the three Terms will be the Logarithm of the fourth required.*

SECT. VII.

Containing a Solution of the Six Cases of Oblique-angled Plain Triangles, together with a Demonstration of the Fundamental Rules.

Theorem the 2d, *or* Axiom the 2d.

In all Plain Triangles,
The Sides are to each other, as are the Sines of their oppofite Angles; that is,



In the Triangle ADE it will be, AD : DE : : *S.* Angle E : *S.* Angle A.

Constr. Produce ED to B (*by the 1ſt Poſt.*) make EB=AD, and let fall the Perpendiculars BC and DF (*by Cor. 7. Prop. 6.*) which will be Sines of their oppoſite Angles E and A (*by Ax. the 1ſt.*) to the Radius EB=AD: Wherefore (*by Prop. the 15th*) it will be,

As EB : ED : : BC : DF ; that is,
As AD : ED : : *S.* ∠E : *S.* ∠A.

This Axiom is uſeful in the Solution of the three firſt Caſes of Oblique Angled Plain Triangles, and may be applied to all the Caſes in Plain Trigonometry, except that wherein the two Sides and the Angle contained are Given, and the three Sides to find the reſt.

Theorem

Theorem the 3d, *or* Axiom the 3d.

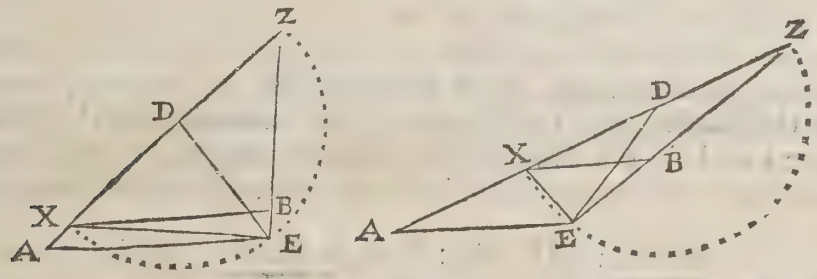
In all Plain Triangles it will be,

As the Sum of any two Sides, is to their Difference, ſo is the Tangent of half the Sum of their oppoſite Angles, to the Tangent of half their Difference; *or*, of the Difference of either of them from the half Sum.

Now the Arch of half Difference added to the half Sum will give the greater of the two Angles, and ſubtracted from half the Sum, will leave the leſſer.

That is, in the Triangle ADE, it will be,

1ſt. AD+DE : AD−DE : : T. $\frac{1}{2}$<ˢ E+A : T. $\frac{1}{2}$<ˢ E−A.



Conſtr. On D as a Center, with the Diſtance of DE, deſcribe the Circle XEZ (*by the 3d Poſt.*) produce AD to Z, (*by the 1ſt Poſt.*) join the Points X and E (*by the 2d Poſt.*) and draw XB parallel to AE (*by the 8th Cor. of Prop. the 5th.*)

Demonſtr. Becauſe D is the Center of the Circle XEZ; therefore are DX, DE, DZ, equal (*by the 15th Def.*) whence AZ is equal to AD+DE (*by the 2d Ax.*) and AX equal to AD−DE (*by the 3d Ax.*)

Again, Becauſe XZ is the Diameter of the Circle XEZ, therefore the Angle XEZ is Right (*by the 3d Cor. of Prop. 9.*) and EZ the Tangent of the Angle EXZ (*by the 1ſt Ax.*) = XED (*by the 6th Prop.*) = $\frac{1}{2}$ EDZ (*by the 9th Prop.*) = $\frac{1}{2}$<ˢ A+E (*by the 1ſt Ax.*) to the Radius XE, and EB is the Tangent of the < EXB (*by the 1ſt Ax.*) = XEA (*by the 3d Prop.*) to the ſame Radius.

And becauſe XB is parallel, AE, it will be (*by the 14th Prop.*)

As ZA : XA : : ZE : BE; that is,

As AD + DE : AD − DE : : T $\frac{1}{2}$ <s A+E : T. < EXB = (*by the 3d Prop.*) < XEA, the Arch of Difference of either from the half Sum

Now, DXE − EXB = (*by the 3d Ax.*) DXB = (*by the 3d Prop.*) DAE and DEX + XEA = (*by the 2d Ax.*) AED.

In such Cases where the Logarithms of two containing Sides are given (which frequently happens, and particularly in the Case for finding the Bearing and Distance between any two inaccessible Places) the following Theorem is very applicable, being easier for Practice, and producing a truer Answer than the former.

Theorem the 4th, *or* Axiom the 4th.

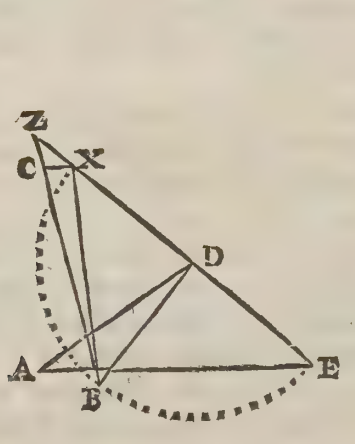
In all Plain Triangles it will be,

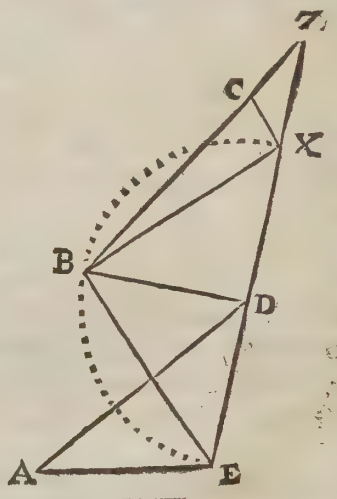
As the lesser of the two Sides, is to the greater, so is the Radius to the Tangent of an Arch, from which taking 45 Degrees, it will be,

As the Radius, is to the Tangent of the remaining Arch; so is the Tangent of half the Sum of the opposite Angles, to the Tangent of half their Difference; that is, in the Triangle ADE, it will be,

As DE : AD : : R : T of an Arch. Again,

As R : T Arch − 45deg. : : $T\frac{1}{2}$ <s A+E : $T\frac{1}{2}$ <s E − A.





Constr.

Conſtr. On D as a Center, with the Diſtance of DE, deſcribe the Semi-circle EBX (*by the 3d Poſt.*) ED being firſt produced to Z, till DZ=DA. Let DB be perpendicular to XE (*by the 6th Cor. of the 6th Prop.*) and draw the Lines BE, BX (*by the 2d Poſt.*) which will be equal (*by the 1ſt Cor. of Prop.* 6.) from X draw XC, perpendicular to BX (*by the 6th Cor. of the 6th Prop.*) then will the Triangles BZE, CZX be ſimilar (*by Prop. the 3d and 4th.*)

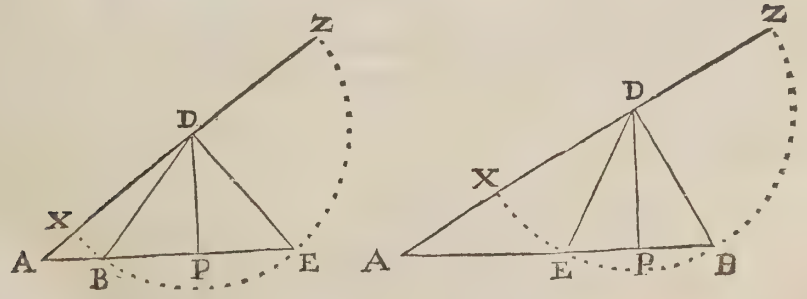
And becauſe the Triangle ZBD is Right-angled at D, it will be (*by the 1ſt Ax.*) (DB=) DE : (DZ =) DA : : R : T. ZBD.
Now ZBD—XBD (=45 Degrees) (*by Cor. the 4th of Prop.* 4.) =CBX. Wherefore,

As R : T. $\angle$ CBX : : (BX=) BE : CX (*by Ax. the 1ſt*) : : EZ : XZ (*by the 14th Prop.*) : : T $\frac{1}{2}$ $\angle^{s}$ E+A : T $\frac{1}{2}$ $\angle^{s}$ E—A (*by the precedent Theorem.*)

Theſe two Axioms *are uſeful in the Solution of the 4th and 5th Caſes of* Oblique-angled Plain Triangles, *where two Sides and the Angle contained are given, to find the reſt.*

Theorem the 5th, *or* Axiom the 5th.

In every Plain Triangle it will be,
As the Baſe, is to the Sum of the other two Sides; ſo is their Difference to the Difference, of the Segments of the Baſe; made by a Perpendicular, let fall from the oppoſite Angle when it falls within, or to the alternate Baſe when it falls without.



That is, in the Triangle ADE, it will be,
As AE : AD + DE : : AD — DE : AB.

Conſtr.

Constr. On D as a Center, with the Distance of DE, describe the Circle XBEZ (*by the 3d Post.*) produce AD to Z (*by the 2d Post.*) join the Points D and B (*by the 1st Post.*) and let fall the Perpendicular DP (*by Cor. the 7th of Prop.* 6.)

Demonstr. Because D is the Center of the Circle XEZ, therefore are DX, DE, DZ, equal, and AZ = AD + DE (*by the 2d. Ax.*) and AX = AD + DE (*by the 3d Ax.*) and because the Point A is without the Circle, AE × AB = AZ × AX (*by Cor. the 2d of Prop.* 16.) Wherefore (*by Prop. the* 12*th*) it will be, AE : AZ : : AX : AB.

That is, AE : AD + DE : : AD − DE : AB, the Difference of the Segments AP and PE in the first Case, and the alternate Base AB in the second.

Now, AE ± AB = $2\left\{\begin{matrix}\text{AP}\\\text{EP}\end{matrix}\right\}$ in the first Case.

And AB ± AE = $2\left\{\begin{matrix}\text{AP}\\\text{EP}\end{matrix}\right\}$ in the second Case.

This and the following Axioms *are of Use in finding the Angles in the sixth and last Case of* Oblique-angled Plain Triangles, *where the three Sides are given.*

Theorem the 6th, *or* Axiom the 6th.

In all Plain Triangles it will be,

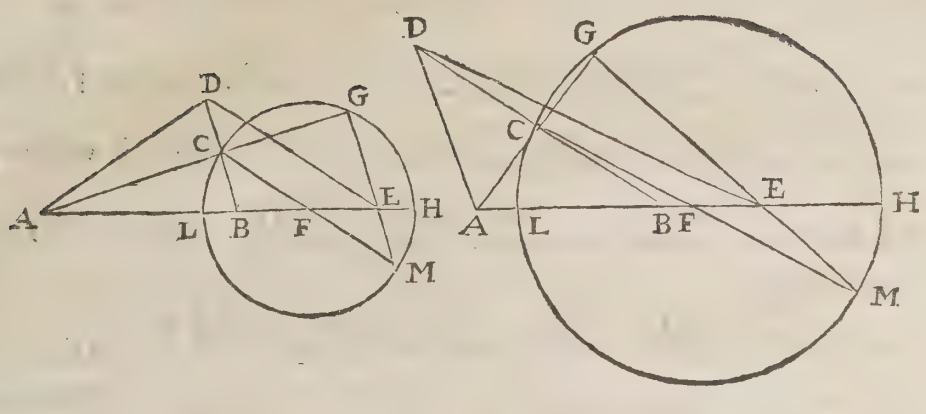
As the Rectangle, or Product of any two contained Sides, is to the Rectangle, or Product of half the Sum of the three Sides, multiplied into the Difference between half the Sum of the three Sides, and the Side opposite to the Angle contained; so is the Square of the Radius, to the Square of the Co-sine of half the contained Angle. That is,

In the Triangle ADE, it will be,

$$\text{AE} \times \text{ED} : \frac{\text{AE}+\text{AD}+\text{DE}}{2} \times \frac{\text{AE}+\text{AD}+\text{DE}}{2} - \text{DE} : : \text{Rq} : c.s.q.$$

$\frac{1}{2}$ the Angle DAE.

Constr.



Conſtruction.

1. Let the Angle DAE be biſected by the Line AC (*by Cor. the 4th of Prop.* 6.) and let DCB be drawn perpendicular to AC (*by Cor. the 7th of Prop.* 6.) then will DC=CB, and AD=AB (*by Prop. the 5th*) and conſequently BE is equal to the Difference of the Sides AE and AD.

2. From C draw CM parallel and equal to DE (*by Cor. the 5th of Prop.* 3.) then will CF = $\frac{1}{2}$ DE. (*by Prop. the 14th*) the Side oppoſite=FM, and BE=FE, equal to half the Difference of the Sides AE, AD, and conſequently AF=half their Sum.

3. Join the Points M and E, and produce ME till it meet AC produced in G, then will MEG be parallel to DCB (*by Cor. the 6th of Prop.* 3.) and conſequently the Angle at G right (*by Cor. the 3d of Prop. the 9th.*)

4. On F as a Center, with the Diſtance of CF = FM = $\frac{1}{2}$ DE, the oppoſite Side, draw the Circle LCHM, which will paſs thro' the Point G (*by Cor. the 3d of Prop. the 9th*) then will AH be equal to half the Sum of the Sides AE, AD, and the oppoſite Side DE; and AL equal to their Difference. Wherefore, (*by the 1ſt Theorem*, or *1ſt Axiom of Trigonometry*) it will be,

As AB : AC :: R : *cs* : : $\angle$ BAC = $\frac{1}{2}$ DAE (*by Conſtr.*)
And AE : AG : : R : *cs*, $\angle$ BAC = $\frac{1}{2}$ DAE (*by the ſame,*)

Wherefore (*by the 12th Prop.*) it will be,
AB × AE : AC × AG : : R*q* . *cs*, *q* $\frac{1}{2}\angle$ DAE.

But, AC×AG=AH×AL (*by Cor. 2d of Prop.* 16.) and AB=AD, by *Conſtruction*. Wherefore,

As

As $AD \times AE : AH \times AL :: Rq : cs, q\frac{1}{2} < DAE$. That is,

As $AD \times AE : \frac{AD+AE+DE}{2} \times \frac{AD+AE-DE}{2} :: Rq : cs, q.\frac{1}{2} DAE$

And because $\frac{AD+AE-DE}{2} = \frac{AD+AE+DE}{2} - DE$, it will be,

As $AD \times AE : \frac{AD+AE+DE}{2} \times \frac{AD+AE+DE}{2} - DE :: Rq : cs : \frac{1}{2} < DAE\, q$.

Put $Z = \frac{AD+AE+DE}{2}$ and $X = \frac{AD+AE+DE}{2} - DE$.

Then it will be, $AD \times AE : Rq :: Z \times X : cs, q\frac{1}{2} < A$. Wherefore, $\frac{R}{AE} \times \frac{R}{DA} \times Z \times X = cs, q\frac{1}{2} < A$. That is,

Because the Difference between the Logarithmic Radius and Logarithm of any other Number, is called the Arithmetical Complement.

1. To the *Arithmetical Complement* of one Side AE, add the *Arithmetical Complement* of the other Side AD. Also, the Logarithm of half the Sum of the three Sides, and the Logarithm of the half Sum leſſened by the Side oppoſite, half the Sum of theſe four Logarithms will be the Co-ſine of half the Angle required.

SECT. VIII.

Containing the Solution of the Six Caſes of Oblique-angled Plain Triangles.

Caſe 1.

Two Sides and an Angle oppoſite to one of them being given, to find the other oppoſite Angle.

Example.

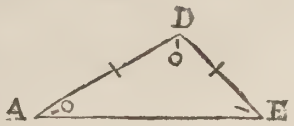
Example.

In the Oblique-angled Plain Triangle ADE,

Are given { AD = 104 — / DE = 70 — / ∠E = 44° 12′ } req. ∠s D and A.

Geometrically.

1. Having drawn the Line AE, make the Angle AED equal to 44° 12′ by the Line of Chords.



2. Make ED equal to 70, by a Line of equal Parts.

3. With the Distance of 104, between the Points of the Compasses, setting one Foot in D, with the other cross the Line AE in A, and join the Points D and A, and the Triangle is constructed; and the Angles D and A may be measured by the Line of Chords.

To find the Angles by Calculation, it will be, by the *2d Theorem* or *Axiom* of *Trigonometry*, AD : DE : : S.∠E : S.∠A. That is,

As the Side opposite to the given Angle A = 104 —	2.0170333
To the Side opposite to the Angle sought = 70 —	1.8450980
So is the Sine of the given Angle E = 44° 12′ —	9.8433335
To the Sine of the required Angle = A 27° 59′ —	9.6714003

The Angles E and A being known, the Angle at D may be found (*by Prop. the 4th*) thus;

From the Sum of the Angles A, D and E = —	180° 00′
Take the Sum of the Angles A and E = —	72 11
Remains the Angle at D = —	107 49

In this Case, if instead of the Angle E, *the Angle* A *had been given*; that is, *if the Side opposite to the given Angle had been the lesser of the two given Sides, then the Case had been doubtful, and the Angle at* D *either Acute or Obtuse, as shall be shewn at large in the 3d Case.*

Case II.

The Angles and one Side of an Oblique-angled Plain Triangle being given, to find either of the other Sides.

Example.

In the Oblique-angled Plain Triangle ADE,

Are given $\left\{\begin{array}{l}\angle A = 27^\circ\ 59' \\ \angle B = 107^\circ\ 49' \\ \angle E = 44^\circ\ 12' \\ AD = 104\end{array}\right\}$ required AE.

Geometrically.

1. Make AD equal to 104, by a Line of equal Parts.

2. Make the Angle at A equal to 44° 12′, and the Angle at D equal to 107° 49′ by the Line of Chords, and continue the Lines AE, DE, till they intersect each other in the Point E, and the Triangle is formed; and AE may be measured by a Line of equal Parts.

To determine its Length by Calculation, it will be, *by Axiom the 2d of Trigonometry*, As S.$\angle$E : S.$\angle$D : : AD : AE. That is,

As the Sine of $\angle$E 44° 12′ oppofite to the given Side	9.8433356
To the Sine of $\angle$D 107° 49′ oppo. to the req. Side	9.9786554
So is the given Side AD = 104 —— —— —— ——	2.0170333
To the required Side AE = 142.02 —— —— ——	2.1523531

After the same manner may the Side DE be found.

Case III.

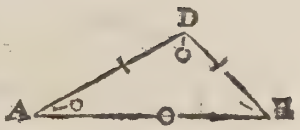
Two Sides and an Angle oppofite to one of them being given, to find the other Side.

In the Oblique-angled Plain Triangle ADE,

Are given $\left\{\begin{array}{l}AD = 104 \\ DE = 70 \\ \angle E = 44^\circ\ 12'\end{array}\right\}$ required AE.

The

The *Geometrical* Conſtruction of this Caſe is exactly the ſame with Caſe the firſt, and AE may be meaſured by a Line of equal Parts.



To find the Angle D it will be as in the firſt Caſe, *viz.*

As AD : DE : : S. Angle E : S. Angle A. That is,

As the given Side AD = 104 —— 2.0170333

To the given Side DE = 70 —— 1.8450980
So is the Sine of the given Angle E = 44° 12′ —— 9.8433356

To the Sine of the Angle at A 27° 59′ —— 9.6714003

Which being obtained, the Angle at D will be found to be 107° 49′, (*by Prop. the 4th.*)

Wherefore to find AE, it will be,

As S. ∠ E : S. ∠ D : : AD : AE. That is,

As the Sine of the Angle E 44° 12′ —— 9.8433356

To the Sine of the Angle D = 107° 49′ its Suppl. 72° 11′ 9.9786554
So is the given Side AD = 104 —— 2.0170333

To the required Side AE = 142.62 —— 2.1523531

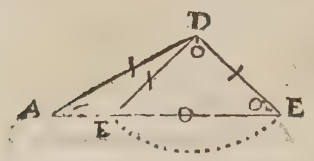
If inſtead of the Angle E *the Angle* A *had been given, the Sides* AD, DE *remaining the ſame, the Caſe had been doubtful; and the third Side* AE *would have had two different Values, as the following Example will ſufficiently ſhew.*

Caſe III.

Example 2.

In the Oblique-angled Plain Triangle ADE,

Are given { AD = 104, DE = 70, ∠A = 27° 59′ } required AE.



The Ambiguity of this Caſe appears from the *Conſtruction* itſelf; for having formed the Angle A, and ſet off AD from A to D, if with the Diſtance of DE, one Foot of the Compaſſes being placed in D, the Arch EE be drawn, it will cut AE in the Points E and E, on the ſame Side of the Point A; and conſequently the Angle D as well as A will have two different Values, and to find each of which it will be, *by the 2d Theorem or 2d Axiom of Trigonometry*,

As DE : AD : : S. $\angle$ A : S. $\angle$ E. That is,

As the given Side DE = 70 —— —— ——	1.8450980
To the given Side AD = 104 —— —— ——	2.0170333
So is the Sine of the Angle A = 27° 59′ —— ——	9.6713716
To the Sine of the Angle E = 44° 12′ —— ——	9.8433069

The Meaſure of the Angle E in the great Triangle AED, but taken from 180° 00′ (becauſe the Triangle EDE is Iſoſceles) leaves 135° 48′ for the Meaſure of the Angle E in the leſſer Triangle ADE; whence, and from the *4th Prop.* the Angle D of the great Triangle ADE, will be 107° 49′, but the Angle D of the leſſer Triangle ADE will be only 16° 13′; whence firſt, to find the Side AE of the great Triangle ADE, it will be,

As S. $\angle$ A : S. $\angle$ D : : DE : AE. That is,

As the Sine of the Angle A = 27° 59′ —— —— ——	9.6713716
To the Sine of $\angle$ D = 107° 49′ its Supplement = 72° 11′	9.9786554
So is the Side ED = 70 —— —— —— —— ——	1.8450980
To the required Side AE = 142.03 —— —— ——	2.1523818

2. To find the Side AE of the leſſer Triangle ADE, it will be,

As S. $\angle$ A : S. $\angle$ D : : DE : AE. That is,

As

As the Sine of the Angle A = 27° 59′ —— —— 9.6713716

To the Sine of the Angle D = 16° 13′ —— —— 9.4460250
So is the given Side ED = 70 —— —— —— 1.8450980

To the Side AE required = 41.66 —— —— 1.6197514

Hence,

If the $\angle$E be $\left\{\begin{matrix}\text{Acute}\\\text{Obtuse}\end{matrix}\right\}$ the $\angle$D is $\left\{\begin{matrix}107\ 49\\16.13\end{matrix}\right\}$ and AE is $\left\{\begin{matrix}142.03\\41.66\end{matrix}\right\}$

If instead of DE, *in the first Example of the third Case, the Side* AE *had been given, the Case would have been likewise doubtful; and* DE *would have had two different Values, as will appear from the following Example.*

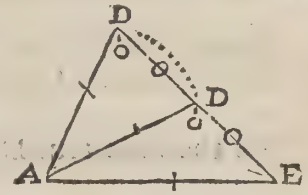
Case III.

Example 3.

In the Oblique-angled Plain Triangle ADE,

Are given $\left\{\begin{matrix}AD = 104\\AE = 142.02\\\angle E = 44°12'\end{matrix}\right\}$ required DE.

Construction.



1. Having drawn AE equal to 142.02 equal Parts, and formed the Angle E equal to 44° 12′ by the Line of Chords, take the Distance 104 the Length of AD, and placing one Foot of the Compasses in A, with the other describe the small Arch DD, which will cut the Side ED, in the Points D and D, on the same Side of the Point E, whence the different Values of the third Side DE may be found, by the Line of equal Parts; but to determine the same by Calculation, it will be,

1. As AD : AE :: S. $\angle$E : S. $\angle$D. That is,

As

As the given Side AD = 104 —— 2.0170333

To the given Side AE = 142.02 —— 2.1523495
So is the Sine of the Angle E = 44° 12′ —— 9.8433356

To the Sine of the Angle D = 72° 11′ —— 9.9786518

The Measure of the Angle at D in the great Triangle ADE, but taken from 180° 00′ (because the Triangle ADD is Isosceles) leaves 107° 49′ for the Angle D in the lesser Triangle ADE; whence the Angle A in the great Triangle will be 63° 37′, and the Angle A in the lesser 27° 59′; wherefore to find the Side DE, in the great Triangle ADE, it will be by the *2d Theorem* or *2d Ax.* of *Trigonometry*.

As S. ∠E : S. ∠A : : AD : DE. That is,

As the Sine of the Angle E 44° 12′ —— 9.8433356

To the Sine of the Angle A = 63° 37′ —— 9.9522310
So is the Side oppofite to Angle E, *viz.* AD = 104 2.0170333

To the required Side DE = 133.64 —— 2.1259287

2. To find the Side DE in the lesser Triangle ADE, it will be,

As S. ∠E : S. ∠A : : AD : DE. That is,

As the Sine of the Angle E = 44° 12′ —— 9.8433356

To the Sine of the Angle A = 27° 59′ —— 9.6713716
So is the Side AD = 104 —— 2.0170333

To the Side req. DE = 70 —— 1.8450693

Hence,

If the ∠D be {Obtuse / Acute} the ∠A is {27.59 / 63.37} and DE is {70.00 / 133.64}

In like manner, if the Sides AE, DE, *and the Angle* A, *were given, then would* AD *be found to be of two different Values; as is manifeſt from the former Conſtructions, by changing the Letters* A *and* E.

4 Caſe

Case IV.

Two Sides and the contained Angle of an Oblique-angled Plain Triangle being given, to find the other oppofite Angle.

Example.

In the Oblique-angled Plain Triangle ADE,

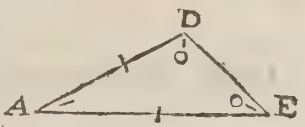
Are given { AE = 142.02, AD = 104, <A = 27° 59′ } req. Angles E and D.

Geometrically.

1. Draw the Line AE, and fet off upon it 142.02 equal Parts.

2. Make the Angle at A equal to 27° 59′ by the Line of Chords.

3. Let AD be made equal to 104 equal Parts.



4. Join the Points D and E, by the Right Line DE, and the Triangle is made; and the Angles at D and E may be measured by the Line of Chords as usually.

AE = 142.02
AD = 104
Sum = 246.02
Differ. = 38.02

The Numerical Solution of this Case depends upon the *3d Theor.* or *3d Ax. of Trigonometry*; for having found the Sum and Difference of the two Sides, fubtract the given Angle from 180°, and take half the Remainder, which will be the half Sum of the unknown Angles (*by the 4th Prop.*) and it will be by the *3d Ax. of Trigonometry*,

<sA+D+E =	180° 00′	
< A =	27 59	
Sum <sD and E =	152 01	
half Sum <s =	76.00½	
As the Sum of the two Sides AE and AD = 246.02		2.3909704
To their Difference = 38.02		1.5800121
So is the Tangent of half the Sum of the Angles at D and E = 76° 00′ 30″		10.6034981
To the Tangent of ½ their Difference, or the Diff. of either of them from the ½ Sum = 31° 48′ 28″		9.7925398

Now

Now

To the half Sum =	76°	00′	30″	From the half Sum =	76°	00′	30″
Add the half Diff. =	31	48	28	Take the half Diff. =	31	48	28
Sum greater ∠ D =	107	48	58	Rem. leſſer ∠ A =	44	12	02

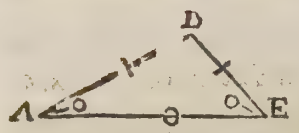
Caſe V.

Given any two Sides and the Angle contain'd, to find the third Side.

Example.

In the Oblique-angled Plain Triangle ADE,

Are given { AD = 104, ∠D = 107° 49′, DE = 70 } required AE.



Make AD equal to 104, the Angle D equal to 107° 49′, the Side DE equal to 70, and draw AE, and the thing is done; and AE may be meaſured as uſual.

This Caſe depends upon the 4*th* and 3*d* for its Solution; but for Variety-ſake, I ſhall apply the 4*th Axiom* for finding the Angles.

As the leſſer of the two Sides DE = 70 —— ——	1.8450980
To the greater Side AD = 104 —— —— ——	2.0170333
So is the Radius ——	10.0000000
To the Tangent of an Arch = 56° 03′ 23″ ——	10.1719353

From which taking away 45° 00′, there will remain 11° 03′ 23″, and it will be,

As the Radius ——— 10.0000000

To the Tangent of 11° 03′ 23″ = ——— 9.2909285
So is the Tangent of the half Sum = 36° 05′ 30″ —— 9.8627214

To the Tangent of half the Diff. = 8° 06′ 25″ —— 9.1536409

Whence the greater Angle E will be 44° 11′ 55″, and the leſſer Angle A equal to 27° 59′ 05″; whence to find the Side EA, it will be by the *2d Axiom of Trigonometry*,

As S. ∠ : A. S. ∠ D : : DE : AE. That is,

As the Sine of the Angle A = 27° 59′ 35″ ——— 9.6713914

To the Sine of the ∠D = 107° 49′ its Sup. 72° 11′ 9.9786554
So is the Side DE = 70 ——— 1.8450980

To the Side AE required, = 142.02 ——— 2.1523620

Caſe VI.

The three Sides being given, to find the Angles.

Example.

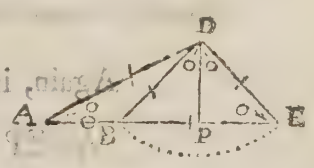
In the Oblique-angled Plain Triangle ADE,

Are given { AE = 142.02, AD = 104, DE = 70 } req. { ∠D, ∠E, ∠A }

Geometrically.

1. Make AE equal to 142, from a Line of equal Parts.

2. With 104, the Length of AD, between the Points of the Compaſſes, ſetting one Foot in A, with the other ſweep a ſmall Arch at D.



3. With 70, the Length of DE, setting one Foot in E, with the other croſs the former Arch in D.

4. From the Interſection D, to the Points A and E, draw the Lines DA, DE, and the Triangle is conſtructed; and the Angles may be meaſured by the Line of Chords as uſual.

But to determine their Quantities by Calculation,

From D let fall the Perpendicular DP, then will the Triangle ADE be reduced into two Right-angled Triangles ADP, EDP, in each of which the Hypothenuſes AD, DE are given, and the Baſes AP, PE, or Segments of the Baſe AE, may be found *by the 5th Axiom of Trigonometry.*

For,

As the Baſe AE = 142.02 ——— 2.1523495

To the Sum of the Sides AD, DE = 174 ——— 2.2405492
So is the Difference of the Sides AD, DE = 34 ——— 1.5314789

To AE the Diff. of the Segments AP, PE = 41.656 1.6196786

Which taken from AE = 142.02 leaves BE = 100.364, the half of which is 50.182 = PE, the leſſer Segment.

This again taken from AE, the whole Baſe = 142.02, leaves 91.838 = AP, the greater Segment.

The Segments AP, PE, or Baſes of the Triangles ADP, EDP, being thus found, the Angles may be eaſily had, by the *6th Caſe* of Right-angled Plain Triangles. For,

In the Triangle ADB it will be,

As AP : AD : : R : Sec. ∠E. That is,

As the Segment AP = 91.838 ——— 1.9630224

To the Side AD = 104 ——— 2.0170333
So is the Radius ——— 10.0000000

To the Secant of the ∠A = 27° 59′ 11″ ——— 10.0540109

Again, in the Triangle EDP, it will be,

As EP : ED : : R : Sec. ∠E. That is,

As

As the Segment PE = 50.182 —— —— —— —— 1.7005480

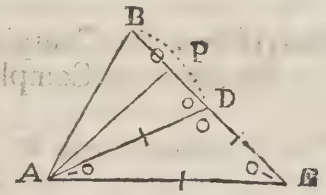
To the Side DE = 70 —— —— —— —— 1.8450980
So is the Radius —— —— —— —— —— —— 10.0000000

To the Secant of the Angle E = 44° 12′ 07″ —— 10.1445500

Now if to the Complement of the Angle A = 62° 00′ 49″ = ADP
Be added the Complement of the Angle E = 45 47 53 = EDP

The Sum will be —— —— —— —— —— —— 107 48 52 = ADE

2. The Angles A, D, E, of the Triangle ADE may be found, if the Perpendicular be let fall from any other Angle, suppose A of the Triangle ADE, upon the oppofite Side ED produced, if need be, for then the Hypothenuſes AE, AD, of the two Right-angled Triangles AEP, ADP, are given, and the Baſes EP, DP may be found, by the *5th Axiom of Trigonometry*, by saying,



As the Baſe ED = 70 —— —— —— —— —— 1.8450980

To the Sum of the two Sides AE, AD 246.02 —— 2.3909704
So is the Difference of the Sides AE, AD = 38.02 —— 1.5800121

To EB the alternate Baſe = 133.624 —— —— 2.1258845

From which taking ED = 70, the Remainder 63.624 will be DB, the half of which is DP = 31.812, the Baſe of the Triangle ADP; whence to find the Angle D, it will be by the *6th Caſe* of Right-angled Plain Triangles.

As PD : R. : : AD : Sec. ∠ D. That is,

As the external Baſe DP = 31.812 —— —— —— 1.5025910

To the Radius —— —— —— —— —— —— 10.0000000
So is the Side AD = 104 —— —— —— —— —— 2.0170333

To the Secant of the Angle D = 72° 11′ 19″ —— 10.5144423

Which taken from 180° 00′, leaves the great Angle D 107° 48′ 41″.

Again, If to the external Segment DP = 31.812, be added the Side DE = 70, the Sum 101.812 will be PE, the Base of the great Triangle APE; whence to find the Angle at E, it will be by the *5th Case*, As EP : EA : : R. Sec. : ∠ E. That is,

As the great Base EP 101.812 — — — — — —	2.0077990
To the Side AE = 142.02 — — — — — — — —	2.1523495
So is Radius — — — — — — — — — — — —	10.0000000
To the Secant of the Angle E = 44° 12′ 07″ — —	10.1445505

Now, if from the Complement of the ∠E = 45° 47′ 53″ = ∠EAP
Be taken the Complement of the ∠D = 17 48 41 = ∠DAP

The Remainder will be — — — — — — 27 59 12 = ∠DAE

3. *After the same manner, and by Help of the fifth* Axiom, *and sixth* Case, *the Angles may be found by letting the Perpendicular fall from the Angle* E, *upon* AD *produced, if need be, but this the intelligent Reader will easily supply himself.*

By the help of the *1st Cor.* of the *6th Trigonometrical Axiom*, may any one Angle as D, of the Triangle ADE, be more readily investigated, after the following Manner,

From half the Sum of the three Sides 158.01, subtract AE the Side opposite to the Angle required, and keep the Remainder 15.19. Then,

To the *Arithmetical Complement* of AD = 104, one of the Sides containing the Angle required	7 9829667
Add the *Arithmetical Complement* of DE = 70, the other containing Side — — — — — —	8.1549020
Also the Logarithm of the half Sum 158.1 — — —	2.1986846
And the Logarithm of the Remainder 15.99 — — —	1.2038485
Half the Sum of these four Logarithms — — — —	19 5404018
Will be the Co-sine of 53° 54′ 20″ ½ — — — —	9.7702009

Which being doubled, gives 107° 48′ 41″, the Angle D required.

The

The Angle D being obtain'd, the Angles A and E may be found by *Case* the 1*st*.

5. By the Help of the same *Axiom* may the Angle D be investigated, after the following Manner,

As the Log of half the Sum of the three Sides 158.01	2.1986846
To the Log. of one of the cont. Sides AD 104 —	2.0170333
So is the Log. of the other contained Side DE 70 —	1.8450980
To a fourth Logarithm — — — —	1.6634467
As this fourth Logarithm — — — —	1.6634467
Is to the Logarithm of the Remainder 15.99 —	1.2038485
So is the Radius — — — —	10.0000000
To a seventh Logarithm — — — —	9.5404018
To which add the Radius — — — —	10.0000000
Half that Sum, *viz.* — — — —	19.5404018
Will be the Co-sine of 53° 54′ 20″ $\frac{1}{2}$ = $\frac{1}{2}$ <D —	9.7702009

Therefore <D = 107° 48′ 41″.

6. By the Help of the same *Axiom* may any one Angle, suppose A of the Triangle ADE, be readily found after the following Manner,

As the Log. of the greater of the 2 cont. Sides AE 142.2	2.1523495
To the Log. of half the Sum of the three Sides 158.01	2.1986846
So is the Log. of half the Sum lessen'd by BC = 88.01 —	1.9445320
To a fourth Logarithm — — — —	1.9908671

Again,

Again,

As the leſſer of the two contained Sides AD = 104 — 2.0170333

To the fourth Logarithm — 1.9908671
So is the Radius — 10.0000000

To a *7th* Log. to which add the Radius — 19.9738338

Half that Sum will be the Co-ſine of 13° 59′ 35″ ½ — 9.9869169

Which being doubled gives 27° 59′ 11″, the Angle at A.

7. By the Help of the ſame *6th Axiom*, may any one Angle, ſuppoſe E of the Triangle ADE, be inveſtigated after the follow-Manner,

As the Radius — 10.0000000

To the Log. of one of the contained Sides AE 142.2 — 2.1523495
So is the Log. of the other contained Side DE = 70 — 1.8450980

To a fourth Logarithm — Neg. 6.0025525

As that fourth Logarithm — Neg. 6.0025525

To the Log. of the half Sum 158.01 — 2.1986846
So is the Log. of the Remainder 54.01 — 1.7324742

To a *7th* Logarithm, to which add the Radius — 19.9337112

Half this Sum will be the Co-ſine of 22° 06′ 03″ ½ — 9.9668556

The double of which 44° 12′ 07″ will be the ∠ at E required.

There are various other Ways for the reſolving this, as well as the former Caſes, but theſe I have ſhewn are more than ſufficient for the Purpoſe.

Each of theſe three laſt Ways may be made Uſe of, for finding the Angles by *Gunter's Scale*; for if you extend the Compaſſes from the firſt to the ſecond, *&c.* againſt the ſeventh Sine will ſtand the Verſed Sine, which gives the Angle required.

Suppoſe

Suppoſe it were required to find the Angle D, by Extention, according to the fifth Rule, proceed thus; extend the Compaſſes from the half Sum (upon the Line of Numbers) to one of the containing Sides (ſuppoſe AD) the Extent will reach from the other containing Side (DE) to a fourth Number.

Extend the Compaſſes from this fourth Number to the Remainder upon the ſame Line, the ſame Extent will reach from the Verſed Sine 00 Deg. to the Angle required upon the ſame Line of Verſed Sines, and ſo for any other.

N. B. *Before the Reader enters upon the* Navigation, *I would adviſe him to read over, and be well acquainted with, the* Circles *of the* Sphere, *the Definitions of which he will meet with in the third Part of the Book.*

PART

PART II.

Of NAVIGATION.

SECT. I.

THE great End and Buſineſs of *Navigation*, is to inſtruct the induſtrious Mariner how he may conduct his Ship the ſhorteſt Way, and in the ſhorteſt Time poſſible, between any two Places aſſigned in Paſſages Navigable.

If the Places propoſed be at no great Diſtance, but ſo that a Ship may ſail within Sight of Land or Soundings, as between *England*, *Holland*, *France*, &c. it is more properly called *Coaſting*; for the Performance of which, there is required only the *Compaſs*, *Lead*, or *Sounding-Line*; and a competent Knowledge of the Nature of the Coaſts.

But how to conduct a Ship through the wide and pathleſs Ocean, where nothing is to be ſeen for a conſiderable Time but Sky and Water, is the proper Buſineſs of *Navigation*, and ſhall be the Subject of the enſuing Treatiſe.

For the due and regular Performance of which are requiſite,

1. A true and perfect Knowledge of the Diſtance and Scituation of the Places, as to the *Latitude*, *Longitude*, *Courſe*, &c.

2. Inſtruments for Directing and Meaſuring the Ship's Way and Courſe ſteered, ſuch as the *Log-Line*, *Half Minute Glaſs*, &c.

3. Inſtruments for making Cœleſtial Obſervations, ſuch as the *Quadrant*, *Fore-ſtaff*, *Azimuth-Compaſs*, &c. to meaſure the *Heights*, *Diſtances*, and *Azimuths* of the *Sun* and *Stars*, for finding the Hour of the Day and Night, Alterations of Latitude, and correcting the Courſe, together with *Charts* or *Maps* for pricking down the Ship's Way.

4. A competent Knowledge of the Nature of *Currents*, of the *Mould* and *Trim* of the Ship, and the Sail she bears, that so due Allowance may be made for *Lee-way*.

5. Skill in the *Navigator*, so that by the Help of these he may be able to know at all Times the Place the Ship is in, how far she hath to sail, and which Way, or upon what Course, to gain her intended Port.

For the obtaining of which there are generally given,

1. The *Latitude* and *Longitude*, both of the Place you are to sail from, and of the Place you are to sail to; by the Help of which may be found,

2. Upon what Course, and how far the Ship must sail.

3. The Account of the Ship's Way obtain'd, by the *Log* and *Compass*.

4. The *Latitude* of the Place the Ship is in by Observation.

How to order, and manage these, so as to give Solutions to the most useful Nautical Problems, *shall be shewn hereafter, and to render them as easy as may be, it is necessary to explain the Terms made use of.*

Def.

1. *Latitude* is the neareſt Distance of any Place from the *Equator*, and is measured by an Arch of the *Meridian*, intercepted between the Place and the *Equator*, and therefore can never exceed 90^{o}, and takes its Denomination according as the Place is situated, either to to the *Northward* or *Southward* of the same.

Wherefore, all Places that lie at the same Distances from, and on the same Side of the *Equator*, are said to lie under the same Parallel of *Latitude*.

Corol. Whence it follows,

1. That if a Ship sail from a $\left\{\begin{matrix} North \\ South \end{matrix}\right\}$ Latitude directly $\left\{\begin{matrix} North \\ South \end{matrix}\right\}$ (which is stiled *Sailing under the same Meridian*) she increases her *Latitude* just so much as is her Distance sail'd.

Wherefore,

If it be reduced into Degrees and Minutes (by allowing 60 Miles to a Degree) and added to the Latitude the Ship sailed from, the Sum will be the Latitude the Ship is in.

Example.

If a Ship from the Latitude of 34° 30′ *North*, ſails 144 Miles directly *North*, and it be required to find the Latitude the Ship is in.

To the Latitude the Ship ſail'd from ——	34° 30′ N.
Add the Diſtance ſailed reduced ——	2 24 Nly.
The Sum will be the Latitude the Ship is in ——	36 54 N

2. If a Ship ſail from a { *North* / *South* } Latitude directly { *South* / *North* } ſhe decreaſes her Latitude juſt as much as is her Diſtance ſail'd.

Wherefore,

If it be firſt reduced and ſubtracted from the Latitude the Ship ſail'd from, the Remainder will be the Latitude the Ship is in.

Example.

If a Ship from the Latitude of 34° 30′ *North*, ſails 144 Miles directly *South*, and it be required to find the Latitude the Ship is in.

From the Latitude the Ship ſail'd from ——	34° 30′ N.
Subtract the Diſtance ſail'd ——	2 24 Sly.
There remains the Latitude the Ship is in ——	32 06 N.

If in this laſt Caſe it ſo happen, that the Diſtance ſail'd be more than the Latitude, the Ship has croſs'd the *Equinoctial Line*, and as the Sailors uſually term it, *depreſs'd one Pole, and rais'd the other*; and the Exceſs of the Diſtance ſail'd above the Latitude, is the Latitude the Ship is in, the contrary Way to what it was before.

Example.

If a Ship from the Latitude 1° 20′ *South*, ſails directly *North*, until her Diſtance ſail'd be 197 Miles, I demand the Latitude the Ship is in.

From the Diſtance ſail'd reduced ——	3° 17′ Nly.
Subtract } the Latitude the Ship { ſail'd from ——	1 20 S.
Remains } { is in ——	1 57 N.

Def.

Def. 2.

Difference of Latitude, is the nearest Distance of two Places lying under the same Meridian, or the nearest Distance of the Parallel of Latitude passing through the same, and is found,

If the Latitudes are the {same / contrary} way by {subtracting / adding} the lesser {from / to} the greater.

Example 1.

Let it be required to find the Difference of Latitude between the *Lizard* and *Cape St. Vincent*.

From / Take	the Latitude of	the *Lizard*	50° 00′ N
		Cape St. Vincent	37 00 N
The Remainder is the Difference of Latitude			13 00

Example 2.

Let it be required to find the Difference of Latitude between *Cape Mount* and *Cape St. Auguſtine*.

To / Add	the Latitude of	*Cape Mount*	5° 23′ N
		Cape St. Auguſtine	8 25 S
The Sum is the Difference of Latitude			13 48

If the Latitude of one Place be given, and the Difference of Latitude between it and a ſecond Place, to find the Latitude of the ſecond Place.

If the Latitude and Difference of Latitude are both the ſame Way, that is, both *Northerly* or both *Southerly*,

To the Latitude given, add the Difference of Latitude made, and the Sum will be the Latitude required.

Example.

Suppoſe a Ship from *Cape Verde* in the Latitude of 14 Degrees 43 Min. *North*, ſails *North Eaſt* until her Difference of Latitude be 4 Deg. 40 Min.

To the Latitude of *Cape Verde*	14° 43′ N	
Add the Difference of Latitude made	4 40 Nly.	
The Sum is the Latitude the Ship is in	19 23 N.	

If the Latitude and Difference of Latitude are contrary, that is, if one be *Northerly* and the other *Southerly*; then,

From the Latitude ſail'd from, take the Difference of Latitude made, and the Remainder will be the Latitude the Ship is in.

Example.

Suppoſe a Ship from *Cape Verde*, in the Latitude of 14° 43′ *North*, ſails *South Eaſt* until her Difference of Latitude be 4° 40′: Then,

From the Latitude of *Cape Verde*	14° 43′ N
Take the Difference of Latitude made	4 40 Sly.
There remains the Latitude the Ship is in	10 03 N

If in this laſt Caſe, the Difference of Latitude exceed the Latitude ſail'd from; then, from the Difference of Latitude made, take the Latitude the Ship ſail'd from, and the Remainder is the Latitude the Ship is in; the contrary Way to what it was before.

Example.

Suppoſe a Ship from the Latitude of 1° 20′ *North*, ſails *South Eaſt* until her Difference of Latitude be 190 Miles, or 3° 10′, and it be required to find the Latitude the Ship is in.

From the Difference of Latitude reduced	3° 10′ Sly.
Take the Latitude the Ship ſailed from	1 20 N
There remains the Latitude the Ship is in	1 50 S

Def. 3.

The *Longitude* of any Place upon the Earth, is meaſured by an Arch of the *Equator*, intercepted between the firſt *Meridian*, and the *Meridian* paſſing through the Place propoſed, or it is equal to the Angle formed by the two *Meridians*.

Def. 4.

Meridional Diſtance, or *Departure*, (which are ſynonymous Terms in *Plain Sailing*, founded upon the *Plain Chart*, whoſe eſſential Property is, that the *Meridians* are parallel to each other, and the Degrees of *Longitude* in every Part equal amongſt themſelves, or to each other; and to the Degrees of *Latitude*, which are each conſtantly repreſented by the ſame ſtrait Line) is the Diſtance of any two

two Places lying under the ſame Parallel, or the Diſtance of the *Meridians* paſſing through two Places lying under different Parallels, number'd in Degrees or Miles of the *Equator*.

Def. 5.

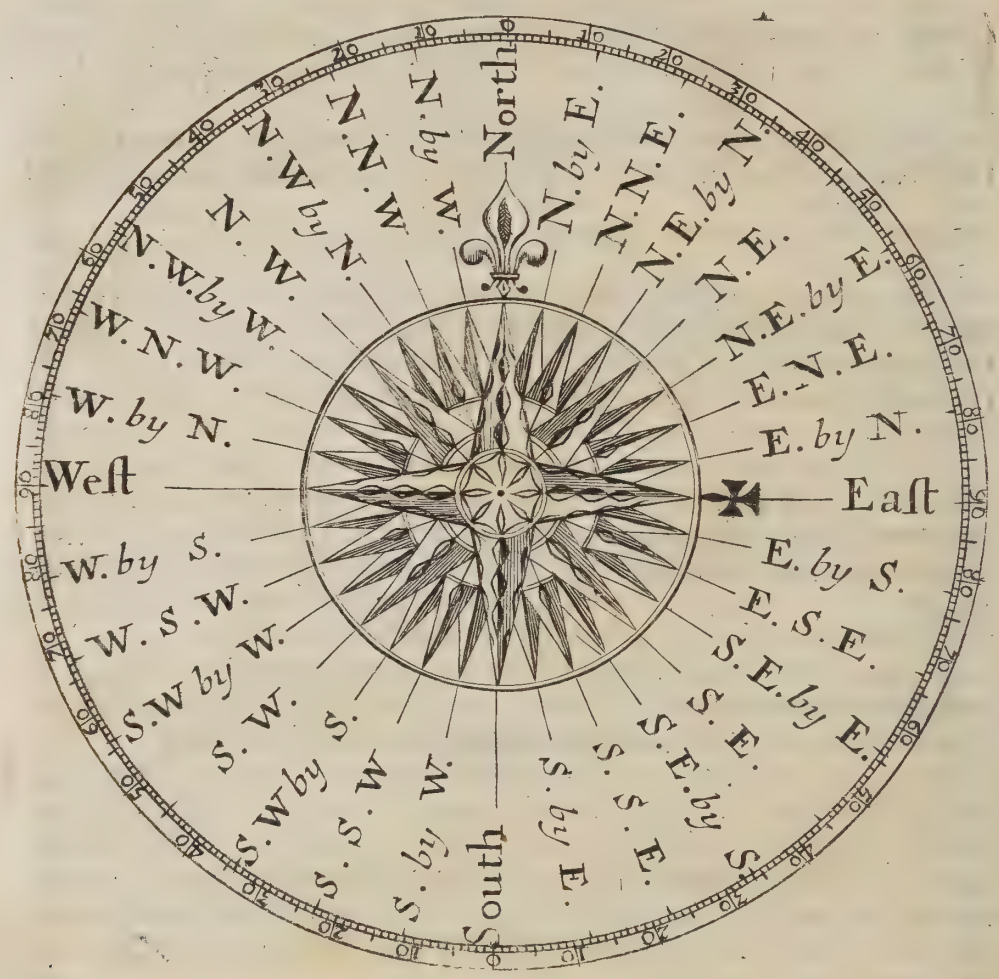
For determining the *Courſe of the Winds*, and to diſcover their *various Alterations* or *Shiftings*, the *Horizon* is uſually divided into 32 equal Parts, by Lines drawn from the Place whereon the Obſerver ſtandeth, (forming Angles with each other of $\frac{1}{32}$ of 360°, or 11° 15′) the four principal of which take their Names from the Places to which they tend, *viz.* that which extends itſelf under the *Meridian* towards the $\left\{\begin{matrix} North \\ South \end{matrix}\right\}$ is called $\left\{\begin{matrix} North \\ South \end{matrix}\right\}$ that which is on the right Hand when the Face is directed *North*, and at Right Angles to the *Meridian Eaſt*; and that which extends itſelf the contrary Way *Weſt*. The others having their Denominations compounded of the principal Lines which are on each Side of it, as may be ſeen in the following Figure: And over which ſoever of theſe Lines the *Courſe of the Wind* is directed, that Wind takes its Name accordingly. And ſince if we move never ſo little to the *Eaſtward* or *Weſtward*, we come under a new *Meridian*; and foraſmuch as all the *Meridians* muſt meet in the Poles, (according to their Nature) it follows, that the *Eaſt* and *Weſt* Line in the laſt Place will not be parallel to the *Eaſt* and *Weſt* Line in the firſt Place, but a little inclined to it, and conſequently the other intermediate Lines in each Station will be inclined to each other, (ſince they always form equal Angles with the *Meridian*.)

Now, If the Diſtances of the *Meridians* be infinitely ſmall, the infinite ſmall Parts of theſe Lines, (proper to each Alteration of Latitude) each with its correſpondent Part, will form Curve Lines upon the Globe, which Lines will form equal Angles with all the *Meridians*, and are the Rhumb Lines deſcribed upon the Globe: And the ſeveral Paths of a Ship that ſails according to the Direction of the Compaſs.

Def. 6.

The *Compaſs* is a Circle made to repreſent the *Horizon*, and divided into 32 equal Parts, by Lines drawn from the Center to the ſeveral equal Diviſions of the Circumference, repreſenting the ſeveral Rhumb-Lines upon the Globe, and by touching the Needle paſſed

paſſed on the Back-ſide with the Magnet or Load-ſtone, a certain Vertue is infuſed into it, which makes it always tend to, and hang nearly parallel with the *Meridian*, and therefore of great Uſe for directing and determining the Ship's Way, and diſcovering the *Courſe* and *Alteration of the Winds.*



Def. 7.

The *Courſe* is the Angle with the Rhumb, upon which the Ship has or muſt ſail, makes with the *Meridian*; ſo that if the Ship ſail upon the ſecond Rhumb, or N. N. E. the *Courſe* is 22° 30′; and ſo for any other, as in the following Table.

A Table

A Table of the Angles which every Rhumb (or Point of the Compass) maketh with the Meridian.

NORTH.	SOUTH.	Points.		D.	M.	NORTH.	SOUTH.
			¼	02	49		
			½	05	37		
			¾	08	26		
North by East	South by East	1		11	15	N. by W.	S. by W.
		1	¼	14	04		
		1	½	16	52		
		1	¾	19	41		
N. N. E.	S. S. E.	2		22	30	N. N. W.	S S. W.
		2	¼	25	19		
		2	½	28	07		
		2	¾	30	56		
N. E. by N.	S. E. by S.	3		33	45	N. W. by N.	S. W. by S.
		3	¼	36	34		
		3	½	39	22		
		3	¾	42	11		
North East	South East	4		45	00	North West	South West
		4	¼	47	49		
		4	½	50	37		
		4	¾	53	26		
N. E. by E.	S. E by E.	5		56	15	N. W. by W.	S. W. by W.
		5	¼	59	04		
		5	½	61	52		
		5	¾	64	41		
E. N. E.	E. S. E.	6		67	30	W. N. W.	W. S. W.
		6	¼	70	19		
		6	½	73	07		
		6	¾	75	56		
East by North	East by South	7		78	45	W. by N.	W. by S.
		7	¼	81	34		
		7	½	84	22		
		7	¾	87	11		
East.	East	8		90	0	West.	West.

Con-

Corol.

Hence it is plain, that if a Ship ſail between the *Meridian* and the *Eaſt* or *Weſt* Line, ſhe alters both her *Latitude* and *Longitude*, and that more or leſs, according as the Rhumb-Line upon which ſhe ſails is nearer to, or farther from the *Meridian*; and in order to determine the juſt Quantity of each, we ſhall premiſe what is abundantly manifeſt from what has been ſaid upon this Subject.

That the *Difference of Latitude*, *Diſtance ſailed*, and *Departure*, form a Right-angled Triangle, the *Courſe* being the Angle oppoſite to the *Departure*; and therefore from any two of theſe, how to find the reſt (by the Help of *Trigonometry*) ſhall be ſhewn in the next *Section*.

But before we conclude this *Section*, it may not be amiſs to give ſome Account of the Method uſed by our Seamen for meaſuring the Ship's Way, or how far ſhe runs in any known Time, which it done by the Help of the *Log-Line* and *Half-minute Glaſs*. The *Log* is only a Piece of Wood, which being tied to a Line and caſt over-board, and the Line veered out as faſt as the Log will carry it away, or rather as the Ship ſails from it, ſhews, according to the Times of its Veering, by the Help of certain Spaces called *Knots*, (which begin at about ten Fathom from the Log, that ſo the Log may be out of the Eddy of the Ship's Wake) how far ſhe has ſailed, and conſequently her Rate of Sailing.

And therefore to determine how many Miles ſhe ſails in an Hour, the Diſtance between Knot and Knot ought to be ſuch a proportional Part of a Sea Mile, (60 of which go to a Degree of the *Meridian*) as Half a Minute, the Time allowed for the Experiment, is of an Hour, which is $\frac{1}{120}$.

Every Degree of the *Meridian* contains (according to Mr. *Norwood*'s Obſervation, with which the *French* nearly agree) 69.54 *Engliſh* Miles, each Mile being by the Statute 1760 Yards, or 5280 Feet.

So that a Degree of the *Meridian* contains 122400 Yards, or 367200 Feet.

And conſequently the true Space between Knot and Knot ought to be 51 Feet.

So that the Diſtance between each Knot being the ſame Part of a Sea-Mile, as the *Half-minute Glaſs* is of an Hour, it follows, that ſo many Knots as ſhe ſails in a Half-minute Glaſs, ſo many Miles ſhe runs in an Hour.

But conſidering that the Ship's Way is really more than is given by the Log; and that it is more ſafe to have the Reckoning ſome-

thing

thing before the Ship, it may not be amiſs to leave out the odd Foot, and make the Space between the Knots, as Mr. *Norwood* has done, to be 50 Feet exactly, tho' our Seamen make the Diſtance but 42 Feet, thinking that 60 *Engliſh* Miles make a Degree, and by common Experience, have been forced to leſſen the *Half-minute Glaſs* about five Seconds, which is but really correcting one Blunder by another; and it were to be wiſh'd, that they would uſe the former Method, ſeeing it is in every reſpect as eaſy, and much more ſure and certain.

SECT. II.

Containing the Solution of the ſeveral Caſes of Plain Sailing, *relating to a* Single Courſe.

Caſe I.

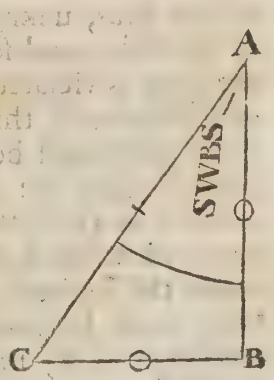
ONE Latitude, Courſe and Diſtance ſailed being given, to find the other Latitude and Departure from the *Meridian*.

Example.

If a Ship from the Latitude 50° 00′ N. ſails S. W. by S. 57.5 Miles, and it be required to find what Latitude ſhe is in, and how far ſhe hath departed from her firſt *Meridian*.

1ſt Geometrically.

Let AB repreſent the *Meridian*, A the Place ſhe departed from.



1. Make the Angle BAC = 3P = 33° 45′.

2. Continue AC till it be equal to the Diſt. ſailed 57,5 Miles, and let fall the Perpendicular CB; then will C be the Place the Ship is in, CB the Departure from the *Meridian*, and AB the Alteration of Latitude: To find either of which *Arithmetically*, tho' there are three ſeveral Ways, yet in this and the following *Caſes*, we ſhall make uſe of one only, and of that which is moſt ready for Practice.

1. To find the Departure from the *Meridian*, it will be,
R : AC : : s, < A : CB, *by Case the 3d of Sect. the 6th of Part the 1st.* That is,

As the Radius	10.0000000
To the Distance sailed 57.5	1.7596678
So is the Sine of the Course 33° 45′	9.7447390
To the Departure 31.95 Miles	1.5044068

And so much is the Ship to the *Westward* of her first *Meridian.*

2. To find the Difference of Latitude, it will be,
R : AC : : s, < C : AB, *by Case the 3d of Sect. the 6th of Part the 1st.* That is,

As the Radius	10.0000000
To the Distance sailed 57.5	1.7596678
So is the Co-sine of the Course 33° 45′	9.9198464
To the Difference of Latitude 47.81	2.6795142

3. To find the Latitude the Ship is in, because she sailed towards the *Equator.*

From the	Latitude she	sail'd from	50° 00′ N.
Subt. the Difference of		has made	0 48 here
There remains the		is in	49 12 N.

By reason sometimes of the Unsteadiness of the Winds, Swelling of the Seas, unknown Currents, setting with or against the Course of the Ship, and several other Accidents, the Estimate Distance by the Log is viciated; and consequently the Difference of Latitude and Departure thus found is naught, and not to be relied on. The Navigator must be sure therefore, so soon as the Sun or Stars appear, to find the Latitude by Observation, according to the Directions in the latter Part of this Treatise; which if found to disagree with the Latitude deduced by this Case, the Difference between the Latitude you departed from, and the last observed Latitude, must be taken: And with this true Difference of Latitude and the Course, find the Distance and Departure by the following *Case.*

Case

Case II.

The Course and Difference of Latitude being given, to find the Distance sailed and Departure.

Example.

A Ship from the Latitude 50° 00′ N. sails S. W. by S. until she be in the Latitude 49° 08′ N. I demand the Distance sailed and Distance from the *Meridian.*

Because the Ship sails from a *North* Latitude *Southerly*,

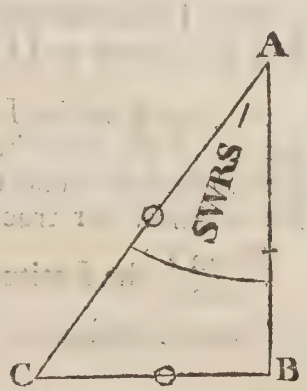
From	the Latitude the Ship sailed	from	50° 00′ N
Subtract		into	49 08 N
There remains the Difference of Latitude			0 52

Therefore,

Geometrically.

1. Make AB equal to the Difference of Latitude 52, and raise the Perpenicular BC.

2. Make the Angle BAC equal to the Course 33° 45′, and continue AC till it meet the Perpendicular in C; then is AC the Distance sailed: To find which,



Arithmetically.

It will be s, ∠ at C : AB : : R : AC, by *Case the 2d of Sect. the 6th of Part the 1st.* That is,

As the Co-sine of the Course 33° 45′	9.9198464
To the Difference of Latitude 52	1.7160033
So is the Radius	10.0000000
To the Distance sailed 62.54 Miles	1.7961569

But to find the Departure it will be, R : AB : : t, ∠ A : BC, by *Case the 1st of Sect. the 6th of Part the 1st.* That is,

As the Radius ——————————— 10.0000000

To the Difference of Latitude 52 ——— 1.7160033
So is the Tangent of the Course 33° 45′ ——— 9.8248926

To the Departure 34.74 Miles ——————— 1.5408959

And so much is she to the *Westward* of her first *Meridian.*

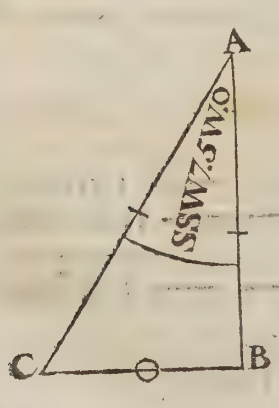
But if having no Reason to suspect the Distance sailed, for the former, or any other Reasons, and are assured in yourself of having made a good Estimate thereof, but mistrust the Care of him that steers, that he has let her *yaw* or fall off, or that you have neglected the Variation, or met with thwart Currents, then with the true Difference of Latitude, and the Distance, find the Course and Departure by the following,

Case III.

The Difference of Latitude and Distance sailed being given, to find the Course and Departure from the *Meridian.*

A Ship from the Latitude 50° 00′ *North*, sails between the *South* and the *West*, until her Distance sail'd be 57.5 Miles, and then by Observation is found to be in the Latitude of 49° 10′ *North*; I desire to know her true Course and Departure?

From } the Latitude the Ship sailed { from ——— 50° 00′ N.
Take } { into ——— 49 10 N.
There remains the Difference of Latitude ——— 0 50



Geometrically.

1. Let AB be made equal to the Difference of Latitude 50.

2. Because the Course is *Southwardly*, on B raise the Perpendicular BC.

3. With the Distance sailed between the Compasses, setting one Foot in A, the Place departed from, with the other cross the Perpendicular in C, which is the Place the Ship is in; and therefore the Angle BAC is the direct Course: To determine which

Arith-

Arithmetically.

It will be AC : R. : : AB : c s, ∠A, by *Case the 6th of Sect. the 6th.*

That is,

As the Diſtance ſailed 57.5	1.7596678
To the Radius	10.0000000
So is the Difference of Latitude 50	1.6989700
To the Co-ſine of the Courſe 29.35 ½	9.9393022

Which, becauſe ſhe ſailed between the *South* and the *Weſt*, is S. S. W. 7° 5′ ½ *Weſterly.*

Now to find the Departure, it will be,

R : AC : : s, ∠A : BC, by *Caſe the 3d of Sect. the 6th of Part the 1ſt.*

That is,

As the Radius	10.0000000
To the Diſtance ſailed 57.5	1.7596678
So is the Sine of the Courſe 29° 35′ ½	9.6935669
To the Departure 28.39 Miles *Weſterly*	1.4532347

If the Difference of Latitude and Departure between the Ship and any other Place be known, then the following *Caſe* is of great Uſe for determining the Rhumb upon which the Ship muſt ſail, and how far.

Caſe IV.

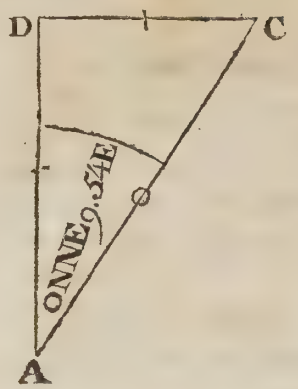
The Difference of Latitude and Departure being given, to find the direct Courſe and Diſtance.

Example.

Admit a Ship in the Latitude 40° 10′ *North*, is bound to a Port in the Latitude 41° 02′ *North*, the Departure between the Places being 33 Miles *Eaſt*, I demand upon what Courſe, and how far ſhe muſt ſail to gain her intended Port?

From the Latitude the Ship	is bound to	41° 02′ N.
Take the Latitude the Ship	ſailed from	40 10 N.
There remains the Difference of Latitude		0 52

Geo-



Geometrically.

1. Make AB equal to 52 Miles, the Difference of Latitude.

2. Raiſe the Perpendicular BC equal to the Departure 59 Miles; then is the Angle BAC the Courſe: To find which,

Arithmetically.

It will be BA : R. : : BC : t, $<$A, by *Caſe the 4th of Sect. the 6th of Part the 1ſt.* That is,

As the Difference of Latitude 52 ——	1.7160033
To the Radius ——	10.0000000
So is the Departure 33 ——	1.5185139
To the Tangent of the true Courſe 32° 24′ ½ ——	9.8025106

Which, becauſe it is between the *North* and the *Eaſt*, is *N. E.* by *N.* 1° 20′ ½ *Northwardly*, or *N. N. E.* 9.54 ½ *Eaſtwardly.*

2. To find the Diſtance the Ship muſt ſail, it will be,

s, $<$A : BC : : R, AC, by *Caſe the 2d of Sect. the 6th.* That is,

As the Sine of the Courſe 32° 24′ ——	9.7290218
To the Departure 33 ——	1.5185139
So is the Radius ——	10.0000000
To the Diſtance ſhe muſt ſail 61.59 ——	1.7894921

If for want of a perfect Knowledge of the Latitude of the Place you was to ſail to, you miſſed the ſame, but made an exact Eſtimate of the Diſtance ſailed by the Log; then may the Latitude of the Place the Ship is in, and the Courſe ſhe has ſailed, be exactly known by the following *Caſe.*

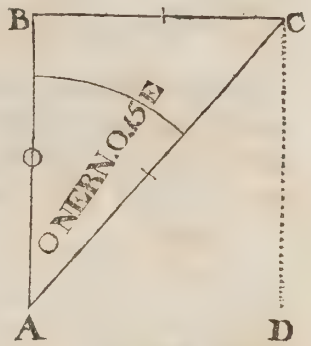
Caſe V.

The Diſtance run and Departure being given, to find the Difference of Latitude and Courſe. *Example.*

Example.

A Ship from the Latitude 40° 10′ *North*, ſails between the *North* and the *Eaſt* 59 Miles, until her Departure be 33 Miles; I demand the Courſe ſhe has ſailed, and the Latitude ſhe is in?

Geometrically.



1. Having drawn the *Meridian* BA, parallel to it, at the Diſtance of 33 Miles the Departure, draw the *Meridian* CD.

2. With the Diſtance ſailed 59 Miles, between the Points of the Compaſſes, ſetting one Foot in A the Place departed from, with the other Foot croſs the *Meridian* CD in the Point C, which will be the Place the Ship is in.

3. From C let fall the Line CB perpendicular to AB, then will CB be the Departure made, and BAC the Angle of the Courſe; to find which by *Calculation*, it will be, by *Caſe the 6th of Sect. the 6th of Part the 1ſt.*

As AC: R. :: BC: S. Angle A. That is,

As the Diſtance ſailed 59	1.7708520
To the Radius	10.0000000
So is the Departure 33	1.5185139
To the Sine of the direct Courſe 34° 00′ ½	9.7476619

Or N. E. by N. 0.15 Eaſterly.

To find the Difference of Latitude made, it will be, by *Caſe the 1ſt of Sect. the 6th of Part the 1ſt*,

t, < A: BC :: R.: AB. That is,

As the Tangent of the Courſe 34° 00′ ½	9.8291295
To the Departure 33	1.5185139
So is the Radius	10.0000000
To the Difference of Latitude 48.9	1.6893844

Now, to find the Latitude the Ship is in, becauſe ſhe ſailed from a *North* Latitude *Northwardly*.

To the Latitude ſhe ſailed from	40° 10′ N.
Add the Difference of Latitude made	0 49 *fere.*
The Sum is the Latitude the Ship is in	40 59 N.

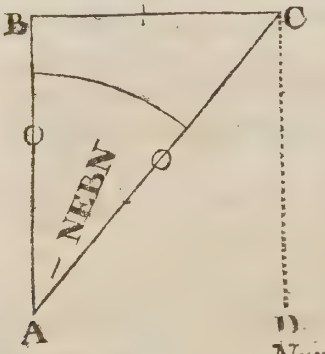
If the Diſtance run, eſtimated by the Log, be not ſufficiently exact, for the Reaſons given in *Caſe* the 1*ſt*, but you are well aſſured of your Courſe, then by the following *Caſe* may be found the true Diſtance ſailed, as alſo the Latitude the Ship is in.

Caſe VI.

One Latitude, Courſe and Departure being given, to find the Latitude the Ship is in, and her Diſtance ſailed.

Example.

A Ship from the Latitude 40° 10′ *North*, ſails N. E. by N. $\frac{1}{4}$ *Eaſtwardly*, until her Departure be 33 Miles; I demand the Diſtance ſailed, and Latitude the Ship is in?



Geometrically.

1. The Angle of the Courſe BAC being made equal to 3P $\frac{1}{4}$, or 36° 34′.

2. With the Departure 33 Miles, draw DC parallel to the *Meridian*, till it meet AC produced in C, then will C be the Place the Ship is in, and AC the Diſtance ſailed. To find which

Numerically, or by Calculation.

By *Caſe the 1ſt of Sect. the 6th*, s, ∠A : BC : : R : AC. That is,

As the Sine of the Courſe 36° 34′	9.7750697
To the Departure 33	1.5185139
So is the Radius	10.0000000
To the Diſtance ſailed 55.39 Miles	1.7434442

But

But for the Difference of Latitude, it will be, by *Case the 1st of Sect. the 6th*, t, ∠A : BC : : R : AB. That is,

As the Tangent of the Course 36° 34′ —— ——	9.8702653
To the Departure 33 —— —— —— —— ——	1.5185139
So is the Radius —— —— —— —— ——	10.0000000
To the Difference of Latitude 44.49 —— ——	1.6482486

Because the Course is *Northerly* and the Latitude *North*,

To the Latitude the Ship sailed from —	40° 10′ N.
Add the Difference of Latitude made —	44 ½
The Sum is the Latitude the Ship is in —	40 54 ½ N.

If the Departure given be not so true as it ought to be, and being well assured of the Course and Distance run, the Place the Ship is in may be gotten by *Case* the *1st* of this *Section*, and then the Course and Distance she must make by *Case the 4th*.

SECT. III.

Being the Application of the Doctrine of Right-angled Plain Triangles *to the Solution of a* Traverse, *or* compound Course.

A *Traverse*, or *Compound Course*, is, when a Ship upon the Shifting of the Winds, sails upon several Courses in any space of Time.

To resolve a *Traverse*, is, so to order and compound the several Courses and Distances obtained by the Log and Compass, that the industrious Mariner may know at all Times the Place the Ship is in, and be able to determine how far, and upon what Course he must sail to gain his intended Port.

I shall give a Solution of one only, and that the most general and useful as may be; which being well understood, will be sufficient for the Learner's Purpose.

Example.

A Ship being at Sea in the Latitude of 37° 10′ N. is bound to a Port in the Latitude 33° 00′ N. the Departure between the Ship and the Place, being 180 Miles W. and therefore the direct Course, by *Case the 4th*, will be SW. by S. 2 Degrees Westerly, and Distance 308 Miles; but finding the Wind variable from the S. by E. to S. and so to the SSW. and W. a small Gale and smooth Water, plies upon these several Courses, with the Distances on each Course obtained by the Log, as follows;

Larboard Tack on board, Wind variable from S. by E. to SSW.
SW. by W. 27 Miles, WSW. ½ W. 30 Miles, W. by S. 25 Miles, W. by N. 18 Miles.

Starboard Tack on board, Wind shifting from SSW. to SW. and WSW.
SSE. 32 Miles, SSE. ¼ Easterly 27 Miles, S. by E. 25 Miles, S. 31 Miles, SSE. 39 Miles.

I demand the Latitude of the Place the Ship is in, and her Departure from the Meridian; as also upon what Course, if possible, and how far she must sail to obtain her desired Port?

Geometrically.

Let A represent the Place the Ship is in, and ASD the Meridian: Make AD equal to the Difference of Latitude 250, and perpendicular thereto, draw PD the Departure equal to 180; then is P the Port the Ship must sail to, AP the Distance, and the Angle PAD the Course. *See the Figure of the* Plain Chart.

But to find the Place of the Ship at the End of the several Courses,

1. Draw the SW. by S. Line, and set off the Distance 27 upon it, from A to *b*, which will be the Place of the Ship at the End of the first Course.

2. From *b*, draw the Line *b c* equal to 30, the second Distance, and parallel to the WSW. ½ W. Line; then will *c* be the Place of the Ship at the End of the second Course.

3. From *c* draw *c d* parallel to the W. by S. Line, and set off upon it 25 Miles from *c* to *d*, which will be the Place of the Ship at the End of the third Course.

And

And proceeding in like manner, by drawing Lines from the Place laſt found parallel to the ſeveral Courſes drawn in the Quadrants, and equal to the ſeveral Diſtances upon each Courſe, will give the ſeveral Places *e f g h i* K, to which the Ship arrived at, and K the Place of the Ship at the End of the ſeveral Runnings.

Laſtly, Having drawn the Lines KA, KH, KR and KP, KA will be the neareſt Diſtance of the Ship from her firſt Port, KAH the Courſe, KH the Departure Weſt, and AH the Difference of Latitude Southwardly the Ship has made; KR the Exceſs of AD above AH, the Difference of Latitude between the Ship and the 2d Port in this Caſe Southwardly, PR the Difference between PD, and KH the Departure Weſt; alſo KP the Diſtance, and the Angle PKR, the Courſe upon which ſhe muſt ſteer, if poſſible, to arrive at it.

To find all which *Trigonometrically*, muſt be found firſt according to *Caſe the 1ſt* of the laſt *Section*. The ſeveral Differences of Latitude and Departure proper to the ſeveral Courſes and Diſtances run, as in the following manner;

1. Courſe SW. by W. Diſtance 27 Miles.

Radius	10.0000000	Radius	10.0000000
Diſtance ſailed 27	1.4313637	Diſtance ſailed 27	1.4313637
s, Courſe 56° 15′	9.9198464	cs, Courſe 56° 15′	9.7447390
Departure 22.45	1.3512101	Diff. Lat. 15.00	1.1761027

2. Courſe WSW. ½ W. Diſtance 30 Miles.

Radius	10.0000000	Radius	10.0000000
Diſtance ſail'd 30	1.4771212	Diſtance ſail'd 30	1.4771212
s, Courſe 73° 07′	9.9808657	cs, Courſe 73° 07′	9.4630323
Departure 28.71	1.4579869	Diff. of Lat. 8.71	0.9401535

After this manner muſt the Difference of Latitude and Departure for every Courſe and Diſtance be found; but I ſhall omit the Operations for Brevity-ſake, as well as for the Improvement of the young Student.

Now, from the ſeveral Differences of Latitude and Departure, thus deduced from the Courſes and Diſtances obtained by the Log and Compaſs, may be found the Latitude of the Place the Ship is

in, and how far ſhe is to the Eaſtward or Weſtward of her firſt Meridian, by diſpoſing of them as in the following Table, called *The* TRAVERSE TABLE, which conſiſts of five Columns; in the firſt of which muſt be placed the ſeveral Courſes and Diſtances, in the other the Differences of Latitude and Departure deduced from each, according as the Courſes are; that is, if the Courſe be North {Eaſtwardly / Weſtwardly} the Difference of Latitude muſt be placed under the Column N. and the Departure under the Columns of {East / West}: but if the Courſe be South {Eaſtwardly / Weſtwardly} then the Departure muſt be placed as before, and the Difference of Latitude under the South: And being thus diſpoſed, it is manifeſt, that the Totals of each Column ſhew the Northings, Southings, Eaſtings and Weſtings, that the Ship has made.

Wherefore, if the Sum of the Northings exceed the Southings, it follows, that the Ship is to the Northward of her firſt Place, and that juſt as much as is the Exceſs; and the contrary, in like manner, if the Sum of the Eaſtings exceed the Weſtings the Ship is to the Eaſtward of her firſt Meridian, otherwiſe to the Weſtward.

The Traverſe Table.		Diff. of Lat.		Departure.	
Courſes.	Diſtances.	N.	S.	E.	W.
1. SW. by W.	27	—	15.00	—	22.45
2. WSW. ½ W.	30	—	8.71	—	28.71
3. W. by S.	25	—	4.88	—	24.52
4. W. by N.	18	3.57	—	—	17.62
5. SSE.	32	—	29.57	12.25	—
6. SSE. ¾ E.	27	—	23.15	13.88	—
7. S. by E.	25	—	24.52	4.88	—
8. South	31	—	31.00	—	—
9. SSE.	39	—	36.04	14.92	—
		3.51	172.87	45.93	93.34
			3.51		45.93
			169.30		47.31

Hence it is manifeſt, that the Ship is 47.31 Miles to the Weſtward, and 169.30 Miles to the Southward of her firſt Place; and therefore the whole Difference of Latitude 2.49 ⅓ Southwardly.

Wherefore,

Becauſe the Latitude of the Place ſhe ſailed from was North,

From the Latitude departed from —— 37° 10′ N.
Take the Difference of Latitude made — 2 49 ⅓ Sly.

There remains the Latitude the Ship is in — 34 20 ⅓ N.

Now, to find the direct Courſe, it will be by *Caſe the 4th.*

As the Difference of Latitude 169.30 —— 2.2287339

To the Departure 47.38 —— 1.6755950
So is the Radius —— 10.0000000

To the Tangent of the Courſe 15° 38′ —— 9.4468611

Or, S. by W. 4° 23′ Weſtwardly.

For the direct Courſe, it will be, by the ſame *Caſe*,

As the Sine of the Courſe 15° 38′ —— 9.4305267

To the Departure 47.38 —— 1.6755950
So is the Radius —— 10.0000000

To the direct Diſtance ſailed 175.8 —— 2.2450683

The next Thing, in order to compleat the *Traverſe*, is, to find the Diſtance of the Ship from her intended Port, and upon what Courſe ſhe muſt ſail, if poſſible, to gain it.

And becauſe the Difference of Latitude between the Place of the Ship at firſt and her intended Port, is greater than the Difference of Latitude made,

From } the Difference of Latitude { firſt given 250 Sly.
Take } { made —— 169 33′ Sly.
Remains } { to make— 80 67 Sly.

Alſo

Alſo becauſe the Departure firſt given, as well as the Departure made, are both the ſame way, their Exceſs ſhall be the Departure the Ship muſt make.

Wherefore,

From	the Departure	firſt given	180° 00′ W.
Take		made	47 38 W.
Remains		to make	132 62 W.

Hence, and by *Caſe the 4th*, the Courſe and Diſtance may be eaſily had.

For,

As the Difference of the Latitude 80.67	1.9067121
To Radius	10.0000000
So is the Departure 132.62	2.1226090
To the Tangent of the Courſe 58° 41′	10.2158969

Which, becauſe the Difference of Latitude is Southerly, and Departure Weſt, is SW. by W. 2° 26′ Weſterly.

Again, for the direct Diſtance it will be,

As the Sine of the Courſe 58° 41′	9.9316143
To the Departure 132.62	2.1226090
So is the Radius	10.0000000
To the direct Diſtance 155.2	2.1909947

A large

A large and very Useful

TABLE

OF

DIFFERENCE

OF

Latitude *and* Departure,

In *Minutes* and *Tenth Parts*,

To every Degree, and Quarter Point of the COMPASS,

For the exact working of a TRAVERSE.

Diff.	1 Deg.		2 Deg.		¼ Point		3 Deg.		4 Deg.		5 Deg.		Diff.
	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	
1	01.0	00.0	01.0	00.0	01.0	00.0	01.0	00.0	01.0	00.1	01.0	00.1	1
2	02.0	00.0	02.0	00.1	02.0	00.1	02.0	00.1	02.0	00.1	02.0	00.2	2
3	03.0	00.1	03.0	00.1	03.0	00.1	03.0	00.2	03.0	00.2	03.0	00.3	3
4	04.0	00.1	04.0	00.1	04.0	00.2	04.0	00.2	04.0	00.3	04.0	00.3	4
5	05.0	00.1	05.0	00.2	05.0	00.2	05.0	00.3	05.0	00.3	05.0	00.4	5
6	06.0	00.1	06.0	00.2	06.0	00.3	06.0	00.3	06.0	00.4	06.0	00.5	6
7	07.0	00.1	07.0	00.2	07.0	00.3	07.0	00.4	07.0	00.5	07.0	00.6	7
8	08.0	00.1	08.0	00.3	08.0	00.4	08.0	00.4	08.0	00.6	08.0	00.7	8
9	09.0	00.2	09.0	00.3	09.0	00.4	09.0	00.5	09.0	00.6	09.0	00.8	9
10	10.0	00.2	10.0	00.4	10.0	00.5	10.0	00.5	10.0	00.7	10.0	00.9	10
11	11.0	00.2	11.0	00.4	11.0	00.5	11.0	00.6	11.0	00.8	11.0	01.0	11
12	12.0	00.2	12.0	00.4	12.0	00.6	12.0	00.6	12.0	00.8	12.0	01.0	12
13	13.0	00.2	13.0	00.5	13.0	00.6	13.0	00.7	13.0	00.9	12.9	01.1	13
14	14.0	00.2	14.0	00.5	14.0	00.7	14.0	00.7	14.0	01.0	13.9	01.2	14
15	15.0	00.3	15.0	00.5	15.0	00.7	15.0	00.8	15.0	01.0	14.9	01.3	15
16	16.0	00.3	16.0	00.6	16.0	00.8	16.0	00.8	16.0	01.1	15.9	01.4	16
17	17.0	00.3	17.0	00.6	17.0	00.8	17.0	00.9	17.0	01.2	16.9	01.5	17
18	18.0	00.3	18.0	00.6	18.0	00.9	18.0	00.9	17.9	01.3	17.9	01.6	18
19	19.0	00.3	19.0	00.7	19.0	00.9	19.0	01.0	18.9	01.3	18.9	01.7	19
20	20.0	00.4	20.0	00.7	20.0	01.0	20.0	01.0	19.9	01.4	19.9	01.7	20
21	21.0	00.4	21.0	00.7	21.0	01.0	21.0	01.1	20.9	01.5	20.9	01.8	21
22	22.0	00.4	22.0	00.8	22.0	01.1	22.0	01.1	21.9	01.5	21.9	01.9	22
23	23.0	00.4	23.0	00.8	23.0	01.1	23.0	01.2	22.9	01.6	22.9	02.0	23
24	24.0	00.4	24.0	00.8	24.0	01.2	24.0	01.3	23.9	01.7	23.9	02.1	24
25	25.0	00.4	25.0	00.9	25.0	01.2	25.0	01.3	24.9	01.7	24.9	02.2	25
26	26.0	00.5	26.0	00.9	26.0	01.3	26.0	01.4	25.9	01.8	25.9	02.3	26
27	27.0	00.5	27.0	00.9	27.0	01.3	27.0	01.4	26.9	01.9	26.9	02.4	27
28	28.0	00.5	28.0	01.0	28.0	01.4	28.0	01.5	27.9	02.0	27.9	02.4	28
29	29.0	00.5	29.0	01.0	29.0	01.4	29.0	01.5	28.9	02.0	28.9	02.5	29
30	30.0	00.5	30.0	01.1	30.0	01.5	30.0	01.6	29.9	02.1	29.9	02.6	30
31	31.0	00.5	31.0	01.1	30.9	01.5	31.0	01.6	30.9	02.2	30.9	02.7	31
32	32.0	00.6	32.0	01.1	31.9	01.6	31.9	01.7	31.9	02.2	31.9	02.8	32
33	33.0	00.6	33.0	01.2	32.9	01.6	32.9	01.7	32.9	02.3	32.9	02.9	33
34	34.0	00.6	34.0	01.2	33.9	01.7	33.9	01.8	33.9	02.4	33.9	03.0	34
35	35.0	00.6	35.0	01.2	34.9	01.7	34.9	01.8	34.9	02.4	34.9	03.1	35
36	36.0	00.6	36.0	01.3	35.9	01.8	35.9	01.9	35.9	02.5	35.9	03.1	36
37	37.0	00.7	37.0	01.3	36.9	01.8	36.9	01.9	36.9	02.6	36.9	03.2	37
38	38.0	00.7	38.0	01.3	37.9	01.9	37.9	02.0	37.9	02.7	37.9	03.3	38
39	39.0	00.7	39.0	01.4	38.9	01.9	38.9	02.0	38.9	02.7	38.9	03.4	39
40	40.0	00.7	40.0	01.4	39.9	02.0	39.9	02.1	39.9	02.8	39.8	03.5	40
41	41.0	00.7	41.0	01.4	40.9	02.0	40.9	02.1	40.9	02.9	40.8	03.6	41
42	42.0	00.7	42.0	01.5	41.9	02.1	41.9	02.2	41.9	02.9	41.8	03.7	42
43	43.0	00.8	43.0	01.5	42.9	02.1	42.9	02.2	42.9	03.0	42.8	03.8	43
44	44.0	00.8	44.0	01.5	43.9	02.2	43.9	02.3	43.9	03.1	43.8	03.8	44
45	45.0	00.8	45.0	01.6	44.9	02.2	44.9	02.4	44.9	03.1	44.8	03.9	45
46	46.0	00.8	46.0	01.6	45.9	02.2	45.9	02.4	45.9	03.2	45.8	04.0	46
47	47.0	00.8	47.0	01.6	46.9	02.3	46.9	02.5	46.9	03.3	46.8	04.1	47
48	48.0	00.8	48.0	01.7	47.9	02.3	47.9	02.5	47.9	03.4	47.8	04.2	48
49	49.0	00.9	49.0	01.7	48.9	02.4	48.9	02.6	48.9	03.4	48.8	04.3	49
50	50.0	00.9	50.0	01.8	49.9	02.4	49.9	02.6	49.9	03.5	49.8	04.4	50
Diff.	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Diff.
	89 Deg.		88 Deg.		7¾ Points		87 Deg.		86 Deg.		85 Deg.		

Diff.	1 Deg.		2 Deg.		¼ Point		3 Deg.		4 Deg.		5 Deg.		Diff.
	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	
51	51.0	00.9	51.0	01.8	50.9	02.5	50.9	02.7	50.9	03.6	50.8	04.4	51
52	52.0	00.9	52.0	01.8	51.9	02.5	51.9	02.7	51.9	03.6	51.8	04.5	52
53	53.0	00.9	53.0	01.8	52.9	02.6	52.9	02.8	52.9	03.7	52.8	04.6	53
54	54.0	00.9	54.0	01.9	53.9	02.6	53.9	02.8	53.9	03.8	53.8	04.7	54
55	55.0	01.0	55.0	01.9	54.9	02.7	54.9	02.9	54.9	03.8	54.8	04.8	55
56	56.0	01.0	56.0	02.0	55.9	02.7	55.9	02.9	55.9	03.9	55.8	04.9	56
57	57.0	01.0	57.0	02.0	56.9	02.8	56.9	03.0	56.9	04.0	56.8	05.0	57
58	58.0	01.0	58.0	02.0	57.9	02.8	57.9	03.0	57.9	04.1	57.8	05.1	58
59	59.0	01.0	59.0	02.1	58.9	02.9	58.9	03.1	58.8	04.1	58.8	05.2	59
60	60.0	01.0	60.0	02.1	59.9	02.9	59.9	03.1	59.8	04.2	59.8	05.2	60
61	61.0	01.1	61.0	02.1	60.9	03.0	60.9	03.2	60.8	04.3	60.8	05.3	61
62	62.0	01.1	62.0	02.2	61.9	03.0	61.9	03.3	61.8	04.3	61.8	05.4	62
63	63.0	01.1	63.0	02.2	62.9	03.1	62.9	03.3	62.8	04.4	62.8	05.5	63
64	64.0	01.1	64.0	02.2	63.9	03.1	63.9	03.4	63.8	04.5	63.8	05.6	64
65	65.0	01.1	65.0	02.3	64.9	03.2	64.9	03.4	64.8	04.5	64.7	05.7	65
66	66.0	01.1	66.0	02.3	65.9	03.2	65.9	03.5	65.8	04.6	65.7	05.8	66
67	67.0	01.2	67.0	02.3	66.9	03.3	66.9	03.5	66.8	04.7	66.7	05.9	67
68	68.0	01.2	67.9	02.4	67.9	03.3	67.9	03.6	67.8	04.8	67.7	05.9	68
69	69.0	01.2	68.9	02.4	68.9	03.4	68.9	03.6	68.8	04.8	68.7	06.0	69
70	70.0	01.2	69.9	02.4	69.9	03.4	69.9	03.7	69.8	04.9	69.7	06.1	70
71	71.0	01.2	70.9	02.5	70.9	03.5	70.9	03.7	70.8	05.0	70.7	06.2	71
72	72.0	01.3	71.9	02.5	71.9	03.5	71.9	03.8	71.8	05.0	71.7	06.3	72
73	73.0	01.3	72.9	02.5	72.9	03.6	72.9	03.8	72.8	05.1	72.7	06.4	73
74	74.0	01.3	73.9	02.6	73.9	03.6	73.9	03.9	73.8	05.2	73.7	06.5	74
75	75.0	01.3	74.9	02.6	74.9	03.7	74.9	03.9	74.8	05.2	74.7	06.6	75
76	76.0	01.3	75.9	02.7	75.9	03.7	75.9	04.0	75.8	05.3	75.7	06.6	76
77	77.0	01.3	76.9	02.7	76.9	03.8	76.9	04.0	76.8	05.4	76.7	06.7	77
78	78.0	01.4	77.9	02.7	77.9	03.8	77.9	04.1	77.8	05.5	77.7	06.8	78
79	79.0	01.4	78.9	02.8	78.9	03.9	78.9	04.1	78.8	05.5	78.7	06.9	79
80	80.0	01.4	79.9	02.8	79.9	03.9	79.9	04.2	79.8	05.6	79.7	07.0	80
81	81.0	01.4	80.9	02.8	80.9	04.0	80.9	04.2	80.8	05.7	80.7	07.1	81
82	82.0	01.4	81.9	02.9	81.9	04.0	81.9	04.3	81.8	05.7	81.7	07.2	82
83	83.0	01.4	82.9	02.9	82.9	04.1	82.9	04.4	82.8	05.8	82.7	07.3	83
84	84.0	01.5	83.9	02.9	83.9	04.1	83.9	04.4	83.8	05.9	83.7	07.3	84
85	85.0	01.5	84.9	03.0	84.9	04.2	84.9	04.5	84.8	05.9	84.7	07.4	85
86	86.0	01.5	85.9	03.0	85.9	04.2	85.9	04.5	85.8	06.0	85.7	07.5	86
87	87.0	01.5	86.9	03.0	86.9	04.3	86.9	04.6	86.8	06.1	86.7	07.6	87
88	88.0	01.5	87.9	03.1	87.9	04.3	87.9	04.6	87.8	06.2	87.7	07.7	88
89	89.0	01.5	88.9	03.1	88.9	04.4	88.9	04.7	88.8	06.2	88.7	07.8	89
90	90.0	01.6	89.9	03.1	89.9	04.4	89.9	04.7	89.8	06.3	89.7	07.9	90
91	91.0	01.6	90.9	03.2	90.9	04.5	90.9	04.8	90.8	06.4	90.7	08.0	91
92	92.0	01.6	91.9	03.2	91.9	04.5	91.9	04.8	91.8	06.4	91.6	08.0	92
93	93.0	01.6	92.9	03.2	92.9	04.6	92.9	04.9	92.8	06.5	92.6	08.1	93
94	94.0	01.6	93.9	03.3	93.9	04.6	93.9	04.9	83.8	06.6	93.6	08.2	94
95	95.0	01.6	94.9	03.3	94.9	04.7	94.9	05.0	94.8	06.6	94.6	08.3	95
96	96.0	01.7	95.9	03.4	95.9	04.7	95.9	05.0	95.8	06.7	95.6	08.4	96
97	97.0	01.7	96.9	03.4	96.9	04.8	96.9	05.1	96.8	06.8	96.6	08.5	97
98	98.0	01.7	97.9	03.4	97.9	04.8	97.9	05.1	97.8	06.9	97.6	08.6	98
99	99.0	01.7	98.9	03.5	98.9	04.9	98.9	05.2	98.8	06.9	98.6	08.7	99
100	100.0	01.7	99.9	03.5	99.9	04.9	99.9	05.2	99.8	07.0	99.6	08.7	100
Diff.	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Diff.
	89 Deg.		88 Deg.		7¾ Point		87 Deg.		86 Deg.		85 Deg.		

Dist.	½ Point		6 Deg.		7 Deg.		8 Deg.		¾ Point.		9 Deg.		Dist.
	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	
1	01.0	00.1	01.0	00.1	01 0	00.1	01.0	00.1	01.0	00.1	01.0	00.2	1
2	02.0	00.2	02.0	00 2	02.0	00.2	02.0	00.3	02.0	00.3	02.0	00.3	2
3	03.0	00.3	03.0	00.3	03.0	00.4	03.0	00.4	03.0	00.4	03.0	00 5	3
4	04.0	00.4	04.0	00 4	04.0	00.5	04.0	00.6	04.0	00.6	03.9	00.6	4
5	05.0	00.5	05.0	00.5	05.0	00.6	04.9	00.7	04.9	00.7	04.9	00.8	5
6	06.0	00.6	06.0	00.6	06.0	00.7	05.9	00 8	05.9	00.9	05.9	00.9	6
7	07.0	00.7	07.0	00.7	06.9	00.8	06.9	01.0	06.9	01.0	06.9	01.1	7
8	08.0	00.8	08.0	00.8	07.9	01.0	07.9	01.1	07.9	01.2	07.9	01.2	8
9	09 0	00.9	08 9	00 9	08.9	01.1	08.9	01.2	08.9	01.3	08.9	01.4	9
10	09.9	01.0	09.9	01.0	09.9	01.2	09.9	01 4	09 9	01 5	09 8	01 6	10
11	10.9	01.1	10.9	01.1	10.9	01.3	10.9	01.5	10.9	01.6	10.9	01.7	11
12	11.9	01.2	11.9	01.2	11.9	01.5	11.9	01.7	11.9	01.8	11.8	01.9	12
13	12.9	01.3	12.9	01.4	12.9	01.6	12.9	01.8	12 9	01.9	12.8	02.0	13
14	13.9	01.4	13.9	01.5	13.9	01.7	13.9	01.9	13.8	02.1	13 8	02.2	14
15	14.9	01.5	14.9	01.6	14.9	01.8	14.8	02.1	14.8	02.2	14.8	02.3	15
16	15.9	01.6	15.9	01.7	15.9	01.9	15 8	02.2	15 8	02 3	15.8	02 5	16
17	16.9	01.7	16.9	01.8	16.9	02.1	16.8	02.4	16.8	02.5	16.8	02.7	17
18	17.9	01.8	17.9	01.9	17.9	02.2	17.8	02 5	17.8	02.6	17.8	02.8	18
19	18.9	01.9	18.9	02.0	18.9	02.3	18.8	02.6	18.8	02.8	18 8	03 0	19
20	19.9	02.0	19.9	02.1	19.8	02.4	19.8	02.8	19 8	02 9	19.7	03.1	20
21	20 9	02 1	20.9	02 2	20.8	02.6	20 8	02.9	20.8	03.1	20.7	03 3	21
22	21.9	02 2	21.9	02.3	21.8	02.7	21.8	03.1	21.8	03.2	21.7	03 4	22
23	22.9	02.2	22 9	02.4	22.8	02 8	22.8	03.2	22.7	03.4	22.7	03.6	23
24	23.9	02.3	23.9	02.5	23.8	02.9	23.8	03.3	23 7	03.5	23.7	03 8	24
25	24.9	02.4	24.9	02.6	24.8	03.0	24.8	03.5	24.7	03.7	24.7	03.9	25
26	25.9	02 5	25.9	02.7	25.8	03.2	25.7	03.6	25.7	03.8	25.7	04.1	26
27	26.9	02.6	26.9	02.8	26.8	03.3	26.7	03.7	26.7	04.0	26.7	04.2	27
28	27.9	02.7	27.8	02.9	27.8	03.4	27 7	03.9	27.7	04.1	27.6	04.4	28
29	28.9	02.8	28.8	03.0	28.8	03.5	28 7	04.0	28.7	04.2	28.6	04.5	29
30	29.8	02.9	29.8	03.1	29.8	03.7	29.7	04.2	29.7	04.4	29.6	04.7	30
31	30.8	03.0	30.8	03.2	30.8	03.8	30.7	04.3	30.7	04.5	30.6	04.9	31
32	31.8	03.1	31.8	03.3	31.8	03 9	31.7	04.4	31.6	04.7	31.6	05.0	32
33	32.8	03.2	32.8	03.4	32 7	04.0	32.7	04.6	32.6	04.8	32.6	05.2	33
34	33.8	03.3	33.8	03.5	33.7	04.1	33.7	04 7	33.6	05.0	33 6	05.3	34
35	34 8	03.4	34.8	03.7	34.7	04.3	34 7	04 9	34.6	05.1	34 6	05.5	35
36	35.8	03.5	35.8	03.8	35.7	04.4	35.6	05.0	35.6	05.3	35.5	05.6	36
37	36.8	03.6	36.8	03.9	36.7	04.5	36.6	05 1	36.6	05.4	36.5	05.8	37
38	37.8	03.7	37.8	04.0	37.7	04.6	37.6	05.3	37.6	05.6	37.5	05.0	38
39	38 8	03.8	38.8	04.1	38.7	04.8	38.6	05.4	38.6	05.7	38.5	06.1	39
40	39.8	03.9	39.8	04.2	39.7	04.9	39.6	05.6	39.5	05.9	39.5	06.3	40
41	40.8	04.0	40.8	04 3	40.7	05.0	40.6	05.7	40.6	06 0	40.5	06.4	41
42	41.8	04.1	41.8	04.4	41.7	05.1	41 6	05 8	41.5	06.2	41 5	06.6	42
43	42.8	04.2	42.8	04.5	42.7	05.2	42.6	06 0	42.5	06.3	42.5	06 7	43
44	43.8	04.3	43.7	04.6	43.7	05.4	43.6	06.1	43 5	06.5	43 5	06.9	44
45	44.8	04.4	44.7	04 7	44 7	05.5	44 6	06.3	44.5	06.6	44 4	07 0	45
46	45.8	04.5	45.7	04.8	45 7	05.6	45 5	06.4	45.5	06.7	45.4	07.2	46
47	46 8	04.6	46.7	04.9	46.6	05.7	46.5	06.5	46.5	06.9	46.4	07.3	47
48	47.8	04.7	47.7	05 0	47.6	05.9	47.5	06.7	47.5	07.0	47 4	07.5	48
49	48 8	04.8	48.7	05.1	48.6	06.0	48.5	06.8	48.5	07.2	48.4	07.7	49
50	49.8	04 9	49.7	05 2	49.6	06 1	49.5	07.0	49.5	07.3	49.4	07.8	50
Dist.	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dist.
	7⅕ Points		84 Deg.		83 Deg.		82 Deg.		7¼ Point		81 Deg.		

Dist.	½ Point.		6 Deg.		7 Deg.		8 Deg.		¾ Point		9 Deg.		Dist.
	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	
51	50.7	05.0	50.7	05.3	50.6	06.2	50.5	07.1	50.4	07.5	50.4	08.0	51
52	51.7	05.1	51.7	05.4	51.6	06.3	51.5	07.2	51.4	07.6	51.4	08.1	52
53	52.7	05.2	52.7	05.5	52.6	06.5	52.5	07.4	52.4	07.8	52.3	08.3	53
54	53.7	05.3	53.7	05.6	53.6	06.6	53.5	07.5	53.4	07.9	53.3	08.4	54
55	54.7	05.4	54.7	05.7	54.6	06.7	54.5	07.6	54.4	08.1	54.3	08.6	55
56	55.7	05.5	55.7	05.8	55.6	06.8	55.5	07.8	55.4	08.2	55.3	08.7	56
57	56.7	05.6	56.7	05.9	56.6	06.9	56.4	07.9	56.4	08.4	56.3	08.9	57
58	57.7	05.7	57.7	06.1	57.6	07.1	57.4	08.1	57.4	08.5	57.3	09.1	58
59	58.7	05.8	58.7	06.2	58.6	07.2	58.4	08.2	58.4	08.7	58.3	09.2	59
60	59.7	05.9	59.7	06.3	59.5	07.3	59.4	08.3	59.3	08.8	59.3	09.4	60
61	60.7	06.0	60.7	06.4	60.5	07.4	60.4	08.5	60.3	08.9	60.2	09.5	61
62	61.7	06.1	61.6	06.5	61.5	07.6	61.4	08.6	61.3	09.1	61.2	09.7	62
63	62.7	06.2	62.6	06.6	62.5	07.7	62.4	08.8	62.3	09.2	62.2	09.9	63
64	63.7	06.3	63.6	06.7	63.5	07.8	63.4	08.9	63.3	09.4	63.2	10.0	64
65	64.7	06.4	64.6	06.8	64.5	07.9	64.4	09.1	64.3	09.5	64.2	10.2	65
66	65.7	06.5	65.6	06.9	65.5	08.0	65.4	09.2	65.3	09.7	65.2	10.3	66
67	66.7	06.6	66.6	07.0	66.5	08.2	66.3	09.3	66.3	09.8	66.2	10.5	67
68	67.7	06.7	67.6	07.1	67.5	08.3	67.3	09.5	67.3	10.0	67.2	10.6	68
69	68.7	06.8	68.6	07.2	68.5	08.4	68.3	09.6	68.2	10.1	68.1	10.8	69
70	69.7	06.9	69.6	07.3	69.5	08.5	69.3	09.7	69.2	10.3	69.1	10.9	70
71	70.6	07.0	70.6	07.4	70.5	08.7	70.3	09.9	70.2	10.4	70.1	11.1	71
72	71.6	07.1	71.6	07.5	71.5	08.8	71.3	10.0	71.2	10.6	71.1	11.3	72
73	72.6	07.1	72.6	07.6	72.4	08.9	72.3	10.2	72.2	10.7	72.1	11.4	73
74	73.6	07.2	73.6	07.7	73.4	09.0	73.3	10.3	73.2	10.9	73.1	11.6	74
75	74.6	07.3	74.6	07.8	74.4	09.1	74.3	10.4	74.2	11.0	74.1	11.7	75
76	75.6	07.4	75.6	07.9	75.4	09.3	75.3	10.6	75.2	11.1	75.1	11.9	76
77	76.6	07.5	76.6	08.0	76.4	09.4	76.2	10.7	76.2	11.3	76.0	12.0	77
78	77.6	07.6	77.6	08.1	77.4	09.5	77.2	10.9	77.1	11.4	77.0	12.2	78
79	78.6	07.7	78.6	08.3	78.4	09.6	78.2	11.0	78.1	11.6	78.0	12.4	79
80	79.6	07.8	79.6	08.4	79.4	09.8	79.2	11.1	79.1	11.7	79.0	12.5	80
81	80.6	07.9	80.5	08.5	80.4	09.9	80.2	11.3	80.1	11.9	80.0	12.7	81
82	81.6	08.0	81.5	08.6	81.4	10.0	81.2	11.4	81.1	12.0	81.0	12.8	82
83	82.6	08.1	82.5	08.7	82.4	10.0	82.2	11.5	82.1	12.2	82.0	13.0	83
84	83.6	08.2	83.5	08.8	83.4	10.1	83.2	11.7	83.1	12.3	83.0	13.1	84
85	84.6	08.3	84.5	08.9	84.4	10.3	84.2	11.8	84.1	12.5	83.9	13.3	85
86	85.6	08.4	85.5	09.0	85.4	10.4	85.2	12.0	85.1	12.6	84.9	13.4	86
87	86.6	08.5	86.5	09.1	86.3	10.5	86.1	12.1	86.0	12.8	85.9	13.6	87
88	87.6	08.6	87.5	09.2	87.3	10.7	87.1	12.2	87.0	12.9	86.9	13.8	88
89	88.6	08.7	88.5	09.3	88.3	10.9	88.1	12.4	88.0	13.1	87.9	13.9	89
90	89.6	08.8	89.5	09.4	89.3	11.0	89.1	12.5	89.0	13.2	88.9	14.1	90
91	90.6	08.9	90.5	09.5	90.3	11.1	90.1	12.7	90.0	13.4	89.9	14.2	91
92	91.6	09.0	91.5	09.6	91.3	11.2	91.1	12.8	91.0	13.5	90.9	14.4	92
93	92.6	09.1	92.5	09.7	92.3	11.3	92.1	12.9	92.0	13.6	91.8	14.5	93
94	93.5	09.2	93.5	09.8	93.3	11.5	93.1	13.1	93.0	13.8	92.8	14.7	94
95	94.5	09.3	94.5	09.9	94.3	11.6	94.1	13.2	94.0	13.9	93.8	14.9	95
96	95.5	09.4	95.5	10.0	95.3	11.7	95.1	13.4	95.0	14.1	94.8	15.0	96
97	96.5	09.5	96.5	10.1	96.3	11.8	96.0	13.5	95.9	14.2	95.8	15.2	97
98	97.5	09.6	97.5	10.2	97.3	12.0	97.0	13.6	96.9	14.4	96.8	15.3	98
99	98.5	09.7	98.5	10.3	98.3	12.1	98.0	13.8	97.9	14.5	97.8	15.5	99
100	99.5	09.8	99.4	10.4	99.2	12.2	99.0	13.9	98.9	14.7	98.8	15.6	100
Dist.	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dist.
	7½ Point.		84 Deg.		83 Deg.		82 Deg.		7¼ Point		81 Deg.		

Dist.	10 Deg.		11 Deg.		1 Point		12 Deg.		13 Deg.		14 Deg.		Dist.
	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	
1	01.0	00.2	01.0	00.2	01.0	00.2	01.0	00.2	01.0	00.2	01.0	00.2	1
2	02.0	00.3	02.0	00.4	02.0	00.4	02.0	00.4	01.9	00.4	01.9	00.5	2
3	02.9	00.5	02.9	00.6	02.9	00.6	02.9	00.6	02.9	00.7	02.9	00.7	3
4	03.9	00.7	03.9	00.8	03.9	00.8	03.9	00.8	03.9	00.9	03.9	01.0	4
5	04.9	00.9	04.9	00.9	04.9	01.0	04.9	01.0	04.9	01.1	04.8	01.2	5
6	05.9	01.0	05.9	01.1	05.9	01.2	05.9	01.2	05.8	01.3	05.8	01.4	6
7	06.9	01.2	06.9	01.3	06.9	01.4	06.8	01.5	06.8	01.6	06.8	01.7	7
8	07.9	01.4	07.8	01.5	07.8	01.6	07.8	01.7	07.8	01.8	07.8	01.9	8
9	08.9	01.6	08.8	01.7	08.8	01.8	08.8	01.9	08.8	02.0	08.7	02.2	9
10	09.8	01.7	09.8	01.9	09.8	01.9	09.8	02.1	09.7	02.2	09.7	02.4	10
11	10.8	01.9	10.8	02.1	10.8	02.1	10.8	02.3	10.7	02.5	10.7	02.7	11
12	11.8	02.1	11.8	02.3	11.8	02.3	11.7	02.5	11.7	02.7	11.6	02.9	12
13	12.8	02.3	12.8	02.5	12.7	02.5	12.7	02.7	12.7	02.9	12.6	03.1	13
14	13.8	02.4	13.7	02.7	13.7	02.7	13.7	02.9	13.6	03.1	13.6	03.4	14
15	14.8	02.6	14.7	02.9	14.7	02.9	14.7	03.1	14.6	03.4	14.5	03.6	15
16	15.7	02.8	15.7	03.0	15.7	03.1	15.6	03.3	15.6	03.6	15.5	03.9	16
17	16.7	02.9	16.7	03.2	16.7	03.3	16.6	03.5	16.6	03.8	16.5	04.1	17
18	17.7	03.1	17.7	03.4	17.7	03.6	17.6	03.7	17.5	04.0	17.5	04.4	18
19	18.7	03.3	18.6	03.6	18.6	03.7	18.6	03.9	18.5	04.2	18.4	04.6	19
20	19.7	03.5	19.6	03.8	19.6	03.9	19.6	04.2	19.5	04.5	19.4	04.8	20
21	20.7	03.6	20.6	04.0	20.6	04.1	20.5	04.4	20.5	04.7	20.4	05.1	21
22	21.7	03.8	21.6	04.2	21.6	04.3	21.5	04.6	21.4	04.9	21.3	05.3	22
23	22.6	04.0	22.6	04.4	22.6	04.5	22.5	04.8	22.4	05.2	22.3	05.6	23
24	23.6	04.2	23.6	04.6	23.5	04.7	23.5	05.0	23.4	05.4	23.3	05.8	24
25	24.6	04.3	24.5	04.8	24.5	04.9	24.4	05.2	24.3	05.6	24.3	06.0	25
26	25.6	04.5	25.5	05.0	25.5	05.1	25.4	05.4	25.3	05.8	25.2	06.3	26
27	26.6	04.7	26.5	05.1	26.5	05.3	26.4	05.6	26.3	06.1	26.2	06.5	27
28	27.6	04.9	27.5	05.3	27.5	05.5	27.4	05.8	27.3	06.3	27.2	06.8	28
29	28.6	05.0	28.5	05.5	28.4	05.7	28.4	06.0	28.2	06.5	28.1	07.0	29
30	29.5	05.2	29.4	05.7	29.4	05.8	29.3	06.2	29.2	06.7	29.1	07.3	30
31	30.5	05.4	30.4	05.9	30.4	06.0	30.3	06.4	30.2	07.0	30.1	07.5	31
32	31.5	05.5	31.4	06.1	31.4	06.2	31.3	06.6	31.2	07.2	31.0	07.7	32
33	32.5	05.7	32.4	06.3	32.4	06.4	32.3	06.9	32.1	07.4	32.0	08.0	33
34	33.5	05.9	33.4	06.5	33.3	06.6	33.2	07.1	33.1	07.6	33.0	08.2	34
35	34.5	06.1	34.4	06.7	34.3	06.8	34.2	07.3	34.1	07.9	34.0	08.5	35
36	35.4	06.2	35.3	06.9	35.3	07.0	35.2	07.5	35.1	08.1	34.9	08.7	36
37	36.4	06.4	36.3	07.1	36.3	07.2	36.2	07.7	36.0	08.3	35.9	09.0	37
38	37.4	06.6	37.3	07.2	37.3	07.4	37.2	07.9	37.0	08.5	36.9	09.2	38
39	38.4	06.8	38.3	07.4	38.2	07.6	38.1	08.1	38.0	08.8	37.8	09.4	39
40	39.4	06.9	39.3	07.6	39.2	07.8	39.1	08.3	39.0	09.0	38.8	09.7	40
41	40.4	07.1	40.2	07.8	40.2	08.0	40.1	08.5	39.9	09.2	39.8	09.9	41
42	41.4	07.3	41.2	08.0	41.2	08.2	41.1	08.7	40.9	09.4	40.7	10.2	42
43	42.3	07.5	42.2	08.2	42.2	08.4	42.1	08.9	41.9	09.7	41.7	10.4	43
44	43.3	07.7	43.2	08.4	43.1	08.6	43.0	09.1	42.9	09.9	42.7	10.6	44
45	44.3	07.8	44.2	08.6	44.1	08.8	44.0	09.4	43.8	10.1	43.7	11.0	45
46	45.3	08.0	45.2	08.8	45.1	09.0	45.0	09.6	44.8	10.3	44.6	11.1	46
47	46.3	08.1	46.1	09.0	46.1	09.2	46.0	09.8	45.8	10.6	45.6	11.4	47
48	47.3	08.3	47.1	09.2	47.1	09.4	47.0	10.0	46.8	10.8	46.6	11.6	48
49	48.3	08.5	48.1	09.3	48.1	09.6	47.9	10.2	47.7	11.0	47.5	11.9	49
50	49.2	08.7	49.1	05.5	49.0	09.8	48.9	10.4	48.7	11.2	48.5	12.1	50
Dist.	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dist.
	80 Point		79 Deg.		7 Points.		78 Deg.		77 Deg.		76 Deg.		

Dist.	10 Deg.		11 Deg.		1 Point.		12 Deg.		13 Deg.		14 Deg		Dist.
	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	
51	50.2	08.8	50.1	09.7	50.0	10.0	50.0	10.6	49.7	11.5	49.5	12.3	51
52	51.2	09.0	51.0	09.9	51.0	10.1	50.9	10 8	50.7	11.7	50.5	12.6	52
53	52.2	09.2	52.0	10.1	52.0	10.3	51 8	11.0	51.6	11.9	51.4	12.8	53
54	53.2	09 4	53 0	10.3	53.0	10.5	52.8	11.2	52.6	12.1	52 4	13 1	54
55	54.2	09.5	54.0	10.5	53.9	10.7	53.8	11.4	53.6	12.4	53.4	13.3	55
56	55.1	09.7	55.0	10.7	54.9	10.9	54.8	11.6	54.5	12.6	54.3	13.5	56
57	56.1	09 9	56 0	10.8	55.9	11.1	55.8	11.8	55 5	12.8	55.3	13.8	57
58	57.1	10.1	56 9	11.1	56.9	11.3	56.7	12.1	56 5	13.0	56.3	14.0	58
59	58.1	10.2	57.9	11.3	57.9	11.5	57.7	12.3	57.5	13 3	57.2	14.3	59
60	59.1	10.4	58.9	11.4	58.8	11.7	58.7	12.5	58.5	13.5	58.2	14.5	60
61	60.1	10 6	59 9	11.6	59.8	11.9	59.7	12.7	59.4	13.7	59.2	14.8	61
62	61.1	10.8	60.9	11.8	60.8	12 1	60.6	12.9	60.4	13 9	60 2	15.0	62
63	62.0	10.9	61.8	12 0	61 8	12.3	61.6	13.1	61.4	14.2	61.1	15.2	63
64	63 0	11.1	62.8	12.2	62.8	12.5	62.6	13.3	62.3	14 4	62 1	15.5	64
65	64.0	11.3	63 8	12.4	63.7	12.7	63.6	13.5	63.3	14 6	63.1	15.7	65
66	65.0	11.5	64.8	12.6	64.7	12.9	64.6	13.7	64.3	14.8	64.0	16.0	66
67	66.0	11.6	65.8	12.8	65 7	13.1	65.5	13.9	65.3	15.1	65.0	16 2	67
68	67.0	11.8	66.7	13.0	66.7	13.3	66.5	14.1	66.2	15.3	66.0	16 4	68
69	68 0	12.0	67 7	13.2	67.7	13.5	67.5	14.3	67 2	15.5	66.9	16.7	69
70	68.9	12 2	68.7	13 4	68.7	13.7	68.5	14.5	68.2	15.7	67.9	16.9	70
71	69.9	12.3	69.7	13.5	69 6	13.9	69.4	14.8	69.2	16.0	68.9	17.2	71
72	70.9	12.5	70.7	13.7	70.6	14.0	70 4	15.0	70.1	16.2	69.9	17.4	72
73	71 9	12.7	71.7	13 9	71.6	14.2	71.4	15.2	71.1	16.4	70.8	17.6	73
74	72.9	12.8	72.6	14.1	72.6	14.4	72.4	15.4	72.1	16.6	71.8	17.9	74
75	73 9	13.0	73.6	14.3	73.6	14.6	73.4	15.6	73.1	16.9	72.8	18.1	75
76	74.8	13.2	74.6	14.5	74.5	14 8	74.3	15.8	74.0	17.1	73.7	18.4	76
77	75.8	13.4	75.6	14.7	75.5	15.0	75.3	16.0	75.0	17.3	74.7	18.6	77
78	76.8	13.5	76.6	14.9	76.5	15.2	76.3	16.2	76.0	17.5	75.7	18.9	78
79	77.8	13.7	77 5	15.1	77.5	15.4	77.3	16.4	77.0	17.8	76.6	19.1	79
80	78 8	13.9	78.5	15.3	78.5	15.6	78 2	16.6	77.9	18.0	77.6	19.2	80
81	79.8	14.1	79.5	15.5	79.4	15.8	79.2	16.8	78 9	18 2	78.6	19.4	81
82	80.8	14.2	80.5	15.6	80.4	16.0	80.2	17.0	79.9	18.4	79.6	19.8	82
83	81.7	14.4	81.5	15.8	81.4	16.2	81.2	17.2	80.9	18.7	80.5	20.1	83
84	82.7	14.6	82.5	16.0	82.4	16.4	82.2	17.5	81.8	18.9	81.5	20.3	84
85	83.7	14.8	83 4	16.2	83.4	16.6	83.1	17.7	82.8	19.1	82.5	20.6	85
86	84 7	14.9	84.4	16.4	84.3	16.8	84 1	17.9	83.8	19.3	83.4	20 8	86
87	85.7	15.1	85.4	16.6	85.3	17.0	85.1	18.1	84.8	19.6	84.4	21.0	87
88	86 7	15.3	86.4	16.8	86.3	17.2	86.1	18.3	85.7	19.8	85.4	21.3	88
89	87.6	15 4	87.4	17 0	87.3	17.4	87.1	18.5	86.7	20.0	86.4	21.5	89
90	88.6	15.6	88.3	17.2	88.3	17.6	88.0	18.7	87.7	20.2	87.3	21.8	90
91	89.6	15.8	89.3	17.4	89.2	17 8	89.0	18.9	88 7	20.5	88.3	22.0	91
92	90.6	16.0	90.3	17.6	90.2	17.9	90.0	19.1	89.6	20 7	89.3	22.2	92
93	91.6	16.1	91.3	17 7	91.2	18.1	91.0	19.3	90.6	20.9	90.2	22.5	93
94	92 6	16.3	92 3	17 9	92.2	18.3	91.9	19.5	91.6	21.1	91.2	22.7	94
95	93.5	16.5	93.3	18.1	93.2	18.5	92.9	19.7	92.6	21.4	92.2	23.0	95
96	94.5	16.7	94.2	18.3	94.2	18.7	93.9	20.0	93.5	21.6	93.1	23.2	96
97	95.5	16.8	95.2	18.5	95.1	18 9	94.9	20 2	94.5	21.8	94.1	23.5	97
98	96 5	17.0	96.2	18.7	96.1	19.1	95.9	20.4	95.5	22.0	95.1	23.7	98
99	97.5	17.2	97.2	18.9	97.1	19 3	96.8	20.6	96.5	22.3	96.1	23.9	99
100	98.5	17.4	98.2	19.1	98.1	19.5	97.8	20.8	97 4	22 5	97 0	24.2	100
Dist.	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dist.
	80 Deg.		79 Deg.		7 Points.		78 Deg.		77 Deg.		76 Deg.		

Diff.	1¼ Deg.		15 Deg.		16 Deg.		1½ Point.		17 Deg.		18 Deg.		Diff.
	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	
1	01.0	00.2	01.0	00.3	01.0	00.3	01.0	00.3	01.0	00.3	00.9	00.3	1
2	01.9	00.5	01.9	00.5	01.9	00.5	01.9	00.6	01.9	00.6	01.9	00.6	2
3	02.9	00.7	02.9	00.8	02.9	00.8	02.9	00.9	02.9	00.9	02.8	00.9	3
4	03.9	01.0	03.9	01.0	03.8	01.1	03.8	01.2	03.8	01.2	03.8	01.2	4
5	04.8	01.2	04.8	01.3	04.8	01.4	04.8	01.5	04.8	01.5	04.8	01.5	5
6	05.8	01.5	05.8	01.5	05.8	01.6	05.8	01.7	05.7	01.7	05.7	01.8	6
7	06.8	01.7	06.8	01.8	06.8	01.9	06.8	02.0	06.7	02.0	06.7	02.2	7
8	07.8	01.9	07.7	02.1	07.7	02.2	07.7	02.3	07.6	02.3	07.6	02.5	8
9	08.7	02.2	08.7	02.3	08.6	02.5	08.6	02.6	08.6	02.6	08.6	02.8	9
10	09.7	02.4	09.7	02.6	09.6	02.8	09.6	02.9	09.6	02.9	09.5	03.1	10
11	10.7	02.8	10.6	02.8	10.6	03.0	10.5	03.2	10.5	03.2	10.5	03.4	11
12	11.6	02.9	11.6	03.1	11.5	03.3	11.5	03.5	11.5	03.5	11.4	03.7	12
13	12.6	03.2	12.6	03.4	12.5	03.6	12.4	03.8	12.4	03.8	12.4	04.0	13
14	13.6	03.4	13.5	03.6	13.5	03.9	13.4	04.1	13.4	04.1	13.3	04.3	14
15	14.5	03.6	14.5	03.9	14.4	04.1	14.4	04.4	14.3	04.4	14.3	04.6	15
16	15.5	04.0	15.5	04.1	15.4	04.4	15.3	04.6	15.3	04.7	15.2	04.9	16
17	16.5	04.1	16.4	04.4	16.7	04.7	16.3	04.9	16.3	05.0	16.2	05.2	17
18	17.5	04.4	17.4	04.7	17.3	05.0	17.2	05.2	17.2	05.3	17.1	05.6	18
19	18.4	04.6	18.4	04.9	18.3	05.2	18.2	05.5	18.2	05.5	18.1	05.9	19
20	19.4	01.9	19.3	05.2	19.2	05.5	19.1	05.8	19.1	05.8	19.0	06.2	20
21	20.4	05.1	20.3	05.4	20.2	05.8	20.1	06.1	20.1	06.1	20.0	06.5	21
22	21.3	05.3	21.2	05.7	21.1	06.1	21.0	06.4	21.0	06.4	20.9	06.8	22
23	22.3	05.6	22.2	06.0	22.1	06.3	22.0	06.7	22.0	06.7	21.9	07.1	23
24	23.3	05.8	23.2	06.2	23.1	06.6	23.0	06.8	22.9	07.0	22.8	07.4	24
25	24.2	06.0	24.1	06.5	24.0	06.9	23.9	07.3	23.9	07.3	23.8	07.7	25
26	25.2	06.3	25.1	06.7	24.9	07.2	24.9	07.5	24.9	07.6	24.7	08.0	26
27	26.2	06.6	26.1	07.0	25.9	07.4	25.8	07.8	25.8	07.9	25.7	08.3	27
28	27.2	06.8	27.0	07.2	26.9	07.7	26.8	08.1	26.8	08.2	26.6	08.6	28
29	28.1	07.0	28.0	07.5	27.8	08.0	27.8	08.4	27.7	08.5	27.6	09.0	29
30	29.1	07.3	29.0	07.8	28.7	08.3	28.7	08.7	28.7	08.8	28.5	09.3	30
31	30.1	07.5	29.9	08.0	29.8	08.5	29.7	09.0	29.6	09.1	29.5	09.6	31
32	31.0	07.9	30.9	08.3	30.7	08.8	30.6	09.3	30.6	09.3	30.4	10.0	32
33	32.0	08.0	31.9	08.5	31.7	09.1	31.6	09.6	31.6	09.6	31.4	10.2	33
34	33.0	08.3	32.8	08.8	32.7	09.4	32.5	09.9	32.5	09.9	32.3	10.5	34
35	33.9	08.5	33.8	09.0	33.6	09.6	33.5	10.2	33.5	10.2	33.3	10.8	35
36	34.9	08.7	34.8	09.3	34.6	09.9	34.4	10.4	34.4	10.5	34.2	11.1	36
37	35.9	09.0	35.7	09.6	35.6	10.2	35.4	10.7	35.4	10.8	35.2	11.4	37
38	36.9	09.2	36.7	09.8	36.5	10.5	36.4	11.0	36.3	11.1	36.1	11.7	38
39	37.8	09.5	37.7	10.1	37.5	10.7	37.3	11.3	37.3	11.4	37.1	12.0	39
40	38.8	09.7	38.6	10.3	38.4	11.0	38.3	11.6	38.2	11.7	38.0	12.4	40
41	39.8	10.0	39.6	10.6	39.4	11.3	39.2	11.9	39.2	12.0	39.0	12.7	41
42	40.7	10.2	40.6	10.9	40.4	11.6	40.2	12.2	40.2	12.3	39.9	13.0	42
43	41.7	10.4	41.5	11.1	41.3	11.8	41.1	12.5	41.1	12.6	40.9	13.3	43
44	42.7	10.7	42.5	11.5	42.3	12.1	42.1	12.8	42.1	12.9	41.8	13.6	44
45	43.6	10.9	43.5	11.6	43.2	12.4	43.1	13.1	43.0	13.1	42.8	13.9	45
46	44.6	11.2	44.4	11.9	44.2	12.7	44.0	13.3	44.0	13.4	43.7	14.2	46
47	45.6	11.4	45.4	12.2	45.2	12.9	45.0	13.6	44.9	13.7	44.7	14.5	47
48	46.6	11.7	46.4	12.4	46.1	13.2	45.9	13.9	45.9	14.0	45.6	14.8	48
49	47.5	11.9	47.3	12.7	47.1	13.5	46.9	14.2	46.9	14.3	46.6	15.1	49
50	48.5	12.1	48.3	12.9	48.1	13.8	47.8	14.5	47.8	14.6	47.5	15.4	50
Diff.	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Diff.
	6¾ Deg.		75 Deg.		74 Deg.		6½ Point		73 Deg.		72 Deg.		

Dist.	1¼ Point.		15 Deg.		16 Deg.		1½ Point.		17 Deg.		18 Deg.		Dist.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
51	49.5	12.4	49.3	13.2	49.0	14.0	48.8	14.8	48.8	14.9	48.5	15.8	51
52	50.4	12.6	50.2	13.5	49.9	14.3	49.7	15.1	49.7	15.2	49.4	16.1	52
53	51.4	12.9	51.2	13.7	50.9	14.6	50.7	15.3	50.7	15.5	50.4	16.4	53
54	52.3	13.1	52.2	14.0	51.9	14.9	51.7	15.7	51.6	15.8	51.3	16.7	54
55	53.3	13.4	53.1	14.2	52.9	15.2	52.6	16.0	52.6	16.1	52.3	17.0	55
56	54.3	13.6	54.1	14.5	53.8	15.4	53.6	16.2	53.5	16.4	53.3	17.3	56
57	55.3	13.8	55.1	14.8	54.8	15.7	54.5	16.5	54.5	16.7	54.2	17.6	57
58	56.3	14.1	56.0	15.0	55.7	16.0	55.5	16.8	55.5	17.0	55.2	17.9	58
59	57.2	14.3	57.0	15.3	56.7	16.3	56.5	17.1	56.4	17.2	56.1	18.2	59
60	58.2	14.6	58.0	15.5	57.7	16.5	57.4	17.4	57.4	17.5	57.1	18.5	60
61	59.2	14.8	58.9	15.8	68.6	16.8	58.4	17.7	58.3	17.8	58.0	18.8	61
62	60.1	15.1	59.9	16.1	69.6	17.1	59.3	18.0	59.3	18.1	59.0	19.2	62
63	61.1	15.3	60.8	16.3	60.5	17.4	60.3	18.3	60.2	18.4	59.9	19.5	63
64	62.1	15.5	61.8	16.6	61.5	17.6	61.2	18.6	61.2	18.7	60.9	19.8	64
65	63.0	15.8	62.8	16.8	62.5	17.9	62.2	18.9	62.2	19.0	61.8	20.1	65
66	64.0	16.0	63.7	17.1	63.4	18.3	63.2	19.2	63.1	19.3	62.8	20.4	66
67	65.0	16.3	64.7	17.4	64.4	18.5	64.1	19.4	64.1	19.6	63.7	20.7	67
68	66.0	16.5	65.7	17.6	65.4	18.7	65.1	19.7	65.0	19.9	64.7	21.0	68
69	66.9	16.8	66.6	17.9	66.3	19.0	66.0	20.0	66.0	20.2	65.6	21.3	69
70	67.9	17.0	67.6	18.1	67.3	19.3	67.0	20.3	66.9	20.5	66.6	21.6	70
71	68.9	17.2	68.6	18.3	68.2	19.6	67.9	20.6	67.9	20.8	67.5	21.9	71
72	69.8	17.5	69.5	18.6	69.2	19.8	68.9	20.9	68.8	21.0	68.5	22.2	72
73	70.8	17.7	70.5	18.9	70.2	20.1	69.8	21.2	69.8	21.3	69.4	22.6	73
74	71.8	18.0	71.5	19.1	71.1	20.4	70.8	21.5	70.8	21.6	70.4	22.9	74
75	72.7	18.2	72.4	19.4	72.1	20.7	71.8	21.8	71.7	21.9	71.3	23.2	75
76	73.7	18.5	73.4	19.7	73.1	20.9	72.7	22.1	72.7	22.2	72.3	23.5	76
77	74.7	18.7	74.4	19.9	74.0	21.2	73.7	22.3	73.6	22.5	73.2	23.8	77
78	75.7	18.9	75.3	20.2	75.0	21.5	74.6	22.6	74.6	22.8	74.2	24.1	78
79	76.6	19.2	76.3	20.4	75.9	21.8	75.6	22.9	75.5	23.1	75.1	24.4	79
80	77.6	19.4	77.3	20.7	76.9	21.0	76.6	23.2	76.5	23.4	76.1	24.7	80
81	78.6	19.7	78.2	21.0	77.9	22.3	77.5	23.5	77.5	23.7	77.0	25.0	81
82	79.5	19.9	79.2	21.2	78.8	22.6	78.5	23.8	78.4	24.0	78.0	25.3	82
83	80.5	20.3	80.2	21.5	79.8	22.9	79.4	24.1	79.4	24.3	79.0	25.6	83
84	81.5	20.4	81.1	21.7	80.8	23.1	80.4	24.4	80.3	24.5	79.9	26.0	84
85	82.4	20.7	82.1	22.0	81.7	23.4	81.3	24.7	81.3	24.8	80.8	26.3	85
86	83.4	20.9	83.1	22.3	82.7	23.7	82.3	25.0	82.2	25.1	81.8	26.6	86
87	84.4	21.1	84.0	22.5	83.6	24.0	83.3	25.2	83.2	25.4	82.7	26.9	87
88	85.4	21.4	85.0	22.8	84.6	24.2	84.2	25.5	84.1	25.7	83.7	27.2	88
89	86.3	21.6	86.0	23.0	85.6	24.5	85.2	25.8	85.1	26.0	84.6	27.5	89
90	87.3	21.9	86.9	23.3	86.5	24.8	86.1	26.1	86.1	26.3	85.6	27.8	90
91	88.3	22.1	87.9	23.5	87.5	25.1	87.1	26.4	87.0	26.6	86.5	28.1	91
92	89.2	22.4	88.9	23.8	88.4	25.3	88.0	26.7	88.0	26.9	87.5	28.4	92
93	90.2	22.6	89.8	24.1	89.4	25.6	89.0	27.0	88.9	27.2	88.4	28.7	93
94	91.2	22.8	90.8	24.3	90.4	25.9	90.0	27.3	89.9	27.5	89.4	29.0	94
95	92.1	23.1	91.8	24.6	91.3	26.2	90.9	27.6	90.8	27.8	90.3	29.3	95
96	93.1	23.3	92.7	24.8	92.3	26.4	91.9	27.9	91.8	28.1	91.3	29.7	95
97	94.1	23.6	93.7	25.1	93.2	26.7	92.8	28.2	92.8	28.4	92.3	30.0	97
98	95.1	23.8	94.7	25.4	94.2	27.0	93.8	28.4	93.7	28.6	93.2	30.3	98
99	96.0	24.1	95.6	25.6	95.2	27.3	94.7	28.7	94.7	28.9	94.2	30.6	99
100	97.0	24.3	96.6	25.9	96.1	27.6	97.7	29.0	95.6	29.2	95.1	30.9	100
Dist.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dist.
	6¾ Point.		75 Deg.		74 Deg.		6½ Point.		73 Deg.		72 Deg.		

Diff.	19 Deg.		1¾ Point		20 Deg.		21 Deg.		22 Deg.		2 Points		Diff.
	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	
1	00.9	00.3	00.9	00.3	00.9	00.3	00.9	00.4	00.9	00.4	00.9	00.4	1
2	01.9	00.6	01.9	00.7	01.9	00.7	01.9	00.7	01.8	00.7	01.8	00.8	2
3	02.8	01.0	02.8	01.0	02.8	01.0	02.8	01.1	02.8	01.1	02.8	01.1	3
4	03.8	01.3	03.8	01.3	03.8	01.4	03.7	01.4	03.7	01.5	03.7	01.5	4
5	04.7	01.6	04.7	01.7	04.7	01.7	04.7	01.8	04.6	01.9	04.6	01.9	5
6	05.7	01.9	05.6	02.0	05.6	02.0	05.6	02.1	05.6	02.2	05.5	02.3	6
7	06.6	02.3	06.6	02.4	06.6	02.4	06.5	02.5	06.5	02.6	06.5	02.7	7
8	07.6	02.6	07.5	02.7	07.5	02.7	07.5	02.9	07.4	03.0	07.4	03.1	8
9	08.5	02.9	08.5	03.0	08.5	03.1	08.4	03.2	08.3	03.4	08.3	03.4	9
10	09.5	03.3	09.4	03.4	09.4	03.4	09.3	03.6	09.3	03.7	09.2	03.8	10
11	10.4	03.6	10.4	03.7	10.3	03.8	10.3	03.9	10.2	04.1	10.2	04.2	11
12	11.3	03.9	11.3	04.0	11.3	04.1	11.2	04.3	11.1	04.5	11.1	04.6	12
13	12.3	04.2	12.2	04.4	12.2	04.4	12.1	04.7	12.0	04.9	12.0	05.0	13
14	13.2	04.6	13.2	04.7	13.2	04.8	13.1	05.0	13.0	05.2	12.9	05.4	14
15	14.2	04.9	14.1	05.1	14.1	05.1	14.0	05.4	13.9	05.6	13.9	05.7	15
16	15.1	05.2	15.1	05.4	15.0	05.4	14.9	05.7	14.8	06.0	14.8	06.1	16
17	16.1	05.5	16.0	05.7	16.0	05.8	15.9	06.1	15.8	06.4	15.7	06.5	17
18	17.0	05.9	16.9	06.1	16.9	06.3	16.8	06.4	16.7	06.7	16.6	06.8	18
19	18.0	06.1	17.9	06.4	17.9	06.5	17.7	06.8	17.6	07.1	17.6	07.3	19
20	18.9	06.5	18.8	06.7	18.8	06.8	18.7	07.2	18.5	07.5	18.5	07.6	20
21	19.9	06.8	19.8	07.1	19.7	07.2	19.6	07.5	19.5	07.9	19.4	08.0	21
22	20.8	07.2	20.7	07.4	20.7	07.5	20.5	07.9	20.4	08.2	20.3	08.4	22
23	21.7	07.5	21.7	07.7	21.6	07.9	21.5	08.2	21.3	08.6	21.2	08.8	23
24	22.7	07.8	22.6	08.1	22.5	08.2	22.4	08.6	22.2	09.0	22.2	09.2	24
25	23.6	08.1	23.5	08.4	23.5	08.5	23.3	09.0	23.2	09.4	23.1	09.6	25
26	24.6	08.4	24.5	08.8	24.4	08.9	24.3	09.3	24.1	09.7	24.0	09.9	26
27	25.5	08.8	25.4	09.1	25.4	09.2	25.2	09.7	25.0	10.1	24.9	10.3	27
28	26.5	09.1	26.4	09.4	26.3	09.6	26.1	10.0	26.0	10.5	25.9	10.7	28
29	27.4	09.4	27.3	09.8	27.2	09.9	27.1	10.4	26.9	10.9	26.8	11.1	29
30	28.4	09.8	28.2	10.1	28.2	10.3	28.0	10.7	27.8	11.2	27.7	11.5	30
31	29.3	10.1	29.2	10.4	29.1	10.6	28.9	11.1	28.7	11.6	28.6	11.9	31
32	30.3	10.4	30.1	10.8	30.1	10.9	29.9	11.5	29.7	12.0	29.6	12.2	32
33	31.2	10.7	31.1	11.1	31.0	11.3	30.8	11.8	30.6	12.4	30.5	12.6	33
34	32.1	11.1	32.0	11.5	31.9	11.6	31.7	12.2	31.5	12.7	31.4	13.0	34
35	33.1	11.4	33.0	11.8	32.9	12.0	32.7	12.5	32.4	13.1	32.3	13.4	35
36	34.0	11.7	33.9	12.1	33.8	12.3	33.6	12.9	33.4	13.5	33.3	13.8	36
37	35.0	12.1	34.8	12.5	34.8	12.6	34.5	13.3	34.3	13.9	34.2	14.2	37
38	35.9	12.4	35.8	12.8	35.7	13.0	35.5	13.6	35.2	14.2	35.1	14.5	38
39	36.9	12.6	36.7	13.1	36.6	13.3	36.4	14.0	36.2	14.6	36.0	14.9	39
40	37.8	13.0	37.7	13.5	37.6	13.7	37.3	14.3	37.1	15.0	36.9	15.3	40
41	38.8	13.3	38.6	13.8	38.5	14.0	38.3	14.7	38.0	15.3	37.9	15.7	41
42	39.7	13.7	39.5	14.1	39.5	14.4	39.2	15.1	38.9	15.7	38.8	16.1	42
43	40.7	14.0	40.5	14.5	40.4	14.7	40.1	15.4	39.9	16.1	39.7	16.5	43
44	41.6	14.3	41.4	14.8	41.3	15.0	41.1	15.8	40.8	16.5	40.6	16.8	44
45	42.6	14.6	42.4	15.2	42.3	15.4	42.0	16.1	41.7	16.8	41.6	17.2	45
46	43.5	15.0	43.3	15.5	43.2	15.7	42.9	16.5	42.6	17.2	42.5	17.6	46
47	44.4	15.3	44.2	15.8	44.2	16.1	43.9	16.8	43.6	17.6	43.4	18.0	47
48	45.4	15.6	45.2	16.2	45.1	16.4	44.8	17.2	44.5	18.0	44.3	18.4	48
49	46.3	15.9	46.1	16.5	46.0	16.8	45.7	17.6	45.4	18.3	45.3	18.7	49
50	47.3	16.3	47.1	16.8	47.0	17.1	46.7	17.9	46.4	18.7	46.2	19.1	50
Diff.	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Diff.
	71 Deg.		6¼ Point		70 Deg.		69 Deg.		68 Deg.		6 Points		

Dist.	19 Deg.		1¾ Point		20 Deg.		21 Deg.		22 Deg.		2 Points.		Dist.
	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	
51	48 2	16.6	48 c	17.2	47 9	17.4	47.6	18 3	47·3	19·1	47·1	19·5	51
52	49·2	16.9	49.0	17.5	48·9	17.8	48.5	18.6	48.2	19.4	48.0	19·9	52
53	50.1	17.3	49·9	17·9	49.8	18.1	49·5	19 0	49.1	19·8	49·0	20·3	53
54	51·1	17.6	50.8	18.2	50.7	18.5	50.4	19·3	50.1	20.2	49 9	20.7	54
55	52.0	17.9	51.8	18.5	51.7	18.8	51.3	19.7	51.0	20.6	50.8	21.0	55
56	52.9	18.2	52.7	18.9	52.6	19.2	52.3	20.1	51·9	21.0	51.7	21.4	56
57	53·9	18.5	53.7	19.2	53.6	19 5	53.2	20.4	52.8	21.3	52.7	21.8	57
58	54.8	18 9	54.6	19·5	54·5	19.8	54·1	20.8	53.8	21.7	53.6	22.2	58
59	55.8	19.2	55·5	19.9	55.4	20.2	55·1	21.1	54·7	22.1	54·5	22.6	59
60	56.7	19·5	56 5	20.2	56.4	20.5	56.0	21.5	55.6	22.5	55·4	23·0	60
61	57·7	19.0	57.4	20.5	57.3	20.9	56.9	21.9	56.5	22.8	56.3	23.3	61
62	58.6	20.2	58 .	20.9	58·3	21.2	57·9	22.2	57.5	23.2	57·3	23.7	62
63	59.6	20.5	59··	21.2	59.2	21.5	58.8	22.6	58.4	23.6	58.2	24·1	63
64	60 5	20.8	60.3	21.6	60.1	21 9	59·7	22.9	59·3	24.0	59·1	24·5	64
65	61.5	21.2	61.2	21.9	61.1	22.2	60 7	23·3	60·3	24·3	60.0	24·9	65
66	62.4	21.5	62.1	22.2	62.0	2·.6	61.6	23.6	61.2	24.7	61.0	25·3	66
67	63 3	21.8	63.1	22.6	63.0	22.9	62.5	24.0	62 1	25 1	61.9	25.6	67
68	64·3	22.1	64.0	22.9	63·9	23·3	63·5	24·4	63.0	25·5	62.8	26.0	68
69	65.2	22.5	65.0	23.2	64.8	23·6	64.4	24.7	64.0	25 8	63.7	26·4	69
70	66.2	22 8	65.9	23 6	65.8	23 9	65·3	25.1	64.9	26.2	64·7	26.8	70
71	67.1	23.1	66.8	23·9	66.7	24·3	66.3	25·4	65.8	26.6	65.6	27.2	71
72	68.1	23 4	67.8	24.2	67·7	24.6	67.2	25.8	66.7	27.0	66.5	27 6	72
73	69.0	23 8	68.7	24.6	68.6	25.0	68.1	26.2	67.7	27.3	67 4	27.9	73
74	70.0	24.1	69.7	24 9	69.5	25.3	69.1	26.5	68.6	27.7	68.4	28.3	74
75	70 9	24.4	70.6	25·3	70.5	25.6	70.0	26.9	69.5	28.1	69·3	28 7	75
76	71·9	24.7	71.6	25.6	71.4	26.0	70.9	27.2	70.5	28.5	70.2	29.1	76
77	72.8	25.1	72.5	25·9	72.4	26.3	71·9	27.6	71 4	28.8	71.1	29.5	77
78	73 7	25.4	73.4	25.3	73·3	26.7	72.8	27.9	72.3	29.2	72.1	29.8	78
79	74 7	25.7	74 4	26 6	74·2	27.0	73·7	28.3	73.2	29 6	73.0	30.2	79
80	75 6	26.0	75·3	26·9	75·2	27.4	74·7	28.7	74·2	30.0	73·9	30.6	80
81	76 6	26 4	76.3	27.3	76·1	27.7	75.6	29.0	75.1	30.3	74.8	31.0	81
82	77·5	26.7	77·2	27.6	77·1	28.0	76·5	29 4	76 0	30.7	75.8	31.4	82
83	78.5	27.0	78 1	28.0	78.0	28.4	77.5	29.7	76.9	31.1	76.7	31.8	83
84	79·4	27.3	79.1	28.3	78·9	28.7	78.4	30.1	77·9	31.5	77.6	32.1	84
85	80.4	27.7	80 1	28.6	79 9	29·1	79 3	30.5	78.8	31 8	78.6	32·5	85
86	81.3	28.0	81.0	29.0	80.8	29.4	80.3	30.8	79.7	32.2	79·4	32.9	86
87	82.3	28.3	81·9	29.3	81.8	29.7	81.2	31.2	80.7	32.6	80.4	33 3	87
88	83.2	28.6	82 8	29.6	82.7	30 1	82.1	31.5	81.6	33.0	81·3	33 7	88
89	84 1	29 0	83 8	30.0	83.6	30 4	83.1	31.9	82.5	33·3	82.2	34·1	89
90	85·1	29·3	84·7	30·3	84 6	30.8	84.0	32.3	83 4	33·7	83·1	34·4	90
91	86.0	29.6	85.7	30.7	85·5	31.1	84.9	32.6	84.4	34.1	84.1	34.8	91
92	87.0	29.9	86.6	31.0	86·4	31·5	85·9	33.0	85.3	34·5	85·0	35·2	92
93	88 0	30.3	87 6	31.3	87.4	31.8	86.8	33·3	86.2	34.8	85·9	35.6	93
94	88.9	30.6	88.5	31.7	88.3	32.1	87.7	33·7	87.2	35.2	86.8	36.0	94
95	89.8	30.9	89 4	32.0	89.3	32.5	88.7	34·0	88.1	35.6	87·8	36·3	95
96	90 8	31.3	90.4	32 3	90.2	32 8	89.6	34·4	89.0	35.9	88.7	36.7	96
97	91.7	31.6	91·3	32.7	91.1	33.2	90.5	34·8	89 9	36.3	89.6	37.1	97
98	92.7	31 9	92.3	33·0	92.1	33.5	91·5	35 1	90 9	36.7	90·3	37.5	98
99	93 6	32.2	93·2	33.3	93.0	33·9	92 4	35.5	91.8	37.1	91·5	37.9	99
100	94·3	32.6	94·2	33.7	9·.0	34.2	93·4	35.8	92 7	37·5	92·4	38·3	100
Dist.	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dist.
	71 Deg.		6¼ Point		70 Deg.		69 Deg.		68 Deg.		6 Points.		

Dist.	23 Deg.		24 Deg.		25 Deg.		2¼ Point		26 Deg		27 Deg.		Dist.
	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	
1	00.9	00.4	00.9	00.4	00.9	00.4	00.9	00.4	00.9	00.4	00.9	00.4	1
2	01.8	00.8	01.8	00.8	01.8	00.8	01.8	00.9	01.8	00.9	01.8	00.9	2
3	02.8	01.2	02.7	01.2	02.7	01.3	02.7	01.3	02.7	01.3	02.7	01.4	3
4	03.7	01.6	03.6	01.6	03.6	01.7	03.6	01.7	03.6	01.7	03.6	01.8	4
5	04.6	01.9	04.6	02.0	04.5	02.1	04.5	02.1	04.5	02.2	04.5	02.3	5
6	05.5	02.3	05.5	02.4	05.4	02.5	05.4	02.6	05.4	02.6	05.3	02.7	6
7	06.4	02.7	06.4	02.8	06.3	03.0	06.3	03.0	06.3	03.1	06.2	03.2	7
8	07.4	03.1	07.3	03.2	07.2	03.4	07.2	03.4	07.2	03.5	07.1	03.6	8
9	08.3	03.5	08.2	03.7	08.0	03.8	08.1	03.8	08.1	03.9	08.0	04.1	9
10	09.2	03.9	09.1	04.1	09.1	04.2	09.0	04.3	09.0	04.4	08.9	04.5	10
11	10.1	04.3	10.0	04.5	10.0	04.6	09.9	04.7	09.9	04.8	09.8	05.0	11
12	11.0	04.7	11.0	04.9	10.9	05.1	10.8	05.1	10.8	05.3	10.7	05.4	12
13	12.0	05.1	11.9	05.3	11.8	05.5	11.7	05.6	11.7	05.7	11.6	05.8	13
14	12.9	05.5	12.8	05.7	12.7	05.9	12.7	05.9	12.6	06.1	12.5	06.4	14
15	13.8	05.9	13.7	06.1	13.6	06.3	13.6	06.4	13.5	06.6	13.4	06.8	15
16	14.7	06.2	14.6	06.5	14.5	06.8	14.5	06.6	14.4	07.0	14.3	07.3	16
17	15.6	06.6	15.5	06.9	15.4	07.2	15.4	07.3	15.3	07.4	15.1	07.7	17
18	16.6	07.0	16.4	07.3	16.3	07.6	16.3	07.7	16.2	07.9	16.0	08.2	18
19	17.5	07.4	17.4	07.7	17.2	08.0	17.2	08.1	17.1	08.3	16.9	08.6	19
20	18.4	07.8	18.3	08.1	18.1	08.4	18.1	08.5	18.0	08.8	17.8	09.1	20
21	19.3	08.2	19.2	08.5	19.0	08.9	19.0	09.0	18.9	09.2	18.7	09.5	21
22	20.2	08.6	20.1	08.9	19.9	09.3	19.9	09.4	19.8	09.9	19.6	10.0	22
23	21.2	09.0	21.0	09.3	20.8	09.7	20.8	09.8	20.7	10.1	20.5	10.4	23
24	22.1	09.4	21.9	09.8	21.7	10.1	21.7	10.3	21.6	10.5	21.4	10.9	24
25	23.0	09.8	22.8	10.2	22.7	10.6	22.6	10.7	22.5	11.0	22.3	11.3	25
26	23.9	10.2	23.7	10.6	23.6	11.0	23.5	11.1	23.4	11.4	23.2	11.8	26
27	24.8	10.5	24.7	11.0	24.5	11.4	24.4	11.5	24.3	11.8	24.1	12.3	27
28	25.8	10.9	25.6	11.4	25.4	11.8	25.3	12.0	25.2	12.3	24.9	12.7	28
29	26.7	11.3	26.5	11.8	26.3	12.3	26.2	12.4	26.1	12.7	25.8	13.2	29
30	27.6	11.7	27.4	12.2	27.2	12.7	27.1	12.8	27.0	13.1	26.7	13.6	30
31	28.5	12.1	28.3	12.6	28.1	13.1	28.0	13.3	27.9	13.6	27.6	14.1	31
32	29.5	12.5	29.2	13.0	29.0	13.5	28.9	13.7	28.7	14.0	28.5	14.5	32
33	30.4	12.9	30.1	13.4	29.9	13.9	29.8	14.1	29.6	14.4	29.4	15.0	33
34	31.3	13.3	31.1	13.8	30.8	14.4	30.7	14.5	30.6	14.9	30.3	15.4	34
35	32.2	13.7	32.0	14.2	31.7	14.8	31.6	15.0	31.5	15.3	31.2	15.9	35
36	33.1	14.1	32.9	14.6	32.6	15.2	32.5	15.4	32.4	15.8	32.1	16.3	36
37	34.1	14.4	33.8	15.0	33.5	15.6	33.4	15.8	33.2	16.2	33.0	16.8	37
38	35.0	14.8	34.7	15.4	34.4	16.0	34.3	16.2	34.1	16.6	33.9	17.2	38
39	35.9	15.2	35.6	15.9	35.3	16.5	35.3	16.7	35.0	17.1	34.7	17.7	39
40	36.8	15.6	36.5	16.3	36.2	16.9	36.2	17.1	35.9	17.5	35.6	18.2	40
41	37.7	16.0	37.5	16.7	37.2	17.3	37.1	17.5	36.8	18.0	36.5	18.6	41
42	38.7	16.4	38.4	17.1	38.1	17.7	38.0	18.0	37.7	18.4	37.4	19.1	42
43	39.6	16.8	39.3	17.5	39.0	18.2	38.9	18.4	38.6	18.8	38.3	19.5	43
44	40.5	17.2	40.2	17.9	39.9	18.6	39.8	18.8	39.5	19.3	39.2	20.0	44
45	41.4	17.6	41.1	18.3	40.8	19.0	40.7	19.2	40.4	19.7	40.1	20.4	45
46	42.3	18.0	42.0	18.7	41.7	19.4	41.6	19.7	41.3	20.2	41.0	20.9	46
47	43.3	18.4	42.9	19.1	42.6	19.9	42.5	20.1	42.2	20.6	41.9	21.3	47
48	44.2	18.8	43.8	19.5	43.5	20.3	43.4	20.5	43.1	21.0	42.8	21.8	48
49	45.1	19.2	44.8	19.9	44.4	20.7	44.3	20.9	44.0	21.5	43.7	22.2	49
50	46.0	19.5	45.7	20.3	45.3	21.1	45.2	21.4	44.9	21.9	44.5	22.7	50
Dist.	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dist.
	67 Deg.		66 Deg		65 Deg		5¾ Point		64 Deg.		63 Deg.		

Diff.	23 Deg.		24 Deg.		25 Deg		2¼ Point		26 Deg.		27 Deg.		Diff.
	Lat	*Dep*	*Lat*	*Dep*	*Lat*	*Dep*	*Lat*	*Dep*	*Lat*	*Dep*	*Lat*	*Dep*	
51	46.9	19.9	46.6	20.7	46.2	21.5	46.1	21.8	45.8	22.3	45.4	23.2	51
52	47 9	20.3	47.5	21.1	47.1	22.0	47.0	22.2	46.7	22.8	46.3	23.6	52
53	48.8	20.7	48.4	21.5	48.0	22.4	47.9	22.7	47.6	23.2	47.2	24.1	53
54	49.7	21.1	49.3	22.0	48.9	22.8	48.8	23.1	48.5	23.7	48.1	24.5	54
55	50 6	21.5	50.2	22.4	49.8	23.2	49.7	23.5	49.4	24.1	49.0	25.0	55
56	51.5	21.9	51.2	23.8	50.7	23 7	50 6	23.9	50.3	24.5	49 9	25.4	56
57	52.5	22.3	52.1	23.2	51.7	24.1	51.5	24.4	51.2	25 0	50.8	25.9	57
58	53.4	22.7	53.0	23.6	52.6	24.5	52.4	24.8	52.1	25 4	51.7	26.3	58
59	54.3	23.0	53.9	24.0	53.5	24.9	53.3	25.2	53.0	25 9	52.6	26.8	59
60	55.2	23 4	54.8	24.4	54.4	25.4	54.2	25.6	53.9	26.3	53.5	27.2	60
61	56.1	23 8	55.7	24 8	55.3	25.8	55.1	26.1	54.8	26.7	54.4	27 7	61
62	57.1	24.2	56.6	25.2	56.2	26.2	56.0	26.5	55.7	27.2	55.2	28.1	62
63	58.0	24.6	57.5	25.6	57.1	26.6	56.9	26.9	56.6	27 6	56.1	28 6	63
64	58.9	25.0	58 5	26.0	58.0	27.0	57.9	27.4	57.5	28.0	57.0	29.1	64
65	59.8	25.4	59.4	26 4	58.9	27.5	58.8	27.8	58.4	28.5	57.9	29.5	65
66	60.7	25.8	60.3	26.8	59.8	27.9	59.7	28.2	59.3	28.9	58.8	30.0	66
67	61.7	26.2	61.2	27.2	60.7	28.3	60 6	28.6	60.2	29.4	59.7	30.4	67
68	62.6	26 6	62.1	27.7	61.6	28.7	61.5	29.1	61.1	29 8	60.6	30.9	68
69	63 5	27.0	63.0	28.1	62.5	29.2	62.4	29 5	62 0	30.2	61.5	31.3	69
70	64.4	27.3	63.9	28.5	63.4	29.6	63.3	29.9	62.9	30.7	62.4	31.8	70
71	65.4	27.7	64.9	28.9	64.3	30 0	64.2	30.4	63.8	31.1	63.3	32.2	71
72	66.3	28.1	65.8	29.3	65.2	30.4	65.1	30.8	64.7	31.6	64.2	32.7	72
73	67.2	28.5	66.7	29.7	66.2	30.8	66.0	31.2	65.6	32.0	65.0	33.1	73
74	68.1	28.9	67.6	30.1	67.1	31.3	66.9	31.6	66.5	32.4	65.9	33.6	74
75	69.0	29.3	68.5	30.5	68.0	31.7	67.8	32.1	67.4	32.9	66.8	34.1	75
76	70.0	29.7	69.4	30.9	68.9	32.1	68.7	32.5	68.3	33.3	677	34.5	76
77	70.9	30.1	70.3	31.3	69.8	32.5	69.6	32.9	69.2	33.7	68.0	35.0	77
78	71.8	30.5	71.2	31.7	70.7	33.0	70.5	33.3	70.1	34.2	69.5	35.4	78
79	72.7	30.9	72.2	32.1	71.6	33.4	71.4	33.8	71.0	34.6	70.4	35.9	79
80	73.6	31.3	73.1	32.5	72.5	33.8	72.3	34.2	71.9	35.1	71.3	36.3	80
81	74 6	31.6	74 0	32.9	73.4	34.2	73.2	34.6	72.8	35.5	72.2	36.8	81
82	75.5	32.0	74.9	33 3	74.3	34.7	74.1	35.1	73.7	35.9	73.1	37.2	82
83	76 4	32.4	75.8	33.8	75.2	35.1	75.0	35.5	74.6	36.4	74.0	37.7	83
84	77.3	32 8	76.7	34.2	76.1	35.5	75.9	35.9	75.5	36.8	74.8	38.1	84
85	78.2	33.2	77 6	34.6	77 0	35.9	76.8	36.3	76.4	37.3	75.7	38.6	85
86	79.2	33.6	78 6	35.0	77.9	36.3	77.7	36.8	77.8	37.7	76.6	39.0	86
87	80.1	34.0	79.5	35.4	78.8	36.8	78.6	37.2	78.2	38.1	77.5	39.5	87
88	81.0	34.4	80 4	35.8	79.7	37.2	79 5	37.6	79.1	38.6	78.4	40.0	88
89	81.9	34.8	81.3	36.2	80.7	37.6	80.5	38.1	80.0	39.0	79.3	40.4	89
90	82.8	35.2	82.2	36.6	81.6	38.0	81.4	38.5	80.9	39.4	80.2	40.9	90
91	83.7	35.6	83.1	37.0	82.5	38.5	82.3	38.9	81.8	39.8	81.1	41.3	91
92	84.7	35.9	84.0	37.4	83.4	38.9	83.2	39.3	82.7	40.3	82.0	41.8	92
93	85.6	36.3	85.0	37.8	84.3	39.3	84.1	39.8	83.6	40.8	82.9	42.2	93
94	86.5	36.7	85.9	38.2	85.2	39.7	85.0	40.2	84.5	41.2	83.8	42.7	94
95	87.4	37.1	86.8	38.6	86.1	40.1	85.9	40.6	85.4	41.6	84.6	43.1	95
96	88.4	37.5	87.7	39.0	87.0	40.6	86.8	41.0	86.3	42.1	85.5	43.6	96
97	89.3	37.9	88.6	39.4	87.9	41.0	87.7	41.5	87.2	42.5	86.4	44.0	97
98	90 2	38.3	89.5	39.9	88.8	41.4	88.6	41.9	88.1	43.0	87.3	44.5	98
99	91.1	38 7	90.4	40.3	89.7	41.8	89.5	42.3	89.0	43.4	88.2	44.9	99
100	92.0	39.1	91.4	40.7	90.6	42.3	90.4	42.7	89.9	43.8	89.1	45.4	100
Diff.	*Dep*	*Lat*	*Dep*	*Lat*	*Dep*	*Lat*	*Dep*	*Lat*	*Dep*	*Lat*	*Dep*	*Lat*	Diff.
	67 Deg.		68 Deg.		65 Deg.		5¾ Point		64 Deg.		63 Deg.		

Diff.	28 Deg.		2½ Point		29 Deg.		30 Deg		2¾ Point		31 Deg.		Diff.
	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	
1	00.9	00.5	00.9	00 5	00.9	00.5	00.9	00 5	00 9	00.5	00.5	00.5	1
2	01.8	00.9	01.8	00 9	01.7	01.0	01.7	01.0	01.7	01.0	01.7	01.0	2
3	02.6	01.4	02.6	01.4	02.6	01.4	02 6	01.5	02.6	01.5	02.6	01.5	3
4	03.5	01.9	03.5	01.9	03.5	01.9	03.5	02.0	03.4	02.1	03.4	02.1	4
5	04.4	02 3	04.4	02.4	04.4	02.4	04.3	02.5	04.3	02 6	04.3	02.6	5
6	05.3	02 8	05.3	02.8	05.2	02.9	05.2	03.0	05.1	03.1	05.1	03.1	6
7	06.2	03.3	06.2	03.3	06.1	03.4	06.1	03.5	06.0	03.6	06 0	03.6	7
8	07.1	03.8	07.1	03.8	07.0	03.9	06.9	04.0	06.9	04.1	06.9	04.1	8
9	07.9	04.2	07.9	04.2	07.9	04.4	07.8	04.5	07.7	04.6	07.7	04.6	9
10	08.8	04.7	08.8	04.7	08.7	04.8	08.7	05.0	08.6	05.1	08.6	05 1	10
11	09 7	05.2	09.7	05.2	09.6	05.3	09 5	05.5	09 4	05.6	09 4	05.7	11
12	10.6	05.6	10.6	05 6	10.5	05.8	10.4	06.0	10.3	06 2	10.3	06.2	12
13	11.5	06.1	11.5	06.1	11.4	06.3	11.3	06.5	11.1	06.7	11.1	06.7	13
14	12.4	06.6	12.3	06.6	12.2	06.8	12.1	07.0	12 0	07.2	12.0	07.2	14
15	13.2	07.0	13.2	07.1	13.1	07 3	13.0	07.5	12.9	07.7	12.9	07.7	15
16	14.1	07.5	14.1	07.5	14.0	07.7	13.9	08.0	13.7	08.2	13 7	08.2	16
17	15.0	08.0	15.0	08.0	14.9	08.2	14.7	08.5	14.6	08.7	14.6	08.8	17
18	15.9	08 4	15.9	08.5	15.7	08.7	15 6	09.0	15 4	09.2	15.4	09.3	18
19	16 8	08.9	16.8	08.9	16.6	09 2	16.4	09.5	16.3	09.8	16 3	09 8	19
20	17.7	09.4	17.6	09.4	17.5	09.7	17.3	10 0	17.1	10.3	17 1	10 3	20
21	18.5	09.9	18.5	09 9	18.4	10.2	18.2	10.5	18.0	10.8	18.0	10.8	21
22	19.4	10.3	19.4	10.3	19.2	10.7	19 0	11.0	18.9	11.3	18.9	11.3	22
23	20.3	10.8	20.3	10.8	20.1	11.1	19.9	11.5	19.7	11.8	19.7	11.8	23
24	21.2	11.3	21.2	11.3	21.0	11.6	20 8	12.0	20.6	12.3	20.6	12.4	24
25	22.1	11.7	22 0	11.8	21.9	12.1	21.6	12.5	21.4	12.8	21.4	12.9	25
26	23.0	12.2	22.9	12.3	22.7	12.6	22.5	13.0	22 3	13.4	22.3	13.4	26
27	23.8	12.7	23.8	12.7	23.6	13.1	23.4	13.5	23.1	13.9	23.1	13 9	27
28	24.7	13 1	24.7	13.2	24.5	13.6	24.2	14.0	24.0	14.4	24.0	14.4	28
29	25.6	13.6	25.6	13.7	25.4	14 1	25.1	14.5	24.9	14.9	24 9	14.9	29
30	26.5	14.1	26.5	14.1	26.2	14 5	26.0	15.0	25.7	15.4	25.7	15.4	30
31	27.4	14.5	27.3	14.6	27.1	15.0	26 8	15.5	26 6	15.9	26.6	16 0	31
32	28.2	15 0	28.2	15.1	28.0	15.5	27.7	16.0	27.4	16.4	27.4	16.5	32
33	29.1	15.5	29.1	15.5	28.9	16.0	28.6	16.5	28.3	17.0	28.3	17.0	33
34	30 0	16.0	30.0	16.0	29.7	16.5	29.5	17.0	29.2	17.5	29.1	17.5	34
35	30.9	16.4	30.9	16 5	30.6	17.0	30.3	17.5	30.0	18.0	30.0	18.0	35
36	31.8	16 9	31.7	17 0	31.5	17.4	31.2	18.0	30.9	18.5	30.9	18.5	36
37	32.7	17.4	32.6	17.4	32.4	17.9	32 0	18.5	31.7	19.0	31.7	19.1	37
38	33.5	17.0	33 5	17.9	33.2	18.4	32.9	19.0	32.5	19.5	32.6	19.6	38
39	34 4	18.3	34.4	18.4	34.1	18.9	33.8	19.5	33.4	20.0	33.4	20.1	39
40	35.3	18.8	35.3	18 9	35.0	19.4	34.6	20.0	34.3	20.6	34.3	20.6	40
41	36.2	19.2	36.1	19.3	35.8	19 9	35.5	20.5	35.2	21.1	35.1	21.1	41
42	37.1	19.7	37.0	19 8	36.7	20.4	36.4	21.0	36.0	21.6	36.0	21.6	42
43	38.0	20.1	37.9	20.3	37.6	20.8	37.2	21.5	36.9	22.1	36.9	22.1	43
44	38 8	20.6	38.8	20.7	38.5	21.3	38.1	22.0	37.7	22.6	37.7	22 6	44
45	39.7	21.1	39.7	21.2	39.3	21.8	39 0	22 5	38.6	23.1	38.6	23.2	45
46	40.6	21.6	40.6	21.7	40.2	22.3	39.8	23.0	39.5	23.6	39.4	23.7	46
47	41.5	22.1	41.4	22.2	41.1	22.8	40.7	23.5	40.3	24.2	40.3	24 2	47
48	42.4	22.5	42.3	22.6	42.0	23.3	41.6	24.0	41.2	24.7	41.1	24.7	48
49	43.3	23.1	43.2	23.2	42.8	23.7	42.4	24.5	42.0	25.2	42.0	25.2	49
50	44.1	23.5	44.1	23.6	43.7	24.2	43.3	25.0	42 9	25.7	42.9	25.7	50
Diff.	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Diff.
	62 Deg.		5⅝ Point		61 Deg		60 Deg.		5¼ Point		59 Deg.		

Dist.	28 Deg.		2½ Points.		29 Deg.		30 Deg.		2¾ Point.		31 Deg.		Dist.
	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	
51	45.0	23.9	45 0	24.0	44.6	24 7	44.2	25.5	43.7	26.2	43.7	26.3	51
52	45.9	24 4	45.9	24.5	45.5	25.2	45.0	26.0	44.6	26 7	44.6	26.8	52
53	46.8	24.9	46.7	25 0	46.3	25.7	45.9	26.5	45.5	27.2	45.4	27.3	53
54	47.7	25.3	47.6	25.5	47.2	26.2	46.8	27.0	46.3	27.8	46.3	27.8	54
55	48 6	25.8	48.5	25.9	48.1	26.7	47.6	27.5	47.2	28.3	47.1	28.3	55
56	49 4	26.3	49.4	26.4	49 0	27.1	48.5	28.0	48.0	28.8	48.0	28.8	56
57	50.3	26.8	50.3	26.9	49.8	27.6	49.4	28.5	48.9	29.3	48.9	29.4	57
58	51.2	27.2	51.2	27.3	50.7	28.1	50.2	29.0	49.7	29.8	49.7	29.9	58
59	52.1	27.7	52.0	27.8	51.6	28.6	51 1	29.5	50.6	30.3	50 6	30.4	59
60	53.0	28.2	52.9	28.3	52.5	29.1	52.0	30.0	51.5	30.8	51.4	30 9	60
61	53 9	28.6	53 8	28.7	53.3	29.6	52.8	30.5	52 3	31.4	52 3	31.4	61
62	54.7	29 1	54.7	29.2	54.2	30.1	53.7	31.0	53.2	31.9	53 1	31.9	62
63	55.6	29.6	55 6	29 7	55.1	30.5	54.6	31.5	54.0	32.4	54 0	32 4	63
64	56.5	30 0	56 4	30.2	56.0	31.0	55.4	32.0	54.9	32.9	54.9	33.0	64
65	57.4	30.5	57.3	30.5	56.8	31.5	56.3	32.5	55 7	33.4	55.7	33.5	65
66	58.3	31.0	58.2	31.1	57.7	32.0	57.2	33.0	56.6	33 9	56.6	34.0	66
67	59.2	31.4	59 1	31.6	58.6	32.5	58.0	33.5	57.5	34.4	57.4	34 5	67
68	60.0	31.9	60.0	32 0	59.5	33.0	58.9	34.0	58.3	35 0	58.3	35.0	68
69	60.9	32.4	60.8	32.5	60.3	33 4	59.7	34.5	59.2	35.5	59.1	35.5	69
70	61.8	32.9	61.7	33.0	61.2	33.9	60.6	35.0	60 0	36.0	60 0	36 0	70
71	62.7	33.3	62.6	33.5	62.1	34 4	61.5	35.5	60.9	36.5	60.9	36.6	71
72	63.6	33 8	63 5	33.9	63.0	34.9	62.3	36.0	61.8	37.0	61.7	37.1	72
73	64.4	34 3	64.4	34.4	63.8	35.4	63.2	36.5	62.6	37.5	62 6	37.6	73
74	65.3	34 7	65.3	34 9	64.7	35.9	64 1	37.0	63.5	38.0	63.4	38.1	74
75	66.2	35.2	66.1	35.4	65 6	36.4	64.9	37.5	64.3	38.6	64.3	38.6	75
76	67.1	35.7	67.0	35.8	66.5	36.8	65.8	38.0	65.2	39.1	65.1	39.1	76
77	68.0	36.1	67 9	36.3	67.3	37.3	66.7	38.5	66.0	39.6	66.0	39.7	77
78	68.9	36.6	68 8	36.8	68.2	37.8	67.5	39.0	66.9	40.1	66.9	40 2	78
79	69.7	37.1	69 7	37.2	69 1	38.3	68.4	39.5	67 8	40.6	67.7	40.7	79
80	70.6	37.6	70.5	37.7	70.0	38.8	69.3	40.0	68.6	41.1	68.6	41.2	80
81	71.5	38.0	71.4	38.2	70.8	39.3	70.1	40.5	69.3	41.6	69 4	41.7	81
82	72.4	38.5	72 3	38.6	71.7	39.7	70 9	41 0	70.3	42.2	70.3	42.2	82
83	73.3	39 0	73.2	39.1	72.6	40 2	71.8	41.5	71.2	42.7	71.1	42.7	83
84	74 2	39.4	74.1	39.6	73.5	40.7	72.7	42.0	72.1	43.2	72.0	43.3	84
85	75.0	39 9	75.0	40.1	74.3	41.2	73 6	42.5	72.9	43.7	72.9	43.8	85
86	75.9	40 4	75.8	40.5	75.2	41.7	74.5	43.0	73.8	44.2	73.7	44.3	86
87	76.8	40.8	76.7	41.0	76.1	42.2	75.3	43.5	74.6	44.7	74.6	44.8	87
88	77.7	41.3	77.6	41.5	77.0	42.7	76.2	44.0	75.5	45.2	75.4	45.3	88
89	78 6	41 8	78.5	41 9	77.8	43.1	77.1	44.5	76.3	45.8	76.3	45.8	89
90	79.5	42 2	79.4	42.4	78.7	43 6	77.9	45.0	77.2	46.3	77.1	46.3	90
91	80.3	42.7	80.2	42.9	79.6	44 1	78 8	45.5	78.1	46 8	78.0	46.9	91
92	81.2	43.2	81.1	43.4	80.5	44.6	79.7	46.0	78.9	47.3	78.9	47.4	92
93	82.1	43.6	82.0	43 8	81.3	45.1	80.5	46.5	79.8	47.8	79.7	47.9	93
94	83.0	44.1	82.9	44 3	82.2	45.6	81.4	47.0	80.6	48.3	80.6	48.4	94
95	83.9	44.6	83 8	44.8	83.1	46.1	82.3	47.5	81.5	48.8	81.4	48.9	95
96	84.8	45.1	84.7	45.2	84.0	46 5	83.1	48.0	82.3	49.3	82.3	49.4	96
97	8.56	45.5	85.5	45.7	84.8	47.0	84.0	48.5	83.2	49.9	83.1	50.0	97
98	86 5	46.0	86.4	45.2	85.7	47.5	84.9	49.0	84.1	50.4	84.0	50.5	98
99	87.4	46 5	87.3	45.7	86.6	48.0	85.7	49.5	84.9	50 9	84 9	51.0	99
100	88.3	46.9	88.2	47.1	87.5	48 5	86.6	50.0	85.8	51.4	85.7	51.5	100
Dist.	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dist.
	62 Deg.		5½ Point.		61 Deg.		60 Deg.		5¼ Point.		59 Deg.		

Diff.	32 Deg.		33 Deg.		3 Points		34 Deg.		35 Deg.		36 Deg.		Diff.
	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	
1	00.8	00.5	00.8	00.5	00.8	00.6	00.8	00.6	00.8	00.6	00.8	00.6	1
2	01.7	01.1	01.7	01.1	01.7	01.1	01.7	01.1	01.6	01.1	01.6	01.2	2
3	02.5	01.6	02.5	01.6	02.5	01.7	02.5	01.7	02.5	01.7	02.4	01.8	3
4	03.4	02.1	03.4	02.2	03.3	02.2	03.3	02.2	03.3	02.3	03.2	02.3	4
5	04.2	02.6	04.2	02.7	04.2	02.8	04.1	02.8	04.1	02.9	04.0	02.9	5
6	05.1	03.2	05.9	03.3	05.0	03.3	05.0	03.4	04.9	03.4	04.8	03.5	6
7	05.9	03.7	05.0	03.8	05.8	03.9	05.8	03.9	05.7	04.0	05.7	04.1	7
8	06.8	04.2	06.7	04.4	06.6	04.4	06.6	04.5	06.5	04.6	06.5	04.7	8
9	07.6	04.8	07.5	04.9	07.5	05.0	07.5	05.0	07.4	05.2	07.3	05.3	9
10	08.5	05.3	08.4	05.4	08.3	05.6	08.3	05.6	08.2	05.7	08.1	05.9	10
11	09.3	05.8	09.2	06.0	09.1	06.1	09.1	06.1	09.0	06.3	08.9	06.5	11
12	10.2	06.4	10.1	06.5	10.0	06.7	09.9	06.7	09.8	06.9	09.7	07.0	12
13	11.0	06.9	10.9	07.1	10.8	07.2	10.8	07.3	10.6	07.5	10.5	07.6	13
14	11.9	07.4	11.7	07.6	11.6	07.8	11.6	07.8	11.5	08.0	11.3	08.2	14
15	12.7	07.9	12.6	08.2	12.5	08.3	12.4	08.4	12.3	08.6	12.1	08.8	15
16	13.6	08.5	13.4	08.7	13.3	08.9	13.3	08.9	13.1	09.2	12.9	09.4	16
17	14.4	09.0	14.3	09.3	14.1	09.4	14.1	09.5	13.9	09.8	13.7	10.0	17
18	15.3	09.5	15.1	09.8	15.0	10.0	15.9	10.1	14.7	10.3	14.6	10.6	18
19	16.1	10.1	15.9	10.3	15.8	10.6	15.7	10.6	15.6	10.9	15.4	11.2	19
20	17.0	10.6	16.8	10.9	16.6	11.1	16.6	11.2	16.4	11.5	16.2	11.8	20
21	17.8	11.1	17.6	11.4	17.5	11.7	17.4	11.7	17.2	12.0	17.0	12.3	21
22	18.6	11.7	18.5	12.0	18.3	12.2	18.2	12.3	18.0	12.6	17.8	12.9	22
23	19.5	12.2	19.3	12.5	19.1	12.8	19.0	12.8	18.8	13.2	18.6	13.5	23
24	20.3	12.7	20.1	13.1	20.0	13.3	19.9	13.4	19.7	13.8	19.4	14.1	24
25	21.2	13.2	21.0	13.6	20.7	13.9	20.7	14.0	20.5	14.3	20.1	14.7	25
26	22.0	13.8	21.8	14.2	21.6	14.4	21.5	14.5	21.3	14.9	21.0	15.3	26
27	22.9	14.3	22.6	14.7	22.4	15.0	22.4	15.1	22.1	15.5	21.8	15.9	27
28	23.7	14.8	23.5	15.2	23.3	15.5	23.2	15.6	22.9	16.1	22.6	16.5	28
29	24.6	15.4	24.3	15.8	24.1	16.1	24.0	16.2	23.8	16.6	23.5	17.0	29
30	25.4	15.9	25.2	16.3	24.9	16.7	24.9	16.8	24.6	17.2	24.3	17.6	30
31	26.3	16.4	26.0	16.9	25.8	17.2	25.7	17.3	25.4	17.8	25.1	18.2	31
32	27.1	17.0	26.8	17.4	26.6	17.8	26.5	17.9	26.2	18.3	25.9	18.8	32
33	28.0	17.5	27.7	18.0	27.4	18.3	27.4	18.4	27.0	18.9	26.7	19.4	33
34	28.8	18.0	28.5	18.5	28.3	18.9	28.2	19.0	27.9	19.5	27.5	20.0	34
35	29.7	18.5	29.4	19.1	29.1	19.4	29.0	19.6	28.7	20.1	28.3	20.6	35
36	30.5	19.1	30.2	19.6	29.9	20.0	29.8	20.1	29.5	20.6	29.1	21.2	36
37	31.4	19.6	31.0	20.1	30.8	20.6	30.7	20.7	30.3	21.2	29.9	21.7	37
38	32.2	20.1	31.9	20.7	31.6	21.1	31.5	21.2	31.1	21.8	30.7	22.3	38
39	33.1	20.7	32.7	21.2	32.4	21.7	32.3	21.8	32.0	22.3	31.5	22.9	39
40	33.9	21.2	33.6	21.8	33.3	22.2	33.2	22.4	32.8	22.9	32.4	23.5	40
41	34.8	21.7	34.4	22.3	34.1	22.8	34.0	22.9	33.6	23.5	33.2	24.1	41
42	35.6	22.3	35.2	22.9	34.9	23.3	34.8	23.5	34.4	24.1	34.0	24.7	42
43	36.5	22.8	36.1	23.4	35.7	23.9	35.6	24.0	35.2	24.6	34.8	25.3	43
44	37.3	23.3	36.9	24.0	36.6	24.4	36.5	24.6	36.0	25.2	35.6	25.9	44
45	38.1	23.8	37.7	24.5	37.4	25.0	37.3	25.2	36.9	25.8	36.4	26.4	45
46	39.0	24.4	38.6	25.0	38.2	25.5	38.1	25.7	37.7	26.4	37.2	27.0	46
47	39.9	24.9	39.4	25.6	39.1	26.1	39.0	26.3	38.5	26.9	38.0	27.6	47
48	40.7	25.4	40.3	26.1	39.9	26.7	39.8	26.8	39.3	27.5	38.8	28.2	48
49	41.5	26.0	41.1	26.7	40.7	27.2	40.6	27.4	40.1	28.1	39.6	28.8	49
50	42.4	26.5	41.9	27.2	41.6	27.8	41.4	28.0	41.0	28.7	40.4	29.4	50
Diff.	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Diff.
	58 Deg.		57 Deg.		5 Points		56 Deg.		55 Deg.		54 Deg.		

Diff.	32 Deg.		33 Deg.		3 Point		34 Deg.		35 Deg.		36 Deg.		Diff.
	Lat	*Dep*	*Lat*	*Dep*	*Lat*	*Dep*	*Lat*	*Dep*	*Lat*	*Dep*	*Lat*	*Dep*	
51	43.2	27.0	42.8	27.8	42.4	28.3	42.3	28.5	41.8	29.2	41.3	30.0	51
52	44.1	27.6	43.6	28.3	43.2	28.9	43.1	29.1	42.6	29.8	42.1	30.6	52
53	44 9	28.1	44 5	28.9	44.1	29.4	43.9	29.6	43 4	30.4	42.9	31.2	53
54	45.8	28.6	45.3	29.4	44.9	39.0	44.5	30.2	44.2	31.0	43.7	31.7	54
55	46.6	29 1	46.1	30.0	45.7	30.6	45.6	30.7	45.1	31.5	44.5	32 3	55
56	47.5	29.7	47.0	30.5	46.6	31.1	46.4	31.3	45.9	32.1	45.2	32.9	56
57	48.3	30.2	47.8	31.0	47.4	31.7	47.3	31.9	46.7	32.7	46.1	33.5	57
58	49.2	30.7	48.7	31.6	48.2	32.2	48.1	32.4	47.5	33.3	46.9	34.1	58
59	50.0	31.3	49.5	32 1	48.0	32.8	48.9	33.0	48.3	33 8	47.7	34 7	59
60	50 9	31.8	50.3	32.7	49.9	33.3	49.7	33.5	49.1	34.4	48.5	35.3	60
61	51.7	32.3	51.2	33.2	50.7	33 9	50.6	34.1	50.0	34.9	49.3	35.9	61
62	52.6	32.9	52.0	33.8	51.5	34 4	51 4	34.7	50.8	35.6	50.2	36 4	62
63	53.4	33.4	52.9	34.3	52.4	35.0	52.2	35.2	51 6	36.1	51.0	37.0	63
64	54.3	33.9	53 7	34 9	53.2	35.5	53.1	35.8	52.4	36.7	51.8	37.6	64
65	55.1	34.4	54.5	35.4	54.0	36.1	53.9	36.3	53.2	37.3	52.6	38.2	65
66	56.0	35.0	55.3	35.9	54.9	36.7	54.7	36.9	54.1	37.9	53.4	38.8	66
67	56.8	35.5	56.2	36.5	55.7	37.2	55.5	37.5	54.9	38.4	54.2	39.4	67
68	57.7	36.0	57.0	37.0	56 5	37.8	56.4	38.0	55 7	39.0	55.0	40.0	68
69	58.5	36.6	57.9	37.6	57.4	38.3	57.2	38 6	56.5	39.6	55.8	40.6	69
70	59.4	37.1	58.7	38.1	58.2	38.9	58.0	39.1	57.3	40.1	56.6	41.1	70
71	60.2	37.6	59 6	38.7	59.0	39.4	58.9	39.7	58.2	40.7	57.4	41.7	71
72	61.0	38.1	60.4	39.2	59.8	40 0	59.7	40.3	59.0	41.3	58.2	42.3	72
73	61.9	38 7	61.2	39.8	60.7	40.6	60.5	40.8	59.8	41 9	59 1	42.9	73
74	62.7	39.2	62.1	40.3	61 5	41.1	61.3	41.4	60.6	42.4	59.9	43.5	74
75	63 6	39.7	62.9	40.8	62.4	41 7	63.2	43.9	61.4	43.0	60 7	44.1	75
76	64.4	40.3	63 8	41.5	63.2	42.2	63.0	42.5	62.3	43.6	61.5	44.7	76
77	65.3	40.8	64.6	41.9	64.0	42.8	63 8	43.0	63.1	44.2	62 3	45.3	77
78	66.1	41.3	65.4	42.5	64.8	43.3	64.7	43.6	63.9	44.7	63.1	45.8	78
79	67.0	41.9	66.3	43.0	65.7	43 9	65.5	44.2	64.7	45.3	63.9	46.4	79
80	67.8	42.4	67.1	43.6	66.5	44 4	66.3	44.7	65.5	45.9	64.7	47.0	80
81	68.7	42.9	68.0	44 1	67.3	45.0	67.1	45.3	66.4	46.5	65.5	47.6	81
82	69.5	43.4	68.8	44.7	68.2	45.5	68.0	45.8	67.2	47.0	66.3	48.2	82
83	70.4	44.0	69.6	45.2	69 0	46.1	68.8	46.4	68.0	47 6	67.1	48.8	83
84	71.2	44.5	70.5	45.8	69.8	46 7	69.6	47.0	68.8	48.2	68.0	49.4	84
85	72.1	45.0	71.3	46.3	70.7	47.2	70 5	47.5	69.6	48.8	68.8	50.0	85
86	72.9	45.6	72 1	46.8	71.5	47.8	71.3	48.1	70.5	49.3	69.6	50.5	86
87	73.8	46.1	73.0	47.3	72.3	48 3	72.1	48.6	71.3	49.9	70.4	51.1	87
88	74.6	46.6	73 8	47.9	73.2	48.9	72.9	49.2	72.1	50.5	71.2	51.7	88
89	75.5	47.2	74.7	48.5	74.0	49.4	73 8	49.8	72 9	51.0	72.0	52.3	89
90	76.3	47.7	75.5	49 0	74.8	50.0	74.6	50.3	73.7	51.6	72.8	52.9	90
91	77.2	48.2	76.3	49.6	75.7	50.6	75.4	50.9	74.5	52.2	73.6	53.5	91
92	78 0	48.7	77.2	50.1	76.5	51.1	76.3	51.4	75.4	52.8	74.4	54.1	92
93	78.9	49.3	78.0	50.6	77.3	51.7	77.1	52.0	76.2	53.3	75.2	54.7	93
94	79.7	49 8	78.9	51.2	78.2	52.2	77.9	52.6	77.0	53 9	76.0	55.2	94
95	80.6	50.3	79.7	51 7	79 0	52.8	78 8	53.1	77.8	54.5	76.9	55.8	95
96	81.4	50.9	80.5	52.3	79.8	53.3	79.6	53.7	78.6	55.1	77.7	56.4	96
97	82.3	51.4	81.4	52.8	80.6	53 9	80.4	54.2	79.5	55.6	78.5	57.0	97
98	83.1	51.9	82.2	53.4	81.5	54.4	81.2	54.8	80.3	56.2	79.3	57.6	98
99	84.0	52.5	83.1	53.9	82.3	54.0	82.1	55.4	81.1	56.8	80.1	58.2	99
100	84.8	53.0	83 9	54.5	83.1	55.6	82.9	55.9	81.9	57 4	80.9	58.8	100
Diff.	*Dep*	*Lat*	*Dep*	*Lat*	*Dep*	*Lat*	*Dep*	*Lat*	*Dep*	*Lat*	*Dep*	*Lat*	Diff.
	58 Deg		57 Deg.		5 Point		56 Deg.		55 Deg.		54 Deg.		

Dist.	3¼ Point,		37 Deg.		38 Deg.		39 Deg.		3½ Points		40 Deg.		Dist.
	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	
1	00.8	00.6	00.8	00.6	00.8	00.6	00.8	00.7	00 8	00.6	00 8	00.6	1
2	01.6	01.2	01.6	01.2	01.6	01 2	01.6	01.3	01.5	01.3	01.5	01.3	2
3	02.4	01.8	02.4	01 8	02.4	01.8	02.3	01.9	02.3	01.9	02.3	01.9	3
4	03.2	02.4	03.2	02.4	03.1	02.5	03.5	02.5	03.1	02.5	03 1	02.6	4
5	04.0	03 0	04.0	03.0	03 9	03.1	03.9	03.1	03.9	03.2	03 8	03 2	5
6	04.8	03.6	04.8	03.6	04.7	03.7	04.6	03 9	04.6	03.8	04.6	03 9	6
7	05.6	04.2	05.6	04.2	05.5	04.3	05.4	04 4	05.4	04.4	05.4	04 5	7
8	06.4	04.8	06.4	04.8	06.3	04.9	06.2	05.0	06.2	05.1	06.1	05.1	8
9	07.2	05.4	07.2	05.4	07.1	05.5	07.0	05 7	07.0	05.7	06.9	05.8	9
10	08.0	06.0	08 0	06.0	07.9	06.2	07.8	06.3	07.7	06.3	07 7	06 4	10
11	08.8	06.6	08.8	06.6	08.7	06.8	08.5	06.9	08 5	07.0	08.4	07.1	11
12	09.6	07.1	09.6	07.2	09.4	07.4	09.3	07 5	09.3	07 6	09 2	07.7	12
13	10.4	07.7	10.4	07.8	10.2	08.1	10.1	08.2	10.0	08.2	10.0	08.4	13
14	11.2	08.3	11.2	08.4	11.0	08.7	10.9	08 8	10.8	03.9	10 7	09.0	14
15	12.0	08.9	12.0	09.0	11.8	09 3	11.6	09.4	11.6	09.5	11.5	09.6	15
16	12.8	09.5	12.8	09.6	12.6	09.8	12.4	10.1	12.4	10.1	12 3	10.3	16
17	13.6	10.1	13.6	10.2	13.4	10 5	13.2	10 7	13.1	10.8	13.0	10.9	17
18	14.5	10.7	14.4	10 8	14.2	11.1	13.9	11.3	13.9	11.4	13.8	11.6	18
19	15.3	11 3	15.2	11 4	15.0	11.7	14.8	12 0	14 7	12.0	14.5	12.2	19
20	16.1	11.9	16.0	12.0	15.8	12.3	15.5	12.6	15.5	12.7	15.3	12.9	20
21	16.9	12.5	16.8	12.6	16.5	12.9	16.3	13.2	16 2	13.3	16.1	13.5	21
22	17.7	13.1	17.6	13.2	17.3	13.5	17.1	13.8	17.0	14.0	16.8	14.1	22
23	18.5	13.7	18.4	13 8	18 1	14.2	17.9	14.5	17.8	14.6	17 6	14.8	23
24	19.3	14.3	19.2	14.4	18.9	14.8	18.6	15.1	18.5	15.2	18.4	15.4	24
25	20.1	14.9	20.0	15.0	19.7	15.4	19.4	15.7	19.3	15.9	19.1	16.1	25
26	20.9	15.5	20 8	15.6	20.5	16.0	20.2	16.4	20.1	16.5	19.9	16.7	26
27	21.7	16.1	21.6	16.2	21.3	16.6	21.0	17.0	20.9	17.1	20.7	17.4	27
28	22.5	16.7	22 4	16.8	22.1	17.2	21 8	17.6	21.6	17.8	21.4	18.0	28
29	23.3	17.3	23.2	17.4	22.8	17.8	22.1	18.3	22.4	18.4	22.2	18.6	29
30	24.1	17.9	24.0	18.0	23.6	18.5	23.3	18.9	23 2	19 0	23.0	19 3	30
31	24.9	18.5	24.8	18.6	24.4	19.1	24.1	19.5	24.0	19.7	23 7	19.9	31
32	25.7	19.1	25 6	19.3	25.2	19.7	24.9	20.1	24.7	20.3	24.5	20.6	32
33	26.5	19.7	26.4	19.9	26.0	20.3	25 6	20.8	25.5	20.9	25.3	21.2	33
34	27.3	20 2	27.1	20.5	26.8	20.9	26.4	21.4	26.3	21.6	26.0	21.9	34
35	28.1	20.8	27.9	21.1	27.6	21.5	27.2	22.0	27.0	22.2	26.8	22 5	35
36	28.9	21.4	28.7	21.7	28.4	22.2	27.7	22.7	27.8	22.8	27.6	23.1	36
37	29.7	22.0	29 5	22.3	29.2	22.8	28.8	23.3	28.6	23.5	28.3	23.8	37
38	30.5	22.6	30.3	22 9	29.9	23.4	29.5	23.9	29 4	24 1	29.1	24.4	38
39	31.3	23.2	31.1	23.5	30.7	24 0	30.3	24.5	30.1	24.7	29.9	25.1	39
40	32.1	23 8	31.9	24.1	31.5	24.6	31.1	25.2	30.9	25.4	30 6	25.7	40
41	32.9	24 4	32.7	24.7	32.3	25.2	31.9	25.8	31.7	26.0	31.4	26.4	41
42	33.7	25.0	33.5	25.3	33.1	25.9	32 6	26.4	32.5	26.6	32.2	27.0	42
43	34.5	25.6	34.3	25.9	33.9	26.5	33.4	27.1	33.2	27.3	32.9	27.6	43
44	35 3	26.2	35.1	26.5	34.7	27.1	34.2	27.7	34.0	27.9	33.7	28.3	44
45	36.1	26.8	35.9	27.1	35.5	27.7	35.0	28.3	34.8	28.5	34.5	28 9	45
46	36.9	27.4	36.7	27.7	36.2	28.3	35 7	29 0	35 6	29 2	35 2	29 6	46
47	37.7	28.0	37.5	28.3	37.0	28.9	36.5	29.6	36.3	29 8	36.0	30.2	47
48	38.5	28.6	38.3	28.9	37.8	29.5	37.8	30.2	37.1	30.4	36.8	30.9	48
49	39.3	29.2	39.1	29.5	38.6	30.2	38.1	30.8	37.9	31.1	37.5	31.5	49
50	40.1	29.8	39 9	30.1	39.4	30.8	38 9	31 5	38 6	31.7	38.3	32.1	50
Dist.	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dist.
	4¾ Point		53 Deg.		52 Deg.		51 Deg.		4½ Point.		50 Deg.		

Dist.	3¼ Point		37 Deg		38 Deg.		39 Deg.		3½ Point		40 Deg		Dist.
	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	
51	41.0	30.4	40.7	30.7	40.2	31.4	39.6	32.1	39.4	32.3	39.1	32.8	51
52	41.8	31.0	41.5	31.3	41.0	32.0	40.4	32.7	40.2	33.0	39.8	33.4	52
53	42.6	31.6	42.3	31.9	41.8	32.6	41.2	33.3	41.0	33.6	40.6	34.1	53
54	43.4	32.2	43.1	32.5	42.5	33.2	42.0	34.0	41.7	34.3	41.4	34.7	54
55	44.2	32.8	43.9	33.1	43.3	33.9	42.7	34.6	42.5	34.9	42.1	35.4	55
56	45.0	33.3	44.7	33.8	44.1	34.5	43.5	35.2	43.3	35.5	42.9	36.0	56
57	45.8	33.9	45.5	34.3	44.9	35.1	44.3	35.9	44.1	36.2	43.7	36.6	57
58	46.6	34.5	46.3	34.9	45.7	35.8	45.1	36.5	44.8	36.8	44.4	37.3	58
59	47.4	35.1	47.1	35.5	46.5	36.3	45.8	37.1	45.6	37.4	45.2	37.9	59
60	48.2	35.7	47.9	36.1	47.3	36.9	46.6	37.8	46.4	38.1	46.0	38.6	60
61	49.0	36.3	48.7	36.7	48.1	37.5	47.4	38.4	47.1	38.7	46.7	39.2	61
62	49.8	36.9	49.5	37.3	48.9	38.2	48.2	39.0	47.9	39.3	47.5	39.9	62
63	50.6	37.5	50.3	37.9	49.6	38.8	49.0	39.6	48.7	40.0	48.3	40.5	63
64	51.4	38.1	51.1	38.5	50.4	39.4	49.7	40.3	49.5	40.6	49.0	41.2	64
65	52.2	38.7	51.9	39.1	51.2	40.0	50.5	40.9	50.2	41.2	49.8	41.8	65
66	53.0	39.3	52.7	39.7	52.0	40.6	51.3	41.5	51.0	41.9	50.5	42.4	66
67	53.8	39.9	53.5	40.3	52.8	41.2	52.1	42.2	51.8	42.5	51.3	43.1	67
68	54.6	40.5	54.3	40.9	53.6	41.9	52.8	42.8	52.6	43.1	52.1	43.7	68
69	55.4	41.1	55.1	41.5	54.4	42.5	53.6	43.4	53.3	43.8	52.9	44.4	69
70	56.2	41.7	55.9	42.1	55.2	43.1	54.4	44.0	54.1	44.4	53.6	45.0	70
71	57.0	42.3	56.7	42.7	55.9	43.7	55.2	44.7	54.9	45.0	54.4	45.6	71
72	57.8	42.9	57.5	43.3	56.7	44.3	55.9	45.3	55.7	45.7	55.1	46.3	72
73	58.6	43.5	58.3	43.9	57.5	44.9	56.7	45.9	56.4	46.3	55.9	46.9	73
74	59.4	44.1	59.1	44.5	58.3	45.6	57.5	46.6	57.2	46.9	56.7	47.6	74
75	60.2	44.7	59.9	45.1	59.2	46.2	58.3	47.2	58.0	47.6	57.4	48.2	75
76	61.0	45.3	60.7	45.7	60.0	46.8	59.1	47.8	58.7	48.2	58.2	48.9	76
77	61.8	45.9	61.5	46.3	60.7	47.4	59.8	48.5	59.5	48.8	59.0	49.5	77
78	62.7	46.5	62.3	46.9	61.5	48.0	60.6	49.1	60.3	49.5	59.7	50.1	78
79	63.5	47.1	63.1	47.5	62.2	48.6	61.4	49.7	61.1	50.1	60.5	50.8	79
80	64.3	47.7	63.9	48.1	63.0	49.3	62.2	50.3	61.8	50.7	61.3	51.4	80
81	65.1	48.3	64.7	48.7	63.8	49.9	62.9	51.0	62.6	51.4	62.0	52.1	81
82	65.9	48.8	65.5	49.3	64.6	50.5	63.7	51.6	63.4	52.0	62.8	52.7	82
83	66.7	49.4	66.3	49.9	65.4	51.1	64.5	52.2	64.2	52.6	63.6	53.4	83
84	67.5	50.0	67.1	50.5	66.2	51.7	65.3	52.9	64.9	53.3	64.3	54.0	84
85	68.3	50.6	67.9	51.1	67.0	52.3	66.1	53.5	65.7	53.9	65.1	54.6	85
86	69.1	51.2	68.7	51.7	67.8	52.9	66.8	54.1	66.5	54.6	65.9	55.3	86
87	69.9	51.8	69.5	52.4	68.6	53.6	67.6	54.8	67.2	55.2	66.6	55.9	87
88	70.7	52.4	70.2	53.0	69.3	54.2	68.4	55.4	68.0	55.8	67.4	56.6	88
89	71.5	53.0	71.1	53.6	70.1	54.8	69.2	56.0	68.8	56.5	68.2	57.2	89
90	72.3	53.6	71.9	54.2	70.9	55.4	69.9	56.6	69.6	57.1	68.9	57.4	90
91	73.1	54.2	72.7	54.8	71.7	56.0	70.7	57.3	70.3	57.7	69.5	58.5	91
92	73.9	54.8	73.5	55.4	72.5	56.6	71.5	57.9	71.1	58.4	70.3	59.1	92
93	74.7	55.4	74.3	56.0	73.3	57.3	72.3	58.5	71.9	59.0	71.2	59.8	93
94	75.5	56.0	75.1	56.6	74.1	57.9	73.0	59.2	72.7	59.6	72.0	60.4	94
95	76.3	56.6	75.9	57.2	74.9	58.5	73.8	59.8	73.4	60.3	72.8	61.1	95
96	77.1	57.2	76.7	57.8	75.6	59.1	74.6	60.4	74.2	60.9	73.5	61.7	96
97	77.9	57.8	77.5	58.4	76.4	59.7	75.4	61.0	75.0	61.5	74.3	62.1	97
98	78.7	58.4	78.3	59.0	77.2	60.3	76.2	61.7	75.7	62.2	75.2	63.0	98
99	79.5	59.0	79.1	59.6	78.0	60.9	76.9	62.3	76.5	62.8	75.8	63.6	99
100	80.3	59.6	79.9	60.2	78.8	61.6	77.7	62.9	77.3	63.4	76.6	64.2	100
Dist.	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dist.
	4¾ Point		53 Deg.		52 Deg.		51 Deg.		4½ Point		50 Deg.		

Diff.	41 Deg.		42 Deg.		3¾ Point		43 Deg.		44 Deg.		4 Point		Diff.
	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	
1	00.7	00.7	00.7	00.7	00.7	00.7	00.7	00.7	00.7	00.7	00.7	00.7	1
2	01.5	01.3	01.5	01.3	01.5	01.3	01.5	01.4	01.4	01.4	01.4	01.4	2
3	02.3	02.0	02.2	02.0	02.2	02.0	02.2	02.0	02.2	02.1	02.1	02.1	3
4	03.0	02.6	03.0	02.7	03.0	02.7	02.9	02.7	02.9	02.8	02.8	02.8	4
5	03.8	03.3	03.7	03.3	03.7	03.4	03.6	03.4	03.6	03.5	03.5	03.5	5
6	04.5	03.9	04.5	04.0	04.4	04.0	04.4	04.1	04.3	04.2	04.2	04.2	6
7	05.3	04.6	05.2	04.7	05.2	04.7	05.1	04.8	05.0	04.9	04.9	04.9	7
8	06.0	05.2	05.9	05.3	05.9	05.4	05.8	05.5	05.7	05.6	05.7	05.7	8
9	06.8	05.9	06.7	06.0	06.7	06.0	06.6	06.1	06.5	06.2	06.4	06.4	9
10	07.5	06.6	07.4	06.7	07.4	06.7	07.3	06.8	07.2	06.9	07.1	07.1	10
11	08.3	07.2	08.2	07.4	08.1	07.4	08.0	07.5	07.9	07.6	07.8	07.8	11
12	09.1	07.9	08.9	08.0	08.9	08.1	08.8	08.2	08.6	08.3	08.5	08.5	12
13	09.8	08.5	09.7	08.7	09.6	08.7	09.5	08.9	09.3	09.0	09.2	09.2	13
14	10.6	09.2	10.4	09.4	10.4	09.4	10.2	09.5	10.1	09.7	09.9	09.9	14
15	11.3	09.8	11.1	10.0	11.1	10.1	11.0	10.2	10.8	10.4	10.6	10.6	15
16	12.1	10.5	11.9	10.7	11.9	10.9	11.7	10.9	11.5	11.1	11.3	11.3	16
17	12.8	11.1	12.6	11.4	12.6	11.4	12.4	11.6	12.2	11.8	12.0	12.0	17
18	13.6	11.8	13.4	12.0	13.3	12.1	13.2	12.3	12.9	12.5	12.7	12.7	18
19	14.3	12.5	14.1	12.7	14.1	12.8	13.9	13.0	13.7	13.2	13.4	13.4	19
20	15.1	13.1	14.9	13.4	14.8	13.4	14.6	13.6	14.4	13.9	14.1	14.1	20
21	15.8	13.8	15.6	14.0	15.6	14.1	15.4	14.3	15.1	14.6	14.8	14.8	21
22	16.6	14.4	16.3	14.7	16.3	14.8	16.1	15.0	15.8	15.3	15.5	15.5	22
23	17.4	15.1	17.1	15.4	17.0	15.4	16.8	15.7	16.5	16.0	16.3	16.3	23
24	18.1	15.7	17.8	16.1	17.8	16.1	17.5	16.4	17.3	16.7	17.0	17.0	24
25	18.9	16.4	18.6	16.7	18.5	16.8	18.3	17.1	18.0	17.4	17.7	17.7	25
26	19.6	17.1	19.3	17.4	19.3	17.4	19.0	17.7	18.7	18.1	18.4	18.4	26
27	20.4	17.7	20.1	18.1	20.0	18.1	19.7	18.4	19.4	18.8	19.1	19.1	27
28	21.1	18.4	20.8	18.7	20.7	18.8	20.5	19.1	20.1	19.4	19.8	19.8	28
29	21.9	19.0	21.5	19.4	21.5	19.5	21.2	19.8	20.9	20.1	20.5	20.5	29
30	22.6	19.7	22.3	20.1	22.2	20.1	21.9	20.5	21.6	20.8	21.2	21.2	30
31	23.4	20.3	23.0	20.7	23.0	20.8	22.6	21.1	22.3	21.5	21.9	21.9	31
32	24.1	21.0	23.8	21.4	23.7	21.5	23.4	21.8	23.0	22.2	22.6	22.6	32
33	24.9	21.6	24.5	22.1	24.4	22.2	24.1	22.5	23.7	22.9	23.3	23.3	33
34	25.6	22.3	25.3	22.7	25.2	22.8	24.9	23.2	24.5	23.6	24.0	24.0	34
35	26.4	23.0	26.0	23.4	25.9	23.5	25.6	23.9	25.2	24.3	24.7	24.7	35
36	27.2	23.6	26.7	24.1	26.7	24.2	26.3	24.5	25.9	25.0	25.4	25.4	36
37	27.9	24.3	27.5	24.7	27.4	24.8	27.0	25.2	26.6	25.7	26.1	26.1	37
38	28.7	24.9	28.2	25.4	28.2	25.5	27.8	25.9	27.3	26.4	26.9	26.9	38
39	29.4	25.6	29.0	26.1	28.9	26.2	28.5	26.6	28.0	27.1	27.6	27.6	39
40	30.2	26.2	29.7	26.8	29.6	26.9	29.2	27.3	28.8	27.8	28.3	28.3	40
41	31.0	26.9	30.5	27.4	30.4	27.5	30.0	28.0	29.5	28.5	29.0	29.0	41
42	31.7	27.5	31.2	28.1	31.1	28.2	30.7	28.6	30.2	29.2	29.7	29.7	42
43	32.5	28.2	31.9	28.8	31.9	28.9	31.4	29.3	30.9	29.9	30.4	30.4	43
44	33.2	28.9	32.7	29.4	32.6	29.5	32.2	30.0	31.6	30.6	31.1	31.1	44
45	34.0	29.5	33.4	30.1	33.3	30.2	32.9	30.7	32.3	31.3	31.8	31.8	45
46	34.7	30.2	34.2	30.8	34.1	30.9	33.6	31.4	33.1	32.0	32.5	32.5	46
47	35.5	30.8	34.9	31.4	34.8	31.6	34.4	32.1	33.8	32.6	33.2	33.2	47
48	36.3	31.5	35.7	32.1	35.6	32.2	35.1	32.7	34.5	33.3	33.9	33.9	48
49	37.0	32.1	36.4	32.8	36.3	32.9	35.8	33.4	35.2	34.0	34.6	34.6	49
50	37.7	32.8	37.2	33.5	37.0	33.6	36.6	34.1	36.0	34.7	35.3	35.3	50
Diff.	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Diff.
	49 Deg.		48 Deg.		4¼ Point		47 Deg.		46 Deg.		4 Point		

Diff.	41 Deg.		42 Deg.		3¾ Point		43 Deg.		44 Deg.		4 Points		Diff.
	Lat	*Dep*	*Lat*	*Dep*	*Lat*	*Dep*	*Lat*	*Dep*	*Lat*	*Dep*	*Lat*	*Dep*	
51	38.5	33.5	37.9	34.1	37.8	34.2	37.3	34.8	36.7	35.4	36.1	36.1	51
52	39.2	34.1	38.6	34.8	38.5	34.9	38.0	35.5	37.4	36.1	36.8	36.8	52
53	40.0	34.8	39.4	35.5	39.3	35.6	38.8	36.1	38.1	36.8	37.5	37.5	53
54	40.8	35.4	40.1	36.1	40.0	36.3	39.5	36.8	38.8	37.5	38.2	38.2	54
55	41.6	36.0	40.9	36.8	40.7	36.9	40.2	37.5	39.6	38.2	38.9	38.9	55
56	42.3	36.7	41.6	37.5	41.5	37.6	41.0	38.2	40.3	38.9	39.6	39.6	56
57	43.0	37.4	42.4	38.1	42.2	38.3	41.7	38.9	41.0	39.6	40.3	40.3	57
58	43.8	38.1	43.1	38.8	43.0	38.9	42.4	39.5	41.7	40.3	41.0	41.0	58
59	44.5	38.7	43.8	39.5	43.7	39.6	43.1	40.2	42.4	41.0	41.7	41.7	59
60	45.2	39.4	44.6	40.1	44.5	40.3	43.8	40.9	43.2	41.7	42.4	42.4	60
61	46.0	40.0	45.3	40.8	45.2	41.0	44.6	41.7	43.9	42.4	43.1	43.1	61
62	46.8	40.7	46.1	41.5	45.9	41.6	45.3	42.3	44.6	43.1	43.8	43.8	62
63	47.6	41.3	46.8	42.2	46.7	42.3	46.1	43.0	45.3	43.8	44.5	44.5	63
64	48.3	42.0	47.5	42.8	47.4	43.0	46.8	43.6	46.0	44.5	45.3	45.3	64
65	49.1	42.6	48.3	43.5	48.2	43.6	47.5	44.3	46.8	45.1	46.0	46.0	65
66	49.8	43.3	49.0	44.2	48.9	44.3	48.3	45.0	47.5	45.8	46.7	46.7	66
67	50.6	44.0	49.8	44.8	49.6	45.0	49.0	45.7	48.2	46.5	47.4	47.4	67
68	51.3	44.6	50.5	45.5	50.4	45.7	49.7	46.4	48.9	47.2	48.1	48.1	68
69	52.1	45.3	51.3	46.2	51.1	46.3	50.5	47.1	49.6	47.9	48.8	48.8	69
70	52.8	45.9	52.0	46.8	51.9	47.0	51.2	47.7	50.3	48.6	49.5	49.5	70
71	53.6	46.6	52.8	47.5	52.6	47.7	51.9	48.4	51.1	49.3	50.2	50.2	71
72	54.3	47.2	53.5	48.2	53.3	48.3	52.7	49.1	51.8	50.0	50.9	50.9	72
73	55.1	47.9	54.2	48.8	54.1	49.0	53.4	49.8	52.5	50.7	51.6	51.6	73
74	55.9	48.5	55.0	49.5	54.8	49.7	54.1	50.5	53.2	51.4	52.3	52.3	74
75	56.6	49.2	55.7	50.2	55.6	50.4	54.8	51.1	53.9	52.1	53.0	53.0	75
76	57.4	49.9	56.5	50.9	56.3	51.0	55.6	51.8	54.7	52.8	53.7	53.7	76
77	58.1	50.5	57.2	51.5	57.1	51.7	56.3	52.5	55.4	53.5	54.4	54.4	77
78	58.9	51.2	58.0	52.1	57.8	52.4	57.0	53.2	56.1	54.2	55.2	55.2	78
79	59.6	51.8	58.7	52.8	58.5	53.1	57.8	53.9	56.8	54.9	55.9	55.9	79
80	60.4	52.5	59.4	53.5	59.3	53.7	58.5	54.6	57.5	55.6	56.6	56.6	80
81	61.1	53.1	60.2	54.2	60.0	54.4	59.2	55.2	58.3	56.3	57.3	57.3	81
82	61.9	53.8	60.9	54.9	60.8	55.1	60.0	55.9	59.0	57.0	58.0	58.0	82
83	62.6	54.5	61.7	55.5	61.5	55.7	60.7	56.6	59.7	57.6	58.7	58.7	83
84	63.4	55.1	62.4	56.2	62.2	56.4	61.4	57.3	60.4	58.3	59.4	59.4	84
85	64.2	55.9	63.2	56.9	63.0	57.1	62.2	58.0	61.1	59.0	60.1	60.1	85
86	64.9	56.4	63.9	57.5	63.7	57.7	63.0	58.6	61.9	59.7	60.8	60.8	86
87	65.7	57.1	64.7	58.2	64.5	58.4	63.6	59.3	62.6	60.4	61.5	61.5	87
88	66.4	57.7	65.4	58.9	65.2	59.1	64.4	60.0	63.3	61.1	62.2	62.2	88
89	67.2	58.4	66.1	59.6	65.9	59.8	65.1	60.7	64.0	61.8	62.9	62.9	89
90	67.9	59.0	66.9	60.2	66.7	60.4	65.8	61.4	64.7	62.5	63.6	63.6	90
91	68.7	59.7	67.6	60.9	67.4	61.1	66.5	62.1	65.5	63.2	64.3	64.3	91
92	69.4	60.4	68.4	61.6	68.2	61.8	67.3	62.7	66.2	63.9	65.0	65.0	92
93	70.2	61.0	69.1	62.2	68.9	62.4	68.0	63.4	66.9	64.6	65.8	65.8	93
94	71.0	61.7	69.9	62.9	69.6	63.1	68.7	64.1	67.6	65.3	66.5	66.5	94
95	71.7	62.3	70.6	63.6	70.4	63.8	69.5	64.8	68.3	66.0	67.2	67.2	95
96	72.5	63.0	71.3	64.2	71.1	64.5	70.2	65.5	69.1	66.7	67.9	67.9	96
97	73.3	63.6	72.1	64.9	71.9	65.1	70.9	66.1	69.8	67.4	68.6	68.6	97
98	74.0	64.3	72.8	65.6	72.6	65.8	71.7	66.8	70.5	68.1	69.3	69.3	98
99	74.7	65.0	73.6	66.2	73.4	66.5	72.4	67.5	71.2	68.8	70.0	70.0	99
100	75.5	65.6	74.3	66.9	74.1	67.2	73.1	68.2	71.9	69.5	70.7	70.7	100
Diff.	*Dep*	*Lat*	*Dep*	*Lat*	*Dep*	*Lat*	*Dep*	*Lat*	*Dep*	*Lat*	*Dep*	*Lat*	Diff.
	49 Deg.		48 Deg.		4¼ Point		47 Deg.		46 Deg.		4 Points		

Of the Nature and Uſe of the preceding Table.

FOR the more ready working of a *Traverſe*, in order to find the Place of the Ship at all Times, the preceding Table has been inſerted, made according to *Caſe the 1ſt* of *Plain Sailing*, ſhewing the Difference of Latitude and Departure, to every Degree, Point and Quarter Point of the Compaſs, the Diſtance from 1 Mile to 100, and may be made to ſerve for any other Diſtance, provided it be firſt divided into Parts not exceeding the Limits of the Table.

At the Head that is in the uppermoſt Rank, are placed the Courſes, beginning from 1 Degree and proceeding on through the ſeveral Degrees, Points and Quarter Points to 45 Degrees, and at the Foot, or in the lowermoſt Rank but one, are placed the Degrees, Points and Quarter Points of the remaining Half of the Quadrant, that is, from 45 Degrees to 90.

In the two outmoſt Columns of each Page are placed the Diſtances, the Table on the Left-hand Page beginning at 1 Mile and continuing to 50; and that on the Right-hand Page beginning where the other left off, *viz.* at 50, and going on to 100.

And in the common Area, that is directly under the Courſe, and againſt the Diſtances, are placed the Differences, or *Alterations* of *Latitude* and *Departure*, correſponding with, or anſwering to the given Courſes and Diſtances.

A few Examples will make it plainer.

Example the 1ſt.

If the Courſe were NE. by N. ½ Eaſterly, Diſtance 76, and the Difference of Latitude and Departure were required?

On the Right-hand Page (becauſe the Diſtance is above 50) find the Diſtance 76; and at the Top (becauſe the Courſe is leſs than 4 Points, or 45 Degrees) look for the Courſe 3 ¼ Points, and right under it, and againſt 76, the Diſtance ſailed will be found in the Column *Lat.* 28.7 for the *Difference of Latitude*, and in the Column *Dep.* 48.2 for the *Departure*.

If the Courſe had been NE. ½ Eaſterly, and the Diſtance as before 76, then the Courſe muſt have been ſought for at the Bottom, and the *Difference of Latitude* would have been found 48.2, and the *Departure* 58.7.

Example

Example the 2d.

Suppoſe the Ship's Courſe NE. by N. ½ Eaſterly, and the Diſtance 176.

On the Right-hand Page at the Top, find out the Courſe 3 ½ Points, and againſt the Diſtance 100, you will find the Difference of Latitude 77.3, and Departure 63.4: Again, Find out 76, the remaining Part of the Diſtance under the Column *Diſt* right againſt it, and in the Column under 3 ½ Points you will find 58.7 for the Difference of Latitude, and 48.2 for the Departure: Now the Sum of this Difference of Latitude, and the Difference of Latitude before found, 136 will be the Difference of Latitude required; alſo the Sum of the Departures 111.6 is the Departure ſought.

Example the 3d.

Suppoſe a Ship ſails South 36° 30′ Eaſterly 45 Miles, and the Difference of Latitude and Departure were required.

Having found the Diſtance ſailed in the Left-hand Page, becauſe the Courſe cannot be found exactly, ſeek the Difference of Latitude 36.4, under the next leſs Degree, *viz.* 36, alſo the Difference of Latitude 35.9 under the next greater Degree, *viz.* 37; then ſay, As the Difference of the Tabular Courſes 60′, to the Difference of the Tabular Differences of Latitude 5, ſo is the Exceſs of the Courſe given 30 above the next leſs Tabular Courſe, to 25, which ſubtracted from 36.4, the Difference of Latitude belonging to 36 Degrees, gives 36.15, the Difference of Latitude ſought: After the ſame Manner the Departure will be found to be 26.75.

In like manner, if the Diſtance were a Fractional Part, then the Difference of Latitude and Departure under the ſame Courſe muſt be found, to the next two extream Diſtances, and Proportion made accordingly.

Example.

Let the Courſe be SSE. ½ Eaſt, and the Diſtance 44.75 Miles.

In the Left-hand Column againſt 44 Miles, the next leſs whole Number to the Diſtance, and under 2 ½ Points find the Difference of Latitude 38.8; likewiſe in the ſame Column againſt 45, the next Integer, find the correſpondent Difference of Latitude 39.7: Then ſay, As 100 to 9, the Difference of the Tabular Numbers, ſo is 75, the Exceſs of the given Diſtance above 44 Miles, to $\frac{675}{1000}$, or $\frac{2}{10}$ almoſt, which therefore added to the Difference of Latitude 38.8, correſponding with 44 Miles, gives 39 5, the true Difference of Latitude required.

In

In like manner for the *Departure*, ſay, As 100 to 5, the *Difference* of the *Tabular Departure*'s anſwering to 44 and 45 Miles, ſo is 75 to $\frac{375}{1000}$, or $\frac{4}{10}$ nearly, which therefore added to the *Departure* 20.7 anſwering to 44 Miles, gives 21.1, the *Departure* ſought.

After the ſame manner may the Difference of Latitude and Departure be found to any given Courſe and Diſtance; and therefore I ſhall only give one ſmall *Traverſe* with its Solution, and leave the Operation to exerciſe the Young Student.

Admit a Ship at Sea in the Latitude of 34° 30′ N. ſails away N. by E. 35, NNE. $\frac{1}{4}$ Eaſt 27, NE. 32, NE. by E. $\frac{1}{2}$ North 40, ENE. $\frac{3}{4}$ Eaſt 50, and it be required to find what Latitude ſhe is in, and how far ſhe is departed from her firſt Meridian?

Having made a Table, and placed the Courſes and Diſtances in the ſubſequent Manner, the Differences of Latitude and Departure will be found as follow.

Courſes.	Diſt.	Diff. of Lat.		Departure.	
		North.	*South.*	*Eaſt.*	*Weſt.*
N. by E.	35	34. 3		6. 8	
NNE. $\frac{1}{4}$ E.	27	24. 4		11. 5	
N. E.	32	22. 6		22. 6	
NE. by E. $\frac{1}{2}$ N.	40	18. 9		35. 3	
ENE. $\frac{3}{4}$ E.	50	12. 1		48. 5	
		112. 3		124.7	

Hence it appears the Ship has departed from her firſt Meridian 124.7 Miles, and altered her Latitude 112.3 Miles.

Wherefore, to find the Latitude ſhe is in, becauſe ſhe ſailed from a *North Latitude Northerly*,

To the Latitude ſailed from —— —— ——	34°	30′
Add the Difference of Latitude reduced ——	1	52 $\frac{1}{3}$
The Sum is the Latitude ſhe is in —— ——	36	25 $\frac{1}{3}$

By the Help of this *Table*, all the ſeveral Caſes of *Plain Sailing* may be eaſily reſolved; but this being a little foreign to the preſent Buſineſs, and not the Deſign for which theſe *Tables* were calculated, we ſhall paſs it by.

SECT.

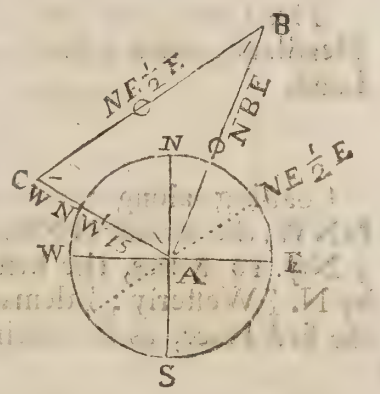
SECT. IV.

Containing the Doctrine of Oblique-angled Plain Triangles, *applied to Problems of* Sailing.

IN this *Section* I ſhall inſert ſuch *Problems* only, as are more immediately uſeful in the *Practice of* Navigation.

Caſe I.

Coaſting along the Shore, I ſaw a Cape of Land, which bore from me N. by E. then I ſtood away W.N.W. 5 Leagues, or 15 Miles, and the ſame Cape bore from me N. E. $\frac{1}{2}$ E. I demand the Diſtance from the Ship in her laſt Station to the Cape?



1. Having drawn the Compaſs NE. SW. let A repreſent the Place of the Ship in her firſt Station, and draw the W. N. W. Line AC equal to 15 Miles; then will C be the Place of the Ship in her ſecond Station.

2. From C draw the Line CB, parallel to the NE. $\frac{1}{2}$ E. Line till it meet the N. by E. Line AB in B; then will B repreſent the Cape of Land, and CB the Diſtance of the Ship from the Cape at her ſecond Station.

To find which by *Calculation*,

In the Triangle ABC are given AC equal to 15 Miles, the Angle A equal to 78 deg. 45 min. equal to 7 Points, equal to the Diſtance between the N. by E. and WNW. the Angle B equal to 39° 22′ $\frac{1}{2}$, equal to 3 $\frac{1}{2}$ Points, equal to the Diſtance between the N. by E. and the NE $\frac{1}{2}$ E. and the Angle C equal to 61 deg. 52 min. $\frac{1}{2}$, equal to 5 $\frac{1}{2}$ Points, equal to the Diſtance between the WNW. and the SW $\frac{1}{2}$ W. Whence, to find the Diſtance CB, it will be *by the 2d Caſe of Sect. the 8th of Trigonometry*,

S. ABC: AC :: S. BAC: CB. That is,

As

As the Sine of the Angle at B = 39° 22′ ½ —— 9.8023585

To the Diſtance run AC = 15 —— 1.1760913
So is the Sine of the Angle at A = 78.45 —— 9.9915739

To the Diſtance between the Ship and the Cape 23.19 1.3653067

If the Diſtance of the Ship in her firſt Station from the Cape be required, it will be by *the 2d Caſe of Sect. the 8th of Trigonometry*,

S. ABC : AC : : S. ACB : AB. That is,

As the Sine of the Angle at B = 39° 22′ ½ —— 9.8023585

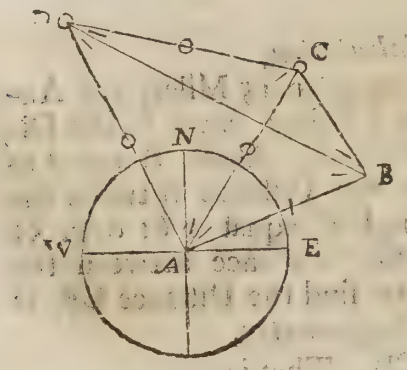
To the Diſtance run AC = 15 —— 1.1760913
So is the Sine of the Angle at C 61° 52′ ½ —— 9.9454298

To the Diſtance required AB = 20.85 Miles —— 1.3191626

This Caſe is of Uſe in finding the Bearing and Diſtance from a Headland, when a Ship is about to take her Departure from the Land.

Caſe II.

Coaſting along the Shore, I ſee two Headlands, the firſt bears NNW. the ſecond NNE. ¾ Eaſterly; then ſtanding away E. by N. ½ Nly. 16 Miles, the firſt bears from me WNW. the ſecond NW. by N. ¼ Weſterly; I demand the Diſtance of each of theſe Ports from the firſt Place, as alſo their Bearing and Diſtance from each other.



Geometrically.

Having drawn the E. by N. ½ N. Line equal to 16 Miles, from the Stations A and B, draw the ſeveral Bearings interſecting each other in the Points D and C, which will repreſent the two Headlands.

Whence,

Whence;

In the Triangle DAB are given,

1. 2. The $\angle \begin{Bmatrix} DAB \\ ABD \end{Bmatrix} = \begin{Bmatrix} 8\ \frac{1}{2} \\ 3\ \frac{1}{2} \end{Bmatrix}$ Points $= \begin{Bmatrix} 95^\circ\ 37'\ \frac{1}{2} \\ 39\ 22\ \frac{1}{2} \end{Bmatrix}$
3. The Distance run AB 16 Miles;

And therefore AD will be easily had, for

s, ADB : AB : : s, ABD : AD. That is,

As the Sine of ADB 45° 00′	9.8494850
To AB 16 Miles	1.2041200
So is the Sine of ABD 39° 22′	9.8022816
To AD 14, 35 Miles	1.1569166

The Distance of the first Headland required.

Again,

In the Triangle ABC are given,

1. 2. The $\angle \begin{Bmatrix} BAC \\ ABC \end{Bmatrix} = \begin{Bmatrix} 3\ \frac{1}{4} \\ 6\ \frac{1}{4} \end{Bmatrix}$ Points $= \begin{Bmatrix} 42^\circ\ 11' \\ 70\ 19 \end{Bmatrix}$ and
3. AB the Distance run 16 Miles.

Wherefore,

s, ACB : AB : : s, ABC : AC. That is,

As the Sine of ACB 67° 30′	9.9656153
To AB 16 Miles	1.2041200
So is the Sine of ABC 70° 19′	9.9738519
To AC 16 31 Miles	1.2123566

The Distance of the Ship from the second Port.

After the same Manner may the Distance of the Headlands D and C, from B, the last Place of the Ship be found, if it be required. But

To find the Bearing and Distance of the Headlands,

In the Triangle DAC are given,

1. DA 14. 35 Miles.
2. AC 16. 31 Miles.
3. The $<$DAC 53° 26′, and therefore it will be,

$AC + AD : AC - AD :: t, \frac{1}{2} ADC + ACD :: t, \frac{1}{2} ADC - ACD.$

That is,

As the Logarithm of 30° 66′	1.4865721
To the Logarithm of 1 96	0.2922561
So is the Tangent of 63° 17′	10.2981626
To the Tangent of 7° 14′	9.1038466

Now 63° 17′ + 7° 14′ = 70° 31′ = $<$ ADC.

And therefore C bears from D East 3° 01′ Northerly; and consequently D from C, West 3° 01′ Southerly.

And for their Distance it will be,

s, ADC : AC :: s, DAC : DC. That is,

As the Sine of ADC 70° 31′	9.9743913
To AC 16. 31	1.2123566
So is the Sine of DAC 53° 26′	9.9048043
To DC 13.89 Miles	1.1427696

The Distance of the two Headlands.

After this Manner may the Mutual Bearings and Distances of any Number of Headlands be found, which are of extraordinary Use for correcting of the *Sea Chart* if need require.

And since I am upon the Business of measuring inaccessible Distances, I cannot but recommend one Way, which, without Dispute, when it can be put in Practice, is the most easy and expeditious Way hitherto known; and that is, by Firing of great Guns.

It

It has been determined many Years ago, by the Reverend and Learned Mr. *Flamstead*, His Majesty's Aſtronomer at *Greenwich*, by ſeveral Experiments made with a large Chamber, let off from the Top of *Shooter's Hill*, which is diſtant from the *Royal Obſervatory* about three Miles, within a ſmall Matter, that Sound travels at the Rate of 1142 Feet in one Second of Time; and this, by repeated Trials, was found to be the ſame, whether the Sound went with or againſt the Wind; or, at leaſt, the Difference was ſo ſmall, as not to be diſcoverable.

A Statute Mile contains 5280 Feet, and conſequently a Sound requires 4.623 Seconds, or 9.246 half Seconds, or 9 $\frac{1}{4}$ half Seconds, to travel an *Engliſh* Mile.

A Degree of a Great Circle, by the moſt accurate Obſervations, contains 69.12 *Engliſh* or Statute Miles.

And becauſe a *Geographical* or *Nautical* Mile is $\frac{1}{60}$ Parts of a Degree, it follows, that an *Engliſh* Mile is to a *Geographical* or *Nautical* Mile as 60 to 69.12.

Wherefore, as 60 to 69.12, ſo is 4.623 Seconds, the Time that Sound requires to travel an *Engliſh* Mile, to 5.326 Seconds, the Time it requires to travel a *Geographical* or *Nautical* Mile.

Again, Becauſe every League contains three *Nautical* Miles, therefore the Sound requires 15.975 Seconds, or 16 Seconds nearly, to travel a *Nautical* Mile.

Wherefore, As 1 to 5.326, ſo is any given Diſtance in Miles to the Time that Sound will require to move over that Diſtance.

Or, As 1 is to 16, ſo is the Diſtance given in Leagues to the Time that the Sound will require to arrive at the given Place.

On the Contrary,

As 5.326 to 1, ſo is the Number of Seconds, between the Time that a Sound is generated, and the Time that it arrives at any given Place, to the Diſtance between thoſe two Places in Miles.

Or, As 16 to 1, ſo is the Number of Seconds, between the Time of its Generation and the Time of its being heard, to the Diſtance between the two Places.

Hence, If the Diſtance between any two Places be given, the Time that Sound will require to travel over that Diſtance may be found: And on the contrary, the Time that a Sound is going between any two Places being known, the Diſtance between thoſe two Places is eaſily had.

And becauſe the Motion of Light is in a Manner inſtantaneous, it moving at the Rate of 10000 Miles, at leaſt, by the beſt Obſervations, in a Second of Time, or whilſt a Pendulum of 39.2 Inches makes one Vibration.

Hence, if the Difference between the Time that an Exploſion is firſt ſeen, and the Time that the firſt Sound of it reaches the Ear, be given, the Diſtance between the Place where the Gun was fired, and the Place of the Obſerver, is eaſily found. For,

As 5.326 to 1, ſo is the Number of Seconds to the Diſtance in *Nautical* Miles. *Or*, As 16 to 1, ſo is the Number of Seconds to the Diſtance in Sea Leagues.

Wherefore, if the Number of Seconds between the Time that the Flaſh of a Gun is ſeen, and the Time that the Report becomes audible, be divided by 5.326, the Quotient will give the Number of Miles of Diſtanee between the Place where the Gun was fired, and the Place of the Obſerver; or if the ſame Number of Seconds be divided by 16, the Quotient will give the Number of Leagues of Diſtance.

Example 1.

Being at Sea, I counted 50 Seconds between the Time that I ſaw the Flaſh of a Gun and heard the Report: I demand the Diſtance.

If 50 Seconds be divided by 5.326, the Quotient 9.388, or 9.4 Miles, will be the Diſtance required. *Or*, If the ſame Number of Seconds, *viz.* 50, be divided by 16, the Quotient 3.125, or 3 $\frac{1}{8}$ Leagues, will give the Leagues of Diſtance; ſo that the Gun when it was fired was 9.4 Miles, or 3 $\frac{1}{8}$ Leagues from the Obſerver.

Example 2.

Being by the Sea-ſide, there was a Ship in the Offing which fired ſome Guns, and obſerving, I found there was 40 Seconds between the Time that I ſaw the Flaſh and heard the Gun: I demand the Diſtance of the Ship from the Shore.

Anſwer.

7 $\frac{1}{2}$ Miles, or 2 $\frac{1}{2}$ Leagues.

If the Obſerver have not a Minute Watch by him, let him take a String, or Piece of Wire, of 32.2 Inches long, and faſten a Plummet

met to one End of it, and hang the other upon a Nail or Peg, or hold it between his Fingers, and count the Numbers of Vibrations between the Flash and the Report, which will give him the Number of Seconds; or, if he has a Mind to be more nice, let him take a String or Wire of 9.8 Inches, and prepare it as before, and the Number of Vibrations in this Case will give the Number of half Seconds of Time.

It is generally found, that the Number of Pulsations of the Artery, or Contractions of the Heart of a Man of a good Habit of Body, is seventy-five in a Minute: Wherefore this being supposed, it follows, that Sound travels at the Rate of one *Nautical* Mile in 6 $\frac{2}{3}$ Pulsations, or one League in 20: And consequently we can never be at a Loss for putting of this Method in Practice; for if the Number of Pulsations between the Time that a Man sees the Explosion of a Gun and hears the Report, be divided by 6 $\frac{2}{3}$ (or 7 for round Number) the Quotient will give the Number of Miles of Distance; or the same Number of Pulsations being divided by 20, will give the same Distance in Leagues.

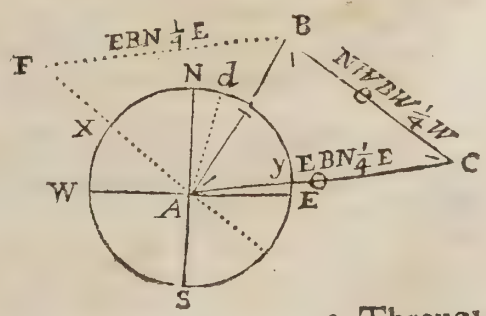
In turning to Windward; to know how near the Wind a Ship will lie, observe what Course she goes on each Tack, then in the Middle between both is the Wind; therefore the half of the Arch or Number of Points between each Course (*viz.* that Arch the Wind is in) is the Distance of the Course from the Wind.

Case III.

Suppose a Ship is bound to a Port that lies NE. by N. $\frac{1}{2}$ Northerly, distant 325 Miles, the Wind being N. by E. and she being able to lie no nearer the Wind than 6 $\frac{1}{4}$ Points, I demand how many Miles she must sail upon her Larboard-Tack, and how many upon her Starboard, to gain her desired Port.

Geometrically.

1. Let A be the Place she sails from, B the Place she is bound to, and *d* A the Course of the Wind.



2. Set off 6 $\frac{1}{4}$ Points on each Side of the Wind to X and Y, and draw AY and AX.

3. Through

3. Through B draw BC parallel to XA, till it meets AY produced, then will AC and CB be the Diſtances required.

To find which *Trigonometically*,

In the Triangle ABC are given,

1ſt. AB 325 Miles.
2d. ∠BAC 4 ¾ Points = 53° 26′
3d. ∠ABC 7 ¾ Points = 87 11 ½
4th. ∠ACB 3 ½ Points = 39 22 ½.

And therefore it will be, *1ſt*,

s, ACB : AB : : s, ABC : AC. That is,

As the Sine of ACB 39° 22′ —— 9.8022816

To AB : 325 Miles —— 2.5118834
So is the Sine of ABC 87° 11′ —— 9.9994750

To AC 51.8 Miles —— 2.7090768

And ſo far muſt ſhe ſail upon her Larboard-Tack, to get as ſoon as poſſible to her deſired Port.

But to find how far ſhe muſt ſail upon her Starboard, it will be

s, ACB : AB : : s, BAC : BC. That is,

As the Sine of ACB 39° 22′ —— 9.8022816

To AB 325 Miles —— 2.5118834
So is the Sine of BAC 53° 26′ —— 9.9048043

To BC 41.6 Miles —— 2.6144061

The Diſtance ſhe muſt ſail upon her Starboard-Tack.

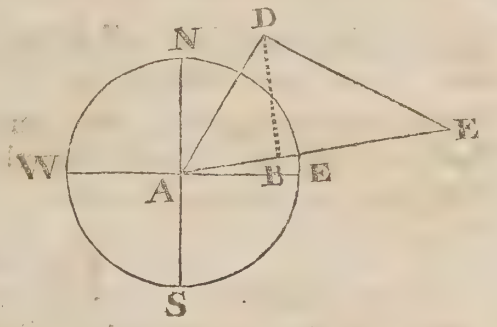
This Problem is of great Uſe in inſtructing the induſtrious Mariner how he may conduct his Ship the ſhorteſt Way, when the Winds are contrary.

Caſe

Case IV.

Suppose two Ports lie NNE. and SSW. distant 300 Miles, and the Wind being at North, a Ship sails from the Southermost, close haul'd upon the Wind 510 Miles, with her Larboard-Tack on board, and then 410 Miles with her Starboard-Tack, and then arrives at the Northermost; I desire to know how near the Wind she makes her Way good upon each Tack.

Let A and D be the two Ports, AE and DE the Distances upon each Tack.



In the Triangle ADE are given,

1. AD the Distance of the Ports, 300.
2. AE the Distance run upon the Larboard-Tack, 510.
3. DE the Distance upon the Starboard, 410.

Now to find the Angle DAE, it will be, (having let fall the Perpendicular DB)

AE : DE + AD : : ED — DA : EB — BA. That is, by

Logarithms.

As the Logarithm of 510 —— 2.7075702

To the Logarithm of 710 —— 2.8512583
So is the Logarithm of 110 —— 2.0413927

To the Logarithm of 153.2 —— 2.1850808

Now, 510 — 153, 2 = 356, 8 = 2 AB.

Therefore, AB = 178. 4, and it will be,
AD : R. : : AB : : c s, DAB is. That is,

As AD 300 —— 2.4771211

To the Radius —— 10.0000000
So is AB 178.4 —— 2.2513948

To the Co-sine of the < A 53. 31 —— 9.7742737

Or,

Or,

To the Arith. Complement of the Side AE 510 —	7.2924298
Add the Arith. Complement of the Side AD 300 —	7.5228787
Alſo the Log. of the half Sum 610 — — — —	2.7853298
And the Log. of the Remainder 200 — — — —	2.3010300
Half the Sum of theſe four Logarithms — —	19.9016683
Will give the Co-ſine of 26° 46′ ½ — — —	9.9508341

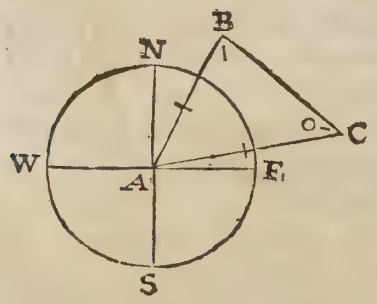
Which being doubled will give 53. 31 for the Angle at A, the ſame as was before found.

Hence it appears, that the Courſe ſhe made upon her Larboard-Tack, was ENE. 8° 31′ Eaſterly; and therefore ſhe made her Way good within 6 ¼ Points.

Caſe V.

Being at Sea, the Wind at North, I ſaw a Headland bearing N. by E. ½ Eaſt, diſtant 3 Leagues, then I ſtood away between the North and Eaſt 8 Leagues with my Larboard-Tacks on board, and then the ſame Headland bore from me Weſt by North; I demand the true Courſe?

Geometrically.



1. Having drawn the Compaſs N. E. S. W. let A repreſent the Place of the Ship at Sea, and draw the N. by E. ½ E. Line AB equal to 9 Miles, then will B repreſent the Headland.

2. From B draw the Line BC parallel to the E. by S. Line, till it interſects AC at the Diſtance of 24 Miles, from the Point A, then will AC be the Ship's Courſe; to find which, in the Triangle ABC are given, AC = 24 Miles, AB = 9 Miles, and the Angle ABC equal to 95° 37′ ½, equal to 8 ½ Points, equal to the Diſtance between the N. by E. ½ E. and the W. by N. whence to find the Angle BAC, by *the 1ſt Caſe of Sect. the 8th of Trigonometry*, it will be,

As

As the Diſtance ſailed AC = 24 ——— 1.3802112

To the Sine of the Angle at B = 95° 37′ ½ ——— 9.9979037
So is the Diſtance given AB = 9 ——— 0.9542425

To the Sine of the Angle at C = 21° 55′ ——— 9.5719350

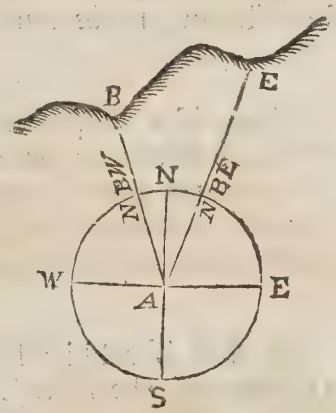
Whence the Angle BAC will be 62° 27′ ½, and conſequently the Courſe is E. by N. 00° 35′ Eaſterly, or 7 Points from the Wind.

Hence, by knowing how near the Ship can lie up to the Wind may the Lee-way be found.

For ſuppoſe ſhe can lie up within ſix Points of the Wind, then in this Caſe ſhe has made one Point Lee-way.

When a Ship is near the Land, as in the former Caſe, the Lee-way may be found much more exactly by the following *Example.*

Suppoſe a Ship at A, lying up N. by W. towards B, but that inſtead of keeping on the Courſe AB, ſhe is carried on in the Line AE, or N. by E.



'Tis evident, that the Point E will always bear the ſame from the Ship, becauſe ſhe keeps on in the ſame Courſe AE, and that the Bearing of every other Point alters.

Hence therefore, the Difference between the Courſe AE, the Ship is carried upon, or N. by E. and the Point of the Compaſs the Ship Capes at, or the Angle BAE equal in this Caſe to two Points, is the Lee-way ſhe makes.

This is of great Uſe when you are about to double a Cape, Headland, *&c.* the Lee-way the Ship makes being known.

In all Caſes, when the Ship is upon a Wind, Allowance muſt be made for her Lee-way; but this being greater or leſs, according to the Growth and Surge of the Sea, Mould and Trim of the Ship, Sail ſhe bears, and Streis of Wind, it will be very proper for the young *Navigator* to get the Aſſiſtance of ſome Able Sailor, with whoſe Help, and a little Experience, his Judgment will be ſo fortified, that he will ſeldom fail of making a good Allowance.

SECT. V.

Of Currents, *and how to allow for them.*

Definition.

CURRENTS are certain Settings of the Stream, by the Means of which, all Bodies moved therein are compelled to alter their Courſe, and ſubmit to the Motion impreſs'd upon them by it.

Whence,

Corol.

1ſt. If a Current ſets with the Courſe of the Ship, it augments her Motion, by as much as is the Drift or Rate of Driving.

Example.

Thus, if a Ship ſails NNE. 20 Miles, in a Current that ſets NNE. 8 Miles in the ſame Time, her true Courſe will be NNE. 28 Miles in the ſame Time: But,

2. If a Current ſets againſt the Ship, it leſſens her Velocity by juſt ſo much as is the Current's Drift.

Example 1.

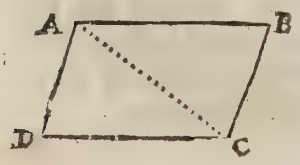
So that if a Ship ſail NE. 49 Miles in a Current that ſets SW. 10 Miles in the ſame Time, then her true Courſe will be NE 39 Miles. But

Example 2.

If in the Time that the Ship ſails NE. 49 Miles, the Current ſets SW. 59 Miles, then the Ship will fall a-ſtern, and her true Courſe would be SW. 10 Miles. But

3. If the Ship thwarts the Current, it not only leſſens or augments her Velocity, but gives her a new Motion, compounded of that of the Ship and Current, as is manifeſt from the following

Lemma.



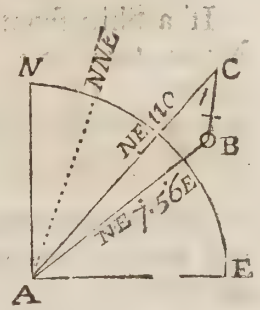
If a Body be agitated by two Motions at the ſame Time, the one with a certain Velocity that will carry it according to the Direction of the Line AB, the Length AB in a certain

certain Space of Time, the other according to the Direction of the Line AD, with a Velocity that will carry it to the Diſtance AD, in the ſame Time, I ſay, that the Body will deſcribe the Diagonal AC, and at the End of that Time will be found in the Point C.

Wherefore,

If a Ship ſails NE. 110 Miles, in a Current that ſets SSW. 30 Miles in the ſame Time, and her true Courſe and Diſtance ſail'd be required.

Geometrically.



1. Set off 4 Points from N. towards E. and draw AC equal to 110 Miles.

2. From C draw CB, parallel to the NNE. Line, and equal to 30 Miles.

3. Draw AB, which will be the Ship's true Courſe: To find which

Trigonometrically.

In the Triangle ABC are

Given $\left\{\begin{matrix} AC\ 110 \\ \angle \text{at } C\ 22^\circ\ 30' \\ BC\ 30 \end{matrix}\right\}$ and it will be, by *Caſe the 4th of Sect.* 8.

$AC+BC : AC-BC :: t, \frac{1}{2}\overline{\angle A+B} : t, \frac{1}{2}\overline{\angle B-A}$. That is,

As the Sum of AC and BC 140 ——— 2.1461280

To their Difference 80 ——— 1.9030900

So is the Tangent of 78° 45′ ——— 10.7013382

To the Tangent of 70° 49′ ——— 10.4583002

Now $78^\circ\ 45' - 70^\circ\ 49' = 7^\circ\ 56' = \angle CAB$.

Hence it appears, that her true Courſe is NE. 7° 56′ Eaſterly.

But for her Diſtance it will be, by *Caſe the 5th of Sect. the 8th.*

$s, \angle A : BC :: s, \angle C : AB$. That is,

As the Sine of the Angle at A 7° 56′ —— —— 9.1399445

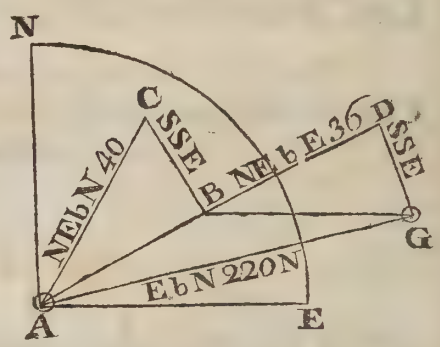
To the Drift of the Current BC 30 —— —— 1.4771212
So is the Sine of the Angle at C 22° 30′ —— —— 9.5828397

To the Distance run 83, 2 Miles —— —— —— 1.9200164

Example 2.

If a Ship from the Latitude 36° 30′ South, sails NE. by N. 40 Miles, then NE. by E. 36 Miles, in a Current that sets SSE. 20 Miles in the Time that the Ship sails 38 Miles, and it be required to find the direct Course and Distance between the Ship and her first Place, as also the Latitude the Ship is in.

1st. Having constructed it according to the Directions given in the former *Case*.



In the Triangle ACB are

Given { AC 40 } to find AB, & { <at C 56° 15′ } the Angles at { BC 21 05 } A and B.

It will be by *Case the 4th of Sect. the 8th.*

AC+BC : AC—CB : : t, ½ <A + <B : t, ½ <B — <A. That is,

As the Sum of AC and BC 61 05 —— —— —— 1.7856857

To their Difference 18 95 —— —— —— —— 1.2776092
So is the Tangent of 61° 52′ ½ —— —— —— —— 10.2720432

To the Tangent of 30° 08′ ½ —— —— —— —— 9.7639667

Now, 61° 52′ ½ — 30° 08′ = 31° 44′ = <CAB, therefore the Ship's first Course is NE. by E. 9° 14′ Eastwardly.

For the Distance, it will be by *Case the 1st of Sect. the 8th,*

s, CAB : CB : : s, ACB : AB. That is,

As

As the Sine of the Angle CAB 31° 44′ —— 9.7209581

To CB the Drift of the Current 21 05 —— 1.3232521
So is the Sine of the Angle at C 56° 15′ —— 9.9198464

To the Distance run 33 28 —— 1.5221404

2. In the Triangle BDG are given,

BD 36
< at D 78° 45′
DG 18 95

} to find BD and the Angles DBG and DGB.

Wherefore,

As the Sum of BD and DG 54 95 —— 1.7399677

To their Difference 17 05 —— 1.2317244
So is the Tangent of 50° 37′ —— 10.0856980

To the Tangent of 20° 42′ —— 9.5774547

Hence, the Angle DBG, will be found 29° 55′; wherefore the Ship's second Course is E. by N. 7° 25′ Eastwardly; and consequently, the Angle ABG = 159° 19′.

But for BG the Distance run, it will be,

As the Sine of the Angle DBG 29° 55′ —— 9.6978741

To DG the Drift of the Current 18 95 —— 1.2776092
So is the Sine of the Angle BDG 78° 45′ —— 9.9915739

To BG the Distance run 37 27 —— 1.5713090

3. In the Triangle ABG are given,

A B 33 28
< ABG 159° 19′
B G 37 27

} whence must be found < BAG and AG, and it will be,

As the Sum of AB and BG 70 55 —— 10.8484970

To their Difference 3, 99 —— 0.6009729
So is the Tangent of 10° 20′ ½ —— 9.2612203

To the Tangent of 00° 36′ —— 8.0136962

Hence

Hence the Angle BAG is 10° 56′, and consequently the Bearing of the Ship from her first Port is E. by N. 2° 20′ Northerly, and for the Distance it will be,

As the Sine of the Angle BAG 10° 56′ —— —— 9.2789911

To the Distance run BG 37 27 —— —— —— 1.5713090
So is the Sine of the Angle ABG 159° 19′ —— —— 9.5480240

To the Distance between the Ship and her Port 69 24— 1.8403419

Now for the Difference of Latitude, it will be,
As the Radius —— —— —— —— —— 10.0000000

To the Distance sailed 69 24 —— —— —— —— 1.8403571
So is the Co-sine of the Course 76° 25′ —— —— 9.3708079

To the Difference of Latitude 16 26 —— —— —— 1.2111650

Hence, because she sail'd from a South Latitude Northerly,
From the Latitude she sail'd from —— —— 36° 30′ Sly.
Take the Difference of Latitude made —— —— 16′ $\frac{2}{3}$ Nly.

Remains the Latitude the Ship is in —— —— 36° 13′ $\frac{2}{3}$ Sly.

Or,
The same Case may be considered, as a Traverse of three Courses,
And then the *Operation* will be as follows,

Courses.	Dist.	Diff. of Lat. N.	Diff. of Lat. S.	Departure. E.	Departure. W.
NE. by N. —— ——	40. 00	33. 26	——	22. 2[illegible]	——
NE. by E. —— ——	36. 00	20. 00	——	29. 9[illegible]	——
S S E. —— —— ——	40. 00	——	36. 95	15. 3	——
		53. 26	36. 95	7. 4[illegible]	East.
		36 95			
		16. 31	North		

Whence

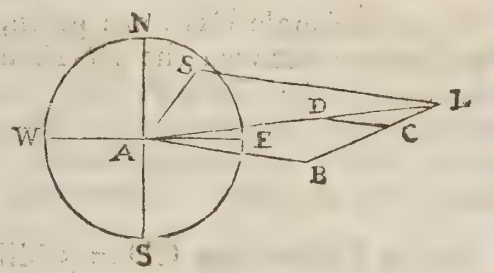
Whence it appears, that the Ship has departed 67.46 Miles from her former *Meridian*; that she has made 16.31 Miles of Difference of Latitude; whence it will be found, that she is in the Latitude of 36° 13′ South; that her direct Course is N. 76° 24′ East, or ENE. 8° 54′ Easterly, and that her direct Distance is 69.36 Miles, agreeing with the former Method of Investigation.

Example 3.

A Ship coming out from Sea in the Night, has Sight of *Scilly* Light, bearing NE. by N. Distance 4 Leagues, it being then Flood Tide, setting ENE. about two Miles an Hour, the Ship running after the Rate of 5 Knots an Hour; I demand upon what Course she must sail, and how far to hit the *Lizard*, it bearing from *Scilly* E. ½ S. Distance 17 Leagues.

Geometrically.

1. Having drawn the Compass N, E, S, W. Let A represent the Place of the Ship at Sea, and draw the NE. by N. Line AS, equal to 12 Miles, then will S represent *Scilly*.



2. From S draw SL equal to 51 Miles, and parallel to the E. ½ S. Line, then will L be the Place of the *Lizard*.

3. Draw LC parallel to the WSW. Line equal to two Miles, and CD equal to five Miles, and parallel to it draw AB, till it meet LC produced in B; then will AB be the Distance required, and the Angle SAB the true Course.

To find which by Calculation,

1. In the Triangle ASL are given, AS equal to 12 Miles, SL equal to 51 Miles, and the Angle ASL equal to 118° 7′ ½, equal to 10 ½ Points, equal to the Distance between the NE. by N. and W. ½ N.

Whence to find *1st* the Angle SAL, it will be, by *the 4th Case of Sect. the 8th of Trogonometry.*

As the Sum of the Sides AS and SL = 63 —— — 1.7993405

To their Difference = 39 —————— 1.5910646
So is the Tangent of ½ the Angles A and L = 30° 56′ 9.7776284

To the Tangent of half their Difference = 20° 21′ 9.5693525

Whence

Whence the Angle A will be found equal to 51° 17′, and consequently the direct Bearing from the Ship to the *Lizard*, is N. 85° 02′ E. or E. by N. 6° 17′ Easterly. And for the Distance it will be,

As the Sine of the Angle at A = 51° 17′ —— 9.8922329

To the Distance from *Scilly* to the *Lizard* = 51 — 1.7075702
So is the Sine of the Angle at S = 118° 7′ ½ —— 9.9454298

To the Dist. between the Ship and the *Lizard* 57 65—1.7607671

Again,

In the Triangle DLC are given, the Angle L equal to 17° 32′, the Distance between the ENE. and the N. 85° 02′ E. the Side CL equal to the Current's Drift in an Hour, equal to two Miles; and the Side CD, the Ship's Distance run in the same Time, equal to five Miles; whence to find the Angle at D, it will be, *by the 1st Case of Sect. the 8th of Trigonometry*,

As the Distance run CD = 5 Miles —— —— 0.6989700

To the Drift of the Current LC = 2 Miles —— 0.3010300
So is the Sine of the Angle at L = 17° 32′ —— —— 9.4789423

To the Sine of the Angle at D = 6° 55′ —— 9.0810023

Hence, because the Angle CDL is equal to BAL, the Course the Ship must steer, is S. 88° 03′ E. And for the Distance it will be, *by Case the 2d of Sect. the 8th of Trigonometry*,

As the Sine of the Angle at B = 155° 33′ —— 9.6168944

To the Distance between the Ship and the *Lizard* 57 65 1.7607993
So is the Sine of the Angle at L = 17° 32′ —— 9.4789423

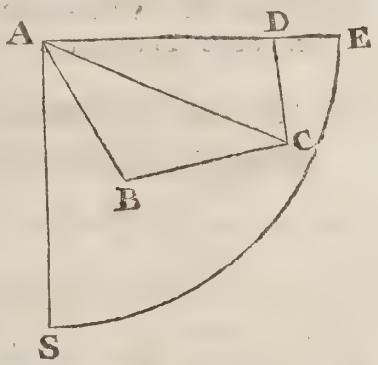
To the Distance she must sail 41.96 Miles —— —— 1.6228472

So that the Ship, in sailing 8 H. 23 M. S. 88 Deg 03 Min. E. will arrive at the *Lizard*.

*

Suppose

Suppose a Ship from a certain Headland, in the Latitude of 34° 00′ North, sails SE. by S. 24 Miles in six Hours, in a Current that sets between the North and the East; and then the Cape is found to bear WNW. and the Ship to be in the Latitude of 33° 45′ N. I demand the Setting and Drift of the Current?



Geometrically.

1. Set off 3 Points from S. towards E. and draw the Line AB equal to 24 Miles.
2. Set off 2 Points from E. towards S. and draw the Line AC.
3. At the Distance of 15 Miles, equal to the Difference of Latitude, draw a parallel to AE, till it intersect AC as in C.
4. From the Intersection C, draw CD parallel to AS, also the Line BC, the Drift and Setting of the Current.

To find which *Arithmetically*,

In the Triangle ADC, Right-angled at D, are given,

$$\left\{\begin{matrix} DC\ 15 \\ \angle A\ 22^\circ\ 30' \end{matrix}\right\} \text{to find AC.}$$

Wherefore, S. $\angle$ A : DC : : R : AC. That is,

As the Sine of the Angle at A 22° 30′ —— ——	9.5828397
To the Difference of Latitude 15 —— ——	1.1760912
So is the Radius —— —— —— —— ——	10.0000000
To the Distance run 39.2 —— —— —— ——	1.5932515

Again, In the Triangle ABC are given,

$$\left\{\begin{matrix} AB\ 24 \\ \angle A\ 33^\circ\ 45' \\ AC\ 39.2 \end{matrix}\right\} \text{to find} \left\{\begin{matrix} \angle C \\ BC \\ \angle B. \end{matrix}\right\}$$

Wherefore,

$AB + AC : AC - AB :: t, \frac{1}{2} \angle B + \angle C : t, \frac{1}{2} \angle B - \angle C.$

As the Sum of the Sides AB and AC 63.2 ——— 1.8007171

To their Difference 15.02 ——— 1.1818436
So is the Tangent of 73° 07′ ½ ——— 10.5180608

To the Tangent of 38° 24′ ½ ——— 9.8991873

Which added to 73° 07′ ½, gives 111° 32′, the Angle at B.

And hence the Current will be found to set E. by N. 00° 58′ Nly.

For BC, the Drift of the Current, it will be,

s, ∠ B : AC :: s, ∠ A : BC. That is,

As the Sine of the Angle at B 111° 32′ ——— 9.9685783

To AC the Distance run 39 02 ——— 1.5932515
So is the Sine of the Angle at A 33° 45′ ——— 9.7447390

To BC, the Drift of the Current in 6 Hours 23.41 ——— 1.3694122

And consequently it sets at the Rate of 3.9 Miles an Hour.

Several other Questions might be proposed relating to Currents, but these being throughly understood, are sufficient for the Purpose.

SECT. VI.

Of the PLAIN CHART.

SINCE a small Part of the Earth's Superficies does differ very little from a Plain, Ingenious Men were first led (for ordinary Uses) to describe the Sea-Coasts between Places not very remote, on Plans, which they call *Charts*, in which they made the Degrees of Longitude equal in all Places to those of Latitude. So that

A *Plain*

A *Plain Chart* is a Repreſentation of ſome ſmall Part of the Superficies of the Terraqueous Globe, in which the *Meridians* are drawn parallel to each other, and conſequently the Degrees of Longitude are equal in all Parts to each other, and alſo equal to thoſe of Latitude, and is generally made to ſerve for ſome particular Voyage; but when made, may ſerve for any Places whoſe Diſtance does not exceed the Limits of the *Chart*.

Wherefore,

If it were required to make a *Chart* that ſhall contain 5 Degrees in Latitude, and $3\frac{1}{2}$ in Longitude.

Or which is the ſame Thing,

Suppoſe it were required to make a *Chart* that ſhall ſerve for a Voyage between the Parallels of 12° 30′ and 17° 30′ North Latitude, and to contain $3°\frac{1}{2}$ of Longitude.

Having pitch'd upon a convenient Length for the Breadth of a Degree, or rather for any proportionable Part of it,

1. Draw the Meridian MN, and prick off the Number of Degrees the *Chart* is to contain, from M towards N. And,

2. At Right-angles to the Meridian MN, from the Point M draw the Line MS, which may repreſent the Parallel of 12° 30′ North, and ſet off the Number of Degrees it muſt contain in Breadth, from M towards S.

3. Through the Point S, draw SQ parallel to MN, as alſo Lines parallel to MN, through the ſeveral Diviſions of the Line MS, which will repreſent ſo many Meridians.

4. The Lines drawn parallel to MS through the equal Diviſions of the Meridian will repreſent ſo many Parallels of Latitude.

5. Each of the Right-angles M,N,Q,S, being divided into 8 equal Parts, the Lines drawn from the angular Points through the ſeveral Diviſions, will be the Rhumb-Lines upon the *Chart*; and are of great Uſe for the ready finding the Bearing of Places, as ſhall be ſhewn hereafter.

Laſtly, Having divided each Degree into as many equal Parts as its Breadth will admit of, the *Chart* is render'd fit for any manner of Uſe that it may be applied to. And,

1. To lay down any Place upon the *Chart*,

If it be by its Latitude and Longitude.

1. Through the Degree in the Meridian anſwering to its Latitude, draw a Parallel of Latitude.

2. Lay a Ruler over the Degrees of Longitude in the Graduated Parallel, and draw a Meridian; where this cuts the Parallel before drawn, is the Place required.

After this Method may any Place be laid down by its Difference of Latitude and Longitude from any Place in the *Chart*, viz. By counting its Difference of Latitude from the Latitude of the given Place, either Northward or Southward, according as the Case requires; and its Difference of Longitude, either Eaſtward or Weſtward, and drawing Lines through the Points found as before.

By the Help of this, may a Map of any particular Country or the Sea-Coaſts be drawn, if the Latitudes and Longitudes of the Places it muſt contain be known; or their Difference from any Place whatſoever.

By the Inverſe Method, *viz.* by drawing Lines parallel to the Meridian, and the Parallel of Latitude paſſing through any Place propoſed, may its Latitude and Longitude be known, and conſequently the Difference of Latitude and Longitude of any two or more Places propoſed.

2. If it be by its Bearing and Diſtance from any Place,

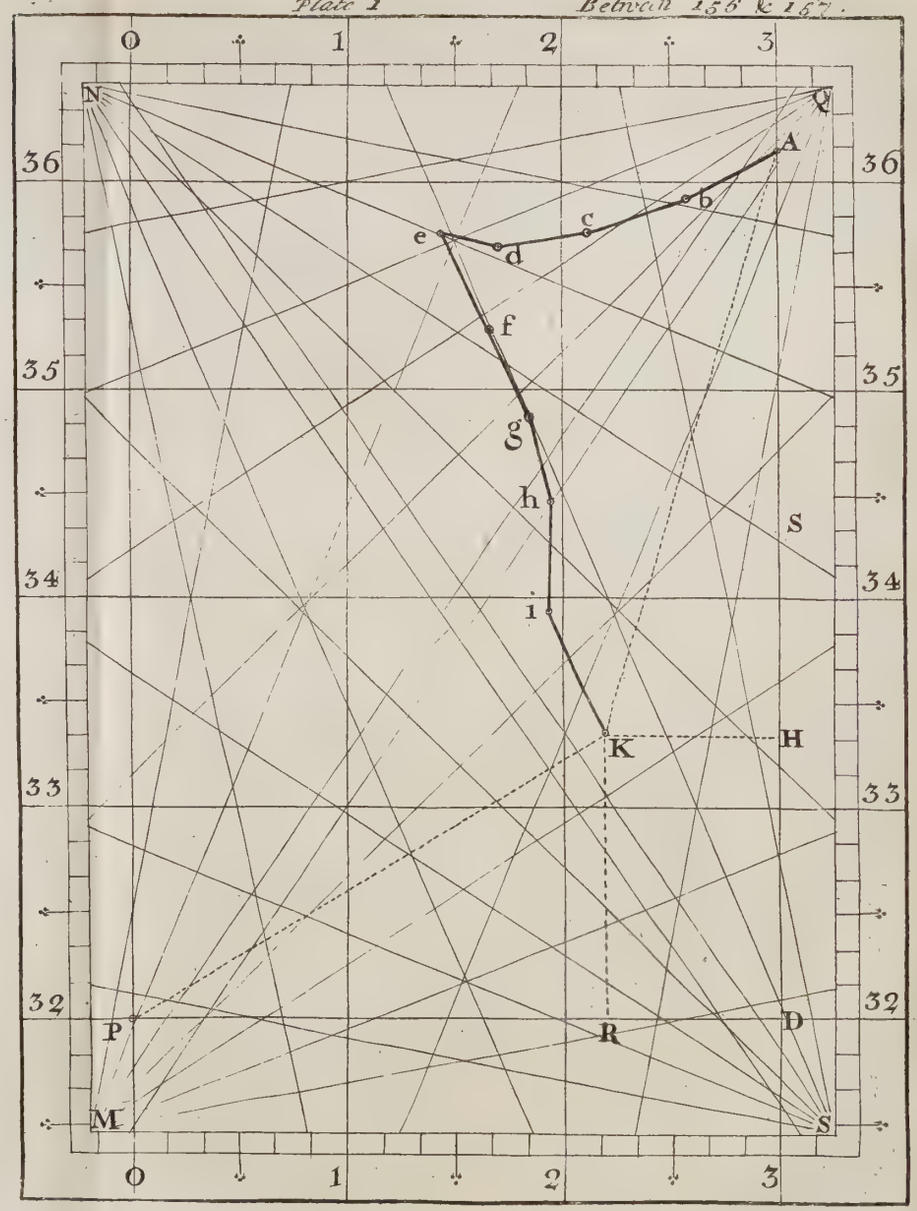
After having found out the Rhumb-Line, anſwering to the Bearing given, through the Place propoſed, and parallel to the Rhumb, draw a Line, and ſet off upon it the Diſtance given in Parts of the Graduated Meridian, and the Point thus found will be the Place required.

By the Inverſed Method, may the Bearing and Diſtance of any two Places in the *Chart* be known, *viz.* by taking their Diſtance betwixt the Points of the Compaſſes, and applying it to the Meridian in order to find the Miles, and by obſerving which of the Rhumb-Lines is parallel to it, in order to diſcover the Bearing.

By the Help of this may the induſtrious Sailor diſcover upon what Courſe, (if poſſible) and how far he muſt ſail, to arrive at his deſired Haven; as alſo to lay down his ſeveral Runnings, and thence find the ſeveral Places that he arrives at in the end; and ſo be able to alter his Courſe a-new, in order to gain any Port in the ſhorteſt Time poſſible. But

Tho' this Way is very expeditious, yet I would not adviſe the Young Student to depend altogether upon it, but rather to keep the Account of the ſeveral Runnings in Numbers, finding the ſeveral Eaſtings, Weſtings, Northings, or Southings, and Tabulizing of them according to the former Methods. and then if at any Time

Plate 1 Betwen 156 & 157.



Time his Curiosity shall lead him to examine how exact he ha been, it will be of great Use to him.

3. If the Bearing and Difference of Latitude be given,

Count the Difference of Latitude either Northward or Southward, (as the Case requires) on the divided Meridian from the given Latitude, and draw the Parallel passing through the same, and lay a Ruler over the Place, and parallel to the Rhumb-Line given, where it cuts the Parallel before drawn, it gives the Place required.

4. If it be by the Distance and Difference of Latitude,

Having drawn the Parallel as before (by the Help of the Difference of Latitude) take the Distance between the Points of the Compasses, and setting one Foot in the Place given, with the other cross the Parallel before drawn, and you'll have the Place required.

5. If it be required to lay it down from the Course and Departure,

Count the Departure in the Graduated Parallel from the Meridian passing through the Place given, either Eastward or Westward, (as the Case requires) and through that Point draw a Meridian, and where the Course intersects, it will be the Place sought.

6. If it be by the Distance and Departure,

By the Help of the Departure, draw the Meridian proper to the Place you would find, then with the Distance of the two Places, between the Points of the Compasses, setting one Foot in the Point given, turn the other about, till it intersect the Meridian last found in the Place sought.

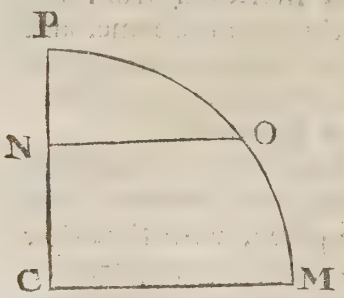
It would be needless to illustrate these Propositions by Examples, since he that understands what went before, will find it no matter of Difficulty to resolve either of these, or any other of the same Nature, with those that have been proposed in Sailing, by the Help of the Chart only.

SECT. VII.

HItherto we have considered the Superficies of the Earth as a Plain, and the Meridians as parallel to each other, and consequently the Degrees of Longitude equal to each other in all Places, and equal to those of Latitude; which tho' it be notoriously false in itself, yet when a Voyage is performed near the Equinoctial, or a large one divided into several small ones, especially if a Man return

return home in or near the oppoſite Rhumb, a tolerable good Account may be kept thereby. For,

That the Degrees of Longitude grow leſs and leſs, the nearer you approach the Pole, (as we have before hinted) is manifeſt from the annex'd Figure; in which P repreſents the Pole, POM a Quadrant of the Meridian, CM the Semidiameter of the Equator, and NO the Semidiameter of any Parallel of Latitude, which is to the Semidiameter of the Equator, as the Sine of the Arch PO, to the Sine of the Arch PM; that is, as the Co-ſine of the Latitude to the Radius.



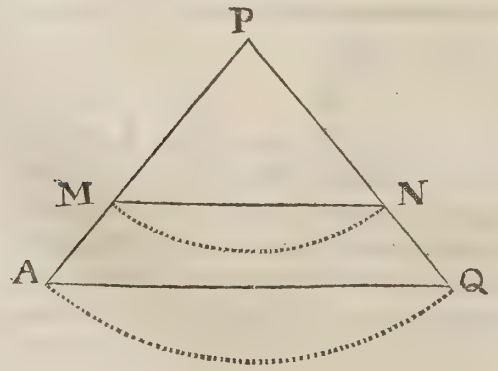
Now, ſince the Circumferences of all Circles are as their Radij directly, and that like Parts have the ſame Proportion with their Wholes, it follows, that the Breadth of a Degree of Longitude, in any Parallel of Latitude, is to the Breadth of a Degree in the Equator, as the Co-ſine of that Latitude is to the Radius; and conſequently ſo are the Miles in one to the Miles in the other.

PROBLEM I.

If the Difference of Longitude between two Places, both in one Parallel or Latitude be given, to find the Diſtance between them; That is,

If a Ship in the Latitude of 49° 30' North or South, ſails directly Eaſt or Weſt, until her Difference of Longitude be 3° 30', and it be required to find her Diſtance ſail'd,

Geometrically.



1. Having drawn the Line AP equal to 90°, make PM equal to the Sine of the Complement of the Latitude, and deſcribe the Arches AQ, and MN.

2. With the Difference of Longit. converted into Miles, taken

taken off of the Line of equal Parts, between the Points of the Compaſſes, ſetting one Foot in A, with the other croſs the Arch AQ in Q, and draw PQ, and where it cuts the Arch MN as in N, draw the Line MN, which will be the Diſtance required.

To find which *Numerically*, it will be,

PA : PM : : AQ : MN. That is,

As the Radius	10.0000000
To the Co-ſine of the Latitude 49° 30′	9.8125444
So are the Miles of Longitude 210	2.3222193
To the Miles of Diſtance run 136.4	2.1347637

PROBLEM II.

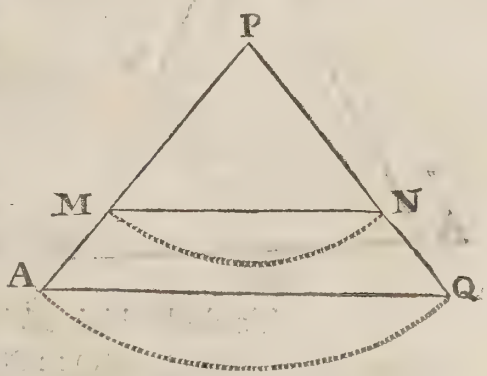
If the Diſtance in any Parallel of Latitude be given, to find the Difference of Longitude.

That is,

If a Ship ſails 156 Miles Eaſt or Weſt in the Latitude of 47° 15′ North or South, and it be required to know how much ſhe has altered her Longitude.

Geometrically.

Having made as before PA equal to the Sine of 90, and PM equal to the Co-ſine of the Latitude, and drawn the Arches MN and AQ, make MN equal to the Diſtance run; and having drawn PN, produce it till it cut the Arch AQ in the Point Q, and draw the Line AQ, which will be the Alteration of Longitude.



And to find it *Numerically*, it will be,

PM : PA : : MN : AQ.

That

That is,

As the Co-fine of the Latitude 47° 15′ ——— 9.8317423

To the Radius ——— 10.0000000

So are the Miles of Diſtance 156 ——— 2.1931246

To the Miles of Longitude 229.8 ——— 2.3613823

Which, converted into Degrees, makes 3° 49′ $\frac{4}{5}$.

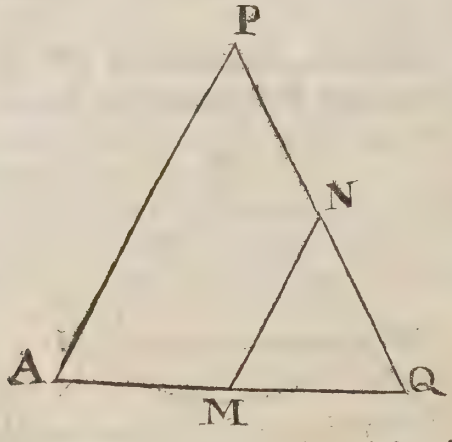
So that in ſailing but 156 Miles Eaſt, in the Latitude of 47° 15′, a Ship will alter her Longitude 3° 49′ $\frac{4}{5}$.

PROBLEM III.

If the Diſtance ſail'd under any Parallel of Latitude, and the Difference of Longitude were given, to find the Latitude.

That is,

If a Ship in ſailing 980 Miles Eaſt or Weſt, alters her Longitude 6° 00′, and the Latitude ſhe is in be required.



Geometrically.

1. Make PQ and PA each equal to 90° of the Line of Sines, and AQ equal to the Miles of Longitude given.

2. Set off the Diſtance run from Q to M, and draw MN parallel to AP, then will NQ be the Sine of the Complement of the Latitude.

To find which *Arithmetically*, it will be,

AQ : MQ : : PQ : NQ. That is,

As the Difference of Longitude in Miles 360 ——— 2.5563025

To the Diſtance run 180 ——— 2.2552725

So is the Radius ——— 10.0000000

To the Co-fine of the Latitude 60° 00′ ——— 9.6989700

So

So that if in the Time that a Ship ſails 180 Miles, ſhe alters her Longitude 6° 00′, ſhe is under the Parallel of 60° 00′, Nth. or Sth.

PROBLEM IV.

Suppoſing two Places under the ſame Parallel of Latitude diſtant 220 Miles, and two other Places under the ſame Meridians with the former, differing in Latitude from the former 4° 20′, and the Diſtance of the two laſt Places were required.

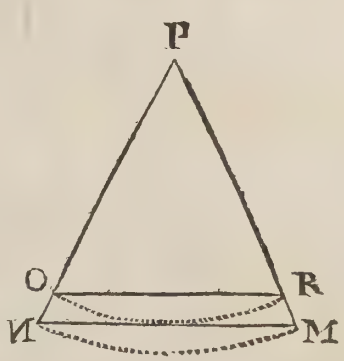
That is,

If two Ships in the Latitude 45° 00′ North, diſtant 220 Miles, ſail both directly North 260 Miles, aud it be required to find their Diſtance aſunder.

Having reduced the Diſtance ſailed into Degrees, and added it to the Latitude they ſailed from, in order to find the Latitude they are in,

Geometrically.

1. Make PN equal to the Sine of the Complement of the Latitude they ſailed from, and PO to the Sine of the Complement of the Latitude they ſailed to, and deſcribe the Arches NM and OR.
2. Let NM be made equal to the Diſtance given, and draw PM till it interſect the Arch OR as in R, and draw OR, which will be the Diſtance required.



And to find it *Numerically*, it will be,

PN : PO : : NM : OR. That is,

As the Co-ſine of the Latitude they ſail'd from 45° 00′	9.8494850
To the Co-ſine of the Latitude they ſail'd to 49° 20′	9.8140192
So is their Diſtance firſt given 220	2.3424227
To their Diſtance required 202.7	2.3069569

Though there are other Queſtions of this Kind, yet he that throughly underſtands what has been ſaid upon this Subject, will find no Difficulty in reſolving any other that can be propoſed.

By the Help of the firſt *Problem* is the following Table made, ſhewing the Miles of Latitude anſwering to a Degree of Longitude in any Parallel, of great Uſe for the ready reſolving all *Problems* of this Kind.

A TABLE *ſhewing how many Miles anſwers to a Degree of* Longitude, *at every Degree of* Latitude.

D. L.	Miles.	D. L.	Miles.	D. L.	Miles.	D. L.	Miles.
1	59. 99	23	55. 23	45	42. 43	67	23. 45
2	59. 96	24	54. 81	46	41. 65	68	22. 48
3	59. 92	25	54. 38	47	40. 92	69	21. 50
4	59. 86	26	53. 93	48	40. 15	70	20. 52
5	59. 77	27	53. 46	49	39. 36	71	19. 54
6	59. 67	28	52. 97	50	38. 57	72	18. 55
7	59. 56	29	52. 47	51	37. 76	73	17. 54
8	59. 42	30	51. 96	52	36. 94	74	16. 53
9	59. 26	31	51. 43	53	36. 11	75	15. 52
10	59. 08	32	50. 88	54	35. 26	76	14. 51
11	58. 89	33	50. 32	55	34. 41	77	13. 50
12	58. 68	34	49. 74	56	33. 55	78	12. 48
13	58. 46	35	49. 15	57	32. 68	79	11. 45
14	58. 22	36	48. 54	58	31. 79	80	10. 42
15	57. 95	37	47. 92	59	30. 90	81	9. 38
16	57. 67	38	47. 28	60	30. 00	82	8. 35
17	57. 37	39	46. 62	61	29. 09	83	7. 32
18	57. 06	40	45. 95	62	28. 17	84	6. 28
19	56. 73	41	45. 28	63	27. 24	85	5. 23
20	56. 38	42	44. 59	64	26. 30	86	4. 18
21	56. 01	43	43. 88	65	25. 36	87	3. 14
22	55. 63	44	43. 16	66	24. 41	88	2. 09
23	55. 23	45	42. 43	67	23. 45	89	1. 05

Though this Table be made only to whole Degrees of Latitude, yet it will not be difficult to make it ſerve for any Latitude whatſoever; by ſaying, As 1 Degree to the Difference between the next greater and leſſer Tabular Latitude, ſo is the Exceſs of the Latitude propoſed above the next leſs Tabular Latitude to a proportionable

tionable Part; which being ſubtracted from the Miles anſwering to the next leſs Tabular Latitude, will give the Miles anſwering to the Latitude propoſed.

Thus,

If it were required to find the Miles anſwering to a Degree of Longitude, in the Latitude of 51° 32′, it will be,

As 1 Degree, or 60 Minutes, is to 82, the Tabular Difference between the Miles anſwering to 51° and 52°, ſo is 32′, the Exceſs of the Latitude given above 51°, to 43.7; which therefore ſubtracted from 37.76, the Miles anſwering to 51°, gives 37.32, the Miles anſwering to 1 Degree, in the Latitude of 51° 32′.

By the Help of this Table, it will be eaſy to convert Degrees of Longitude into Miles of Diſtance, or Miles of Eaſting or Weſting into Degrees of Longitude, in any Latitude whatſoever.

For,

1. If it were required to convert Degrees of Longitude into Miles of Diſtance, it will be,

As 1 Degree to the Miles anſwering thereto in the Latitude propoſed, ſo is the Difference of Longitude to the Miles of Diſtance.

Wherefore,

If two Places lie Eaſt and Weſt of each other, in the Latitude of 43° 20′, whoſe Difference of Longitude is 2° 27′, and it be required to find the Diſtance between them, having found out the Miles anſwering to 1 Degree, in the Latitude of 43° 20′, *viz.* 43.64, it will be,

As 60 Minutes to 43.64, ſo is 2° 27′, or 147′ Minutes, to 106.92 Miles, the Diſtance between them.

Again,

2. If it were required to convert Miles of Eaſting and Weſting into Degrees of Longitude.

Having found out in the Table the Miles anſwering to a Degree of Longitude, in the Parallel under which you ſail, it will be,

As the Tabular Miles to 1 Degree, ſo are the Miles you have ſail'd, to the Degrees of Longitude:

Thus,

Thus,

Suppoſe I have ſailed 104 Miles, under the Parallel of 43° 30′, and it be required to know how much I have altered my Longitude, it will be,

As 43.52, the Miles anſwering to 1 Degree, in the Latitude of 43° 30′, to 104 the Miles of Diſtance, ſo is 1 Degree to 2° 23′, the Difference or Alteration of Longitude, and ſo for the reſt.

By the Help of this Table, may all the Problems propoſed in this Section be eaſily reſolved; but this I leave to Exerciſe the Ingenuity of the Young Student.

SECT. VIII.

Of Mercator's *Sailing.*

WHoever conceives the Superficies of the *Terraqueous Globe* to be inveſted with a *Rete* or *Net*, compoſed of Meridians and Parallels, cannot but eaſily apprehend, when the ſame comes to be extended on a Plane (and ſo diſtorted, as to make the Meridians become parallel to each other, and the Degrees of Longitude every where equal to each other, and to thoſe of Latitude) that very great and notorious Errors will unavoidably follow.

For ſince (as we have already proved) that the Parallels decreaſe in the Proportion of the Sines of their Diſtances from the Pole; or, which is the ſame Thing, That a Degree of Longitude under any Parallel, is to a Degree of Longitude under the Equator, as the Radius to the Secant of its Diſtance from the Equator. The Repreſentation of Places in this Chart remote from the Equator, will be ſo diſtorted, that (for inſtance) an Iſland in the Latitude of 60 Degrees (where the Radius of the Parallel is but half ſo great as that of the Equator) would have its Length from Eaſt to Weſt, in compariſon of its Breadth from North to South, repreſented in a double Proportion of what indeed it is; (as Mr. *Wright* very well obſerves in his *Correction of Errors in Navigation.*

To

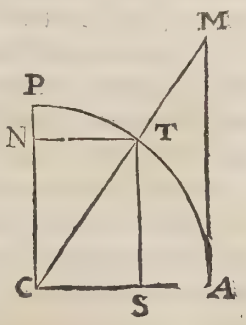
To rectify which, amongſt many other Inconveniences, he there takes Notice of, and to contrive a Projection *in plano*, that might be liable to none of theſe, was the great Study of many of the Antients; and tho' a Method was hinted by *Ptolomy* himſelf, yet it was never happily accompliſhed, 'till undertaken by our ingenious Country-man, Mr. *Edward Wright*.

Which he did by letting the Meridians remain parallel as before, and protracting the Degrees remote from the Equator, in like Proportion with thoſe of Longitude.

That is,

By making a Degree of Latitude ſo protracted, to a Degree of Latitude (which is every where equal to a Degree of Longitude in the Equator) as the Radius to the Sine of the Diſtance of ſuch Parallel from the Pole; or, as the Secant of its Diſtance from the Equator. The Reaſon of which is manifeſt from the following Figure.

Where the Arch PTA repreſents a Quadrant of the Meridian, P the Pole, CA the Semidiameter of the Equator, and NT the Semidiameter of any Parallel of Latitude, which is to the Semidiameter of the Equator, as the Sine of the Arch PT is to the Sine of the Arch PA; that is, as CS to CA, as hath been already proved.



From A draw AM parallel to CP, or perpendicular to CA, till it meet CT produced in M.

And becauſe NT equal to CS, is ſuppoſed every where equal to CA, and that CS is to CA, as CT is to CM; when CS becomes equal to CA, CT will become every where equal to CM; that is, the Semidiameter of the Meridian in any Parallel, is equal to the Secant of the Diſtance of ſuch Parallel from the Equator.

Whence it follows,

1. That a Degree of the enlarged Meridian, is every where equal to the Secant of the Arch contained between it and the Equator.

2. That its Diſtance from the Equator, ſhall be equal to the Sum of all the Secants contained between it and the Equator.

3. That the Diſtance of any two Parallels on the ſame Side of the Equator, ſhall be equal to the Difference of the Sum of the Secants

cants of all the Arches contained between each of the Parallels and the Equator. But

4. That the Diſtance between any two Parallels on contrary Sides of the Equator, is equal to the Sum of the Sum of all the Secants, contained between each of the Parallels and the Equator.

The Eſſential Property of this incomparable Projection (which in moſt Caſes is preferable even to the *Globe* itſelf) being thus found out and demonſtrated, the next Thing requiſite, was a ready Way for finding the Sum of the Secants of any given Arch.

This Mr. *Wright* did by the continual Addition of the Secants of 1′, 2′, 3′, 4′, 5′, *&c.* and thence formed a Table for the eaſy Diviſion of the Meridian Line, which was ſufficiently exact for the Mariners Uſe.

But how to compute independently the Meridional Parts anſwering to any given Latitude, or the Interval between any two, was not known, till after the Meridian Line, or Sum of the Secant, was found to be analogous to a Scale of Logarithmic Tangents, of half the Complements of the Latitudes.

Which was firſt diſcover'd by Chance, and afterwards proved by Mr. *James Gregory*, in his *Exercitationes Geometricæ*; but with ſuch a Train of Conſequences, and Complication of Proportions, that the Evidence of the Demonſtrations was in a great Meaſure loſt, and the Reader tired before he attained it.

After which, the Sagacious Dr. *Halley*, having very well obſerved, that in the *Stereographick Projection* on the Plane of the Equator, the Rhumb-Lines are proportional Spirals about the Pole, and that thence the Differences of Longitude, or Arches of the Equator, are the Exponents or Logarithms of the *Rationes*, of the ſeveral Diſtances of the Pole from the Curve, terminated by Lines drawn from the ſame Pole, to the ſeveral Diviſions of the Equator, which Diſtances are the Tangents of half the Complement of the Latitude in the ſame Projection: It follows, that the Differences of Longitude, are the Logarithms of the *Rationes* of thoſe Angles one to another.

And ſince the *Nautical* Meridian is no other than a Table of the Longitudes, anſwering to each Minute of Latitude on the Rhumb-Line, making an Angle of 45 Degrees with the Meridian, it is manifeſt, that the Meridian is no other than a Scale of the Logarithmic Tangents of the Half Complements of the Latitude.

Now,

Now, Since in every Point of any Rhumb-Line the Difference of Latitude is to the Meridian Diſtance, as the Radius to the Tangent of the Angle that Rhumb makes with the Meridian; and that thoſe equal Meridian Diſtances are every where to the Differences of Longitude, as the Radius to the Secant of the Latitude; it follows, that the Differences of Longitude on any Rhumb, are Logarithms of the ſame Tangents, but of a different Species, being proportioned to each other, as are the Tangents of the Angles made with the Meridian.

Hence,

From any Table of Logarithmic Tangents, it will be eaſy to find the Number of equal Miles of Longitude, anſwering to the Diſtance of any two Points, in the Meridian of the true Chart, and conſequently to make a Table for the ready Diviſion of the ſame. For,

Having determined the Angle which the Rhumb Line makes with the Meridian, under which the Differences of the Logarithmic Tangent are the true Differences of Longitude to every Degree or Minute of Latitude; it will be, as the Tangent of that Rhumb to the Tangent of the Rhumb under which the Differences of Longitude anſwer to the Diviſions of the Meridian Line; ſo is the Difference of the Logarithmic Tangents of half the Complements of any two Latitudes propoſed, to the Difference of Longitude on the propoſed Rhumb, or the Meridional Difference of Latitude between the ſame Places on the true Chart; and ſo is the Logarithmic Tangent of half the Complement of the Latitude of any Place, to the Diſtance of the ſame Place from the Equator.

Now, If we put *Unity* for one Minute of Longitude, as in the Common Meridian Line; becauſe the Length of the Arch of one Minute, is 000290888, *&c.* and the Index of the *Briggs*'s Logarithms (from which *Ulacy*'s Tables are made) 230258, *&c.* it will be, as 230258, *&c.* to 290888, *&c.* ſo is the Radius to 1.2633114, *&c.* the Natural Tangent of 51° 38′ 9″, the Angle which the Rhumb-Line makes with the Meridian; under which the Common Tables of Tangents are the true Differences of Longitudes.

Wherefore,

As 1263 to 1 Minute, ſo is the Difference of the Logarithmic Tangents of half the Complements of the Latitudes of any two Places, to their Meridional Difference of Latitude in Minutes.

Hence,

Hence, and from what has been said before, it follows, That the Difference of the Logarithmic Tangents, or Tangent Complements, of half the Complements of the Latitudes of any two Places, on the same Side the Equator, divided by 1263, will give their Meridional Difference of Latitude.

2. That the Sum of the Differences of the Tangent of 45 Degrees, and the Tangents, or Co-tangents of half the Complements of the Latitudes of two Places on contrary Sides of the Equator, divided by 1263, will give their Meridional Difference of Latitude.

3. That the Difference of the Logarithmic Tangent, or Co-tangent of half the Complement of the Latitude, and the Tangent of 45 Degrees, divided by 1263, will be the Meridional Difference of Latitude in Minutes.

Or, which is the same Thing,

To half the given Latitude add 45 Degrees, and having found out the Logarithmic Tangent of the Sum, reject the Characteristick, and divide the Remainder by 1263, and the Quotient will be the Meridional Parts of the Latitude proposed.

Hence we are taught a ready Way to find the Meridional Parts contained between any two Places, or to form a Table for the easy Division of the Meridian Line in the true Chart, by the Help of the Logarithmic Tangents only.

How the Meridional Parts of any Portion of the inlarged Meridian Line may be found, from the Length of the Arch itself, by a direct Method, may be seen in *Page* 255, &c. *of the 1st Part of any System.*

A

CORRECT

TABLE

OF

Meridional Parts,

TO

Every Degree and Minute of *Latitude*.

L. M	0 min.	1 min.	2 min.	3 min.	4 min.	5 min.	6 min.	L. M
0	0.0	60.0	120.0	180.1	240.2	300.4	360.7	0
1	1.0	61.0	121.0	181.1	241.2	301.4	361.7	1
2	2.0	62.0	122.0	182.1	242.2	302.4	362.7	2
3	3.0	63.0	123.0	183.1	243.2	303.4	363.7	3
4	4.0	64.0	124.0	184.1	244.2	304.4	364.7	4
5	5.0	65.0	125.0	185.1	245.2	305.4	365.7	5
6	6.0	66.0	126.0	186.1	246.2	306.4	366.7	6
7	7.0	67.0	127.0	187.1	247.2	307.4	367.7	7
8	8.0	68.0	128.0	188.1	248.2	308.4	368.7	8
9	9.0	69.0	129.0	189.1	249.2	309.4	369.7	9
10	10.0	70.0	130.0	190.1	250.2	310.4	370.7	10
11	11.0	71.0	131.0	191.1	251.2	311.4	371.7	11
12	12.0	72.0	132.0	192.1	252.2	312.4	372.7	12
13	13.0	73.0	133.0	193.1	253.2	313.4	373.7	13
14	14.0	74.0	134.0	194.1	254.2	314.4	374.7	14
15	15.0	75.0	135.0	195.1	255.2	315.4	375.7	15
16	16.0	76.0	136.0	196.1	256.2	316.4	376.8	16
17	17.0	77.0	137.0	197.1	257.2	317.5	377.8	17
18	18.0	78.0	138.0	198.1	258.2	318.5	378.8	18
19	19.0	79.0	139.0	199.1	259.2	319.5	379.8	19
20	20.0	80.0	140.0	200.1	260.2	320.5	380.8	20
21	21.0	81.0	141.0	201.1	261.3	321.5	381.8	21
22	22.0	82.0	142.0	202.1	262.3	322.5	382.8	22
23	23.0	83.0	143.0	203.1	263.3	323.5	383.8	23
24	24.0	84.0	144.0	204.1	264.3	324.5	384.8	24
25	25.0	85.0	145.0	205.1	265.3	325.5	385.8	25
26	26.0	86.0	146.0	206.1	266.3	326.5	386.8	26
27	27.0	87.0	147.0	207.1	267.3	327.5	387.8	27
28	28.0	88.0	148.0	208.1	268.3	328.5	388.8	28
29	29.0	89.0	149.0	209.1	269.3	329.5	389.8	29
M	min.	min.	min.	min.	min.	min.	min.	M
L.	0	1	2	3	4	5	6	L.

L.	0	1	2	3	4	5	6	L.
M	min.	min.	min.	min.	min.	min.	min.	M
30	30 0	90.0	150.0	210.1	270.3	330.5	390.8	30
31	31.0	91.0	151.0	211.1	271.3	331.5	391.8	31
32	32.0	92.0	152.0	212.1	272.3	332.5	392 9	32
33	33.0	93 0	153.1	213.1	273.3	333 5	393 9	33
34	34.0	94.0	154.1	214.1	274 3	334.5	394.9	34
35	35 0	95.0	155.1	215 1	275.3	335.5	395.9	35
36	36.0	96.0	156.1	216.1	276.3	336.5	396.9	36
37	37.0	97.0	157.1	217.1	277.3	337.5	397.9	37
38	38.0	98.0	158.1	218.1	278.3	338.5	398.9	38
39	39.0	99.0	159.1	219.1	279.3	339.6	399.9	39
40	40.0	100.0	160.1	220.2	280.3	340.6	400.9	40
41	41.0	101.0	161.1	221.2	281.3	341.6	401.9	41
42	42.0	102.0	162.1	222.2	282.3	342 6	402.9	42
43	43.0	103.0	163.1	223.2	283.3	343.6	403.9	43
44	44.0	104.0	164 1	224.2	284.3	344.6	404.9	44
45	45.0	105.0	165.1	225.2	285.3	345.6	405.9	45
46	46.0	106 0	166.1	226.2	286.3	346.6	406.9	46
47	47.0	107.0	167.1	227.2	287 3	347.6	407.0	47
48	48.0	108.0	168 1	228.2	288.3	348.6	408.0	48
49	49.0	109.0	169.1	229.2	289.3	349.6	409.0	49
50	50.0	110.0	170.1	230.2	290.3	350.6	410.0	50
51	51.0	111.0	171.1	231.2	291.3	351.6	411.0	51
52	52.0	112.0	172.1	232 2	292.4	352.6	412.0	52
53	53.0	113.0	173.1	233.2	293.4	353.6	413.0	53
54	54.0	114.0	174.1	234.2	294.4	354.6	414.0	54
55	55.0	115.0	175.1	235 2	295.4	355.6	415 0	55
56	56.0	116 0	176.1	236.2	296 4	356.6	416.0	56
57	57 0	117.0	177.1	237.2	297.4	357.6	417.0	57
58	58.0	118.0	178.1	238.2	298 4	358.6	418.0	58
59	59.0	119.0	179.1	239.2	299.4	359.7	419.0	59
M	min	min.	min.	min.	min.	min.	min.	M
L.	0		2	3	4	5	6	L.

L.	7	8	9	10	11	12	13	L.
M	min.	min.	min.	min.	min.	min.	min.	M
0	421.0	481.6	542.2	603.1	664.1	725.3	786.8	0
1	422.1	482.6	543.3	604.1	665.1	726.3	787.8	1
2	423.1	483.6	544.3	605.1	666.1	727.4	788.8	2
3	424.1	484.6	545.3	606.1	667.1	728.4	789.9	3
4	425.1	485.6	546.3	607.1	668.2	729.4	790.9	4
5	426.1	486.6	547.3	608.1	669.2	730.4	791.9	5
6	427.1	487.6	548.3	609.2	670.2	731.5	792.9	6
7	428.1	488.6	549.3	610.2	671.2	732.5	793.0	7
8	429.1	489.6	550.3	611.2	672.2	733.5	794.0	8
9	430.1	490.7	551.3	612.2	673.3	734.5	795.0	9
10	431.1	491.7	552.4	613.2	674.3	735.5	796.0	10
11	432.1	492.7	553.4	614.2	675.3	736.6	797.0	11
12	433.1	493.7	554.4	615.3	676.3	737.6	798.1	12
13	434.1	494.7	555.4	616.3	677.3	738.6	799.1	13
14	435.2	495.7	556.4	617.3	678.4	739.6	800.1	14
15	436.2	496.7	557.4	618.3	679.4	740.7	801.2	15
16	437.2	497.7	558.4	619.3	680.4	741.7	802.2	16
17	438.2	498.7	559.5	620.3	681.4	742.7	803.2	17
18	439.2	499.8	560.5	621.4	682.4	743.7	804.2	18
19	440.2	500.8	561.5	622.4	683.5	744.8	805.3	19
20	441.2	501.8	562.5	623.4	684.5	745.8	806.3	20
21	442.2	502.8	563.5	624.4	685.5	746.8	807.3	21
22	443.2	503.8	564.5	625.4	686.5	747.8	808.3	22
23	444.2	504.8	565.5	626.4	687.5	748.9	809.4	23
24	445.2	505.8	566.5	627.5	688.6	749.9	810.4	24
25	446.2	506.8	567.6	628.5	689.6	750.9	811.4	25
26	447.3	507.8	568.6	629.5	690.6	751.9	812.5	26
27	448.3	508.8	569.6	630.5	691.6	752.9	813.5	27
28	449.3	509.9	570.6	631.5	692.6	753.9	814.5	28
29	450.3	510.9	571.6	632.5	693.7	754.0	815.5	29
M	min	min.	min.	min.	min.	min.	min.	M
L.	7	8	9	10	11	12	13	L.

L. M	7 min.	8 min.	9 min.	10 min.	11 min.	12 min.	13 min.	L. M
30	451.3	511 9	572.6	633 6	694.7	756.0	817.6	30
31	452.3	512.9	573.6	634 6	695.7	757.0	818.6	31
32	453.3	513.9	574.7	635.6	696.7	758.1	819.7	32
33	454.3	514.9	575.7	636.6	697 7	759.1	820.7	33
34	455.3	515.9	576.7	637.6	698 8	760 1	821.7	34
35	456.3	516.9	577.7	638.6	699 8	761.1	822.7	35
36	457.3	517.9	578.7	6397	700.8	762.2	823.8	36
37	458.4	518.0	579 7	640.7	701.8	763.2	824.8	37
38	459.4	519.0	580.7	641.7	702.8	764.2	825.8	38
39	460 4	520.0	581.8	642.7	703.9	765.2	826.9	39
40	461.4	521.0	582.8	643.7	704.9	766.3	827.9	40
41	462.4	523.0	583 8	644.7	7059	767.3	828.9	41
42	463.4	5240	584.8	645.8	706.9	768.3	829.9	42
43	464.4	525.0	585.8	646.8	707.0	769.3	830.0	43
44	465.4	526.0	586.8	647.8	708.0	770.4	832.0	44
45	466 4	527.1	587.8	648.8	709.0	771.4	833.0	45
46	467.4	528.1	588.9	649.8	711.0	772.4	834.1	46
47	468.4	529.1	589 9	650.9	712.0	773.4	835.1	47
48	469.5	530.1	590.9	651.9	713.1	774.5	836.1	48
49	470.5	531.1	591.9	652.9	714.1	775.5	837.2	49
50	471.5	532.1	592.9	653.9	715.1	776.5	838.2	50
51	472 5	533.1	593.9	654.9	716.1	777.5	839.2	51
52	473.5	534.1	594.9	6559	717.1	778.6	840.2	52
53	474.5	535.1	596.0	656.9	718.2	779.6	841.3	53
54	475.5	536.2	597.0	657.0	719.2	780.6	842.3	54
55	476.5	537.2	598.0	658.0	720.2	781.6	843.3	55
56	477.5	538.2	599.0	660.0	721.2	782.7	844.4	56
57	478.5	539.2	600.0	661.0	722.3	783.7	845.4	57
58	479.5	540.2	601.0	662.0	723.3	784.7	846.4	58
59	480.6	541.2	602.0	663.1	724.3	785.8	847.5	59
M	min.	min.	min.	min.	min.	min.	min.	M
L.	7	8	9	10	11	12	13	L.

L.	14	15	16	17	18	19	20	L.
M	min.	min.	min.	min.	min.	min.	min.	M
0	848 5	910.5	972.7	1035.3	1098.2	1161.5	1225.1	0
1	849.5	911.5	973.8	1036.3	1099.3	1162.5	1226.2	1
2	850.5	912.5	974.8	1037.4	1100.3	1163.6	1227.3	2
3	851.6	913.6	975 8	1038.4	1101 4	1164.7	1228.3	3
4	852.6	914 6	976.9	1039 5	1102.4	1165.7	1229.4	4
5	853.6	915.6	977.9	1040.5	1103.5	1166.8	1230.5	5
6	854.7	916.7	978.0	1041.6	1104.5	1167.8	1231.5	6
7	855.7	917.7	980.0	1042.6	1105.6	1168.9	1232.6	7
8	856 7	918.7	981.1	1043.7	1106 6	1169.0	1233.7	8
9	857 8	919.8	982.1	1044.7	1107 7	1171.0	1234.7	9
10	858.8	920.8	983.1	1045.8	1108.7	1172 1	1235.8	10
11	859.8	921.9	984.2	1046.8	1109.8	1173.1	1236.9	11
12	860.9	922.9	985.2	1047.9	1110.8	11742	1237.9	12
13	861.9	923 9	986.3	1048.9	1111.9	1175 2	1238.0	13
14	862.9	925 0	987.3	1049 0	1112.9	1176.3	12400	14
15	863.0	926.0	988.3	1051.0	1113 0	1177.4	1241.1	15
16	864.0	927.0	989.4	1052.0	1115.1	1178.4	1242.2	16
17	866.0	928.1	990.4	1053.1	1116.1	1179 5	1243.2	17
18	867.0	929.1	991.5	1054.1	1117.2	1180.5	1244.3	18
19	868.1	930.1	992.5	1055.2	1118.2	1181.6	1245.4	19
20	869.1	931.2	993.5	1056.2	1119.3	11827	1246.4	20
21	870.1	932.2	994.6	1057.3	1120.3	11837	1247.5	21
22	871 2	933.3	995.6	1058.3	1121.4	1184.8	1248.6	22
23	872.2	934.3	996.7	1059.4	1122.4	1185.8	1249.6	23
24	873.2	935 3	997.7	1060 4	1123.5	1186.9	1250.7	24
25	874 3	936 4	998.8	1061.5	1124 5	1187.0	1251.8	25
26	875.3	937.4	999.8	1062.5	1125.6	1189.0	1252.8	26
27	876.3	938.4	1000.8	1063.6	1126.6	1190.1	1253.9	27
28	877.4	939 5	1001.9	1064.6	1127.7	1191.1	1254.0	28
29	878.4	940.5	1002.9	10657	1128.8	1192.2	1256.0	29
M	min.	min.	min.	min	min.	min	min	M
L.	14	15	16	17	18	19	20	L.

L.	14	15	16	17	18	19	20	L.
M	min.	min.	min.	min.	min.	min.	min.	M
30	879 4	941.6	1004 0	1066 7	1129.8	1193.2	1257.1	30
31	880.5	942.6	1005 0	1067.8	1130.9	1194.3	1258.2	31
32	881.5	943.6	1006.1	1068.8	1131.9	11954	1259.3	32
33	882.5	9447	1007.1	10699	1132.0	1196.4	1260.3	33
34	883 6	945 7	1008.1	1070.9	1134.0	1197.5	1261.4	34
35	884.6	946.7	1009.2	1071.0	1135.1	1198.6	1262 5	35
36	885.6	947.8	1010.2	1073.0	1136.1	11996	1263.5	36
37	886.7	948 8	1011.3	1074.1	1137.2	1200.7	1264.6	37
38	887.7	949.9	1012.3	1075 1	1138.2	1201.8	1265.7	38
39	888.7	950 9	1013.4	1076.2	1139.3	1202 9	1266.7	39
40	889.8	951.9	1014 4	1077.2	1140.4	1203.9	1267.8	40
41	890.8	952.0	1015.5	1078.3	1141.4	1204.9	1268 9	41
42	891.8	954.0	1016.5	1079.3	1142.5	1206.0	1269.9	42
43	892.9	955.1	1017 5	1080.4	1143.5	1207.1	1271.0	43
44	893.9	956.1	1018.6	1081.4	1144.6	1208.1	1272.1	44
45	894.9	957.1	1019.6	1082.5	1145.6	1209.2	1273.1	45
46	896.0	958.2	1020.7	1083.5	1146.7	1210.3	1274.2	46
47	897 0	959.2	1021 7	1084 6	1147.7	1211.3	12753	47
48	898 0	960.2	1022 8	1085.6	1148.8	1212.4	1276.4	48
49	899.1	961.3	1023.8	1086.7	11499	12134	1277.4	49
50	900.1	962.3	1024.9	1087.7	1150.9	12145	1278.5	50
51	901.1	963.4	10259	1088 8	1151.0	12156	1279.6	51
52	902.2	964.4	1026.9	1089.8	1152.0	1216.6	1280.6	52
53	903.2	965.4	1027.0	1090 9	1153.1	1217.7	1281.7	53
54	904.3	966.5	1029.0	1091.9	1155 1	12188	1282 8	54
55	905.3	967.5	1030.1	1092.0	1156.2	1219.8	1283 8	55
56	906 3	968.6	1031.1	1094.0	1157.3	1220.9	1284.9	56
57	907.4	969.6	1032.2	1095.0	1158.3	1221.9	1285.0	57
58	908.4	970.6	10332	1096.1	1159.4	1223.0	1287.1	58
59	909.4	971.7	1034.3	1097.2	1160.4	1224.1	1288 1	59
M	min.	min.	min.	min.	min.	min.	min.	M
L	14	15	16	17	18	19	20	L

L.	21	22	23	24	25	26	27	L.
M	min.	min.	min.	min.	min.	min.	min.	M
0	1289.2	1353.7	1418.6	1484.1	1550.0	1616.5	1683.5	0
1	1290.3	1354.8	1419.7	1485.2	1551.1	1617.6	1684.6	1
2	1291.3	1355.8	1420.8	1486.2	1552.2	1618.7	1685.8	2
3	1292.4	1356.9	1421.9	1487.3	1553.3	1619.8	1686.9	3
4	1293.5	1358.0	1422.0	1488.4	1554.4	1620.9	1688.0	4
5	1294.6	1359.1	1424.1	1489.5	1555.5	1622.0	1689.1	5
6	1295.6	1360.2	1425.2	1490.6	1556.6	1623.2	1690.3	6
7	1296.7	1361.2	1426.2	1491.7	1557.7	1624.3	1691.4	7
8	1297.8	1362.3	1427.3	1492.8	1558.8	1625.4	1692.5	8
9	1298.8	1363.4	1428.4	1493.9	1559.9	1626.5	1693.6	9
10	1299.9	1364.5	1429.5	1495.0	1561.0	1627.6	1694.7	10
11	1300.0	1365.6	1430.6	1496.1	1562.1	1628.7	1695.9	11
12	1302.1	1366.6	1431.7	1497.2	1563.2	1629.8	1697.0	12
13	1303.1	1367.7	1432.8	1498.3	1564.4	1630.9	1698.1	13
14	1304.2	1368.8	1433.9	1499.4	1565.5	1632.1	1699.2	14
15	1305.3	1369.9	1434.9	1500.5	1566.6	1633.2	1700.4	15
16	1306.4	1370.0	1436.0	1501.6	1567.7	1634.3	1701.5	16
17	1307.4	1372.0	1437.1	1502.7	1568.8	1635.4	1702.6	17
18	1308.5	1373.1	1438.2	1503.8	1569.9	1636.5	1703.7	18
19	1309.6	1374.2	1439.3	1504.9	1571.0	1637.6	1704.9	19
20	1310.6	1375.3	1440.4	1505.0	1572.1	1638.8	1706.0	20
21	1311.7	1376.4	1441.5	1507.1	1573.2	1639.9	1707.1	21
22	1312.8	1377.4	1442.6	1508.2	1574.3	1641.0	1708.2	22
23	1313.9	1378.5	1443.7	1509.3	1575.4	1642.1	1709.4	23
24	1314.9	1379.6	1444.7	1510.4	1576.5	1643.2	1710.5	24
25	1316.0	1380.7	1445.8	1511.5	1577.6	1644.3	1711.6	25
26	1317.1	1381.8	1446.9	1512.6	1578.7	1645.5	1712.8	26
27	1318.2	1382.9	1448.0	1513.7	1579.8	1646.6	1713.9	27
28	1319.2	1383.9	1449.1	1514.8	1580.9	1647.7	1715.0	28
29	1320.3	1385.0	1450.2	1515.9	1582.1	1648.8	1716.1	29
M	min.	min.	min.	min.	min.	min.	min.	M
L.	21	22	23	24	25	26	27	L.

L M	21 min.	22 min.	23 min.	24 min.	25 min.	26 min.	27 min.	L M
30	1321.4	1386.1	1451.3	1517.0	1583 2	1649.9	1717.3	30
31	1322 5	1387.2	1452.4	1518.1	1584.3	1651.0	1718.4	31
32	1323.5	1388.3	1453.5	1519.2	1585 4	1652 2	1719.5	32
33	1324.6	1389.3	1454 6	1520.3	1586 5	1653.3	1720.6	33
34	1325.7	1390.4	1455 6	1521.4	1587.6	1654 4	1721.8	34
35	1326 8	1391.5	1456.7	1522.5	1588.7	1655.5	1722.9	35
36	1327.8	1392.6	1457.8	1523.6	1589 8	1656.6	1724.0	36
37	1328.9	1393 7	1458.9	1524.7	1590.9	1657.7	1725.2	37
38	1329.0	1394 8	1460.0	1525.8	1592 0	1658.9	1726.3	38
39	1331.1	1395.8	1461.1	1526.9	1593.1	1660 0	1727.4	39
40	1332.1	1396.9	1462.2	1528 0	1594.3	1661.1	1728.5	40
41	1333.2	1398 0	1463.3	1529.1	1595.4	1662.2	1729.7	41
42	1334.3	1399.1	1464.4	1530.2	1596.5	1663.3	1730.8	42
43	1335.4	1400.2	1465.5	1531.3	1597.6	1664.5	1731.9	43
44	1336.4	1401.3	1466.6	1532.4	1598.7	1665.6	1733.1	44
45	1337.5	1402.3	1467.7	1533.5	1599.8	1666.7	1734.2	45
46	1338 6	1403.4	1468.7	1534.6	1600.9	1667.8	1735.3	46
47	1339.7	1404 5	1469.8	1535.7	1602.0	1668.9	1736.5	47
48	1340 8	1405.6	1470.9	1536.8	1603.1	1670.1	1737.6	48
49	1341.8	1406 7	1472.0	1537.9	1604.2	1671.2	1738.7	49
50	1342.9	1407.8	1473.1	1539.0	1605.4	1672.3	1739.8	50
51	1343.0	1408.9	1474 2	1540.1	1606 5	1673 4	1741.0	51
52	1345.1	1409.9	1475.3	1541.2	1607.6	1674.5	1742.1	52
53	1346.1	1411.0	1476.4	1542 3	1608.7	1675.7	1743.2	53
54	1347.2	1412.1	1477.5	1543.4	1609 8	1676.8	1744.4	54
55	1348.3	1413.2	1478 6	1544.5	1610.9	1677.9	1745.5	55
56	1349.4	1414 3	1479.7	1545.6	1612.0	1679.0	1746.6	56
57	1350.4	1415 4	1480 8	1546 7	1613.1	1680 2	1747.8	57
58	1351 5	1416.5	1481.9	1547.8	1614 2	1681.3	1748.9	58
59	1352.6	1417.5	1482.0	1548.9	1615 4	1682.4	1750.0	59
M L.	min. 21	min. 22	min. 23	min. 24	min. 25	min 26	min. 27	M L.

L.	28	29	30	31	32	33	34	L.
M	min.	min.	min.	min.	min.	min.	min.	M
0	1751.2	1819.4	1888.4	1958 0	2028.4	2099.5	2171.5	0
1	1752 3	1820.6	1889.5	1959.2	2029.6	2100.7	2172.7	1
2	1753.4	1821.7	1890.7	1960.3	2030.7	2101.9	2173.9	2
3	1754.6	1822.9	1891.8	1961 5	2031.9	2103 1	2175.1	3
4	1755.7	1824 0	1893.0	1962.7	2033.1	2104.3	2176.3	4
5	1756.8	1825.2	1894.2	1963 8	2034.3	2105.5	2177.5	5
6	1758.0	1826.3	1895.4	1965.0	2035.5	2106.7	2178.7	6
7	1759.1	1827.4	1896.5	1966.2	2036.6	2107.9	2179.9	7
8	1760.2	1828.6	1897.6	1967.4	2037.8	2108.1	2181.1	8
9	1761.4	1829.7	1898.8	1968.5	2039.0	2110.3	2182.3	9
10	1762.5	1830.9	1899 9	1969.7	2040.2	2111.5	2183.6	10
11	1763.6	1832.0	1901.1	1970.9	2041.4	2112.7	2184 8	11
12	1764.8	1833.2	1902.2	1972.0	2042.5	2113.9	2186.0	12
13	1765.9	1834.3	1903.4	1973 2	2043.7	2115.0	2187.2	13
14	1767.0	1835.5	1904.6	1974.4	2044 9	2116.2	2188.4	14
15	1768.2	1836.6	1905.7	1975.5	2046.1	2117.4	2189.6	15
16	1769.3	1837.8	1906.9	1976.7	2047.3	2118.6	2190.8	16
17	1770.4	1838.9	1908.0	1977.9	2048.5	2119.8	2192.0	17
18	1771.6	1840.0	1909 2	1979.0	2049.6	2121.0	2193.2	18
19	1772.7	1841.2	1910.3	1980.2	2050.8	2122.2	2194.4	19
20	1773.8	1842.3	1911.5	1981.4	2052.0	2123.4	2195.7	20
21	1775.0	1843.5	1912.7	1982.6	2053.2	2124.6	2196.9	21
22	1776.1	1844.6	1913 8	1983.7	2054.4	2125.8	2198.1	22
23	1777.3	1845.8	1915.0	1984.9	2055.6	2127.0	2199.3	23
24	1778 4	1846.9	1916.1	1986.0	2056.7	2128 2	2200.5	24
25	1779.5	1848.1	1917.3	1987.2	2057.9	2129.4	2201.7	25
26	1780.7	1849.2	1918.5	1988.4	2059.1	2130.6	2202.9	26
27	1781.8	1850.4	1919.6	1989.6	2060.3	2131.8	2204.1	27
28	1782.9	1851.5	1920.8	1990.8	2061.5	2133.0	2205.3	28
29	1784.1	1852.7	1920.9	1991.9	2062.7	2134.2	2206.6	29
M	min.	min.	min.	min.	min.	min.	min.	M
L	28	29	30	31	32	33	34	L.

L.	28	29	30	31	32	33	34	L.
M	min.	min.	min.	min.	min.	min.	min.	M
30	1785.2	1853 8	1923 1	1993.1	2063.9	2135.4	2207.8	30
31	1786.4	1855.0	1924.3	1994.3	2065.0	2136.6	2209.0	31
32	1787.5	1856.1	19254	1995.5	2066.2	2137.7	2210.2	32
33	1788.6	1857 3	19266	1996.6	2067.4	2139.0	2211.4	33
34	1789.8	1858.4	1927.7	1997.8	2068.6	2140.2	2212.6	34
35	1790.9	18596	1928.9	1999.0	2069.8	2141.4	2213.8	35
36	1792.0	18607	1930.1	2000.1	2071.0	2142.6	2215.1	36
37	1793 2	1861.9	1931.2	2001.3	2072.2	2143.8	2216.3	37
38	1794.3	1863.0	1932.4	2002.5	2073.3	2145.0	2217.5	38
39	1795.5	18642	1933.6	2003.7	2074.5	2146.2	2218.7	39
40	1796.6	1865.3	1934.7	2004.8	2075.7	2147.4	2219.9	40
41	1797.7	1866.5	1935.9	2006.0	2076.9	2148.6	2221.1	41
42	1798.9	1867.6	1937.1	2007.2	2078.1	2149.8	2222.4	42
43	1800.0	1868 8	1938.2	2008.4	2079.3	2151.0	2223.6	43
44	1801.2	1869.9	1939.4	2009.5	2080.5	2152.2	2224.8	44
45	1802 3	1871.1	1940.8	2010.7	2081.7	2153.4	2226.0	45
46	1803.4	1872.2	1941.7	2011.9	2082.9	2154.6	2227.2	46
47	1804.6	1873.4	1942.9	2013.1	2084.0	2155.8	2228.4	47
48	1805.7	1874.5	1944.0	2014.2	2085.2	2157.0	2229.7	48
49	1806.9	1875.7	1945.2	2015.4	2086.4	2158.2	2230.9	49
50	1808.0	1876.8	1946.4	2016.6	2087.6	2159.4	2232.1	50
51	1809.2	1878.0	1947.5	2017.8	2088.8	2160.6	2233.3	51
52	1810.3	1879.1	1948.7	2019.0	2090.0	2161.8	2234.5	52
53	1811.4	1880.3	1949.9	2020.1	2091.2	2163.0	2235.7	53
54	1812.6	1881.5	1951.0	2021.3	2092.4	2164.2	2237.0	54
55	18137	1882.6	1952.2	2022.5	2093.6	2165.5	2238.2	55
56	1814.9	1883.8	1953.3	2023.7	2094.8	2166.7	2239.4	56
57	1816.0	1884.9	1954.5	2024.8	2096.0	2167.9	2240.6	57
58	1817.2	1886.1	1955.7	2026.0	2097.1	2169.1	2241.8	58
59	1818.3	1887.2	1956.8	2027.2	2098.3	2170.3	2243.1	59
M	min.	min.	min.	min.	min.	min.	min.	M
L.	28	29	30	31	32	33	34	L.

L.	35	36	37	38	39	40	41	L.
M	min.	min.	min.	min.	min.	min.	min.	M
0	2244.3	2318.0	2392.6	2468.3	2544.9	2622.7	2701.6	0
1	2245.5	2319.2	2393.9	2469.5	2546.2	2624.0	2702.9	1
2	2246.7	2320.5	2395.1	2470.8	2547.5	2625.3	2704.2	2
3	2248.0	2321.7	2396.4	2472.1	2548.8	2626.6	2705.6	3
4	2249.2	2322.9	2397.6	2473.3	2550.1	2627.9	2706.9	4
5	2250.4	2324.2	2398.9	2474.6	2551.4	2629.2	2708.2	5
6	2251.6	2325.4	2400.1	2475.9	2552.7	2630.5	2709.6	6
7	2252.8	2326.6	2401.4	2477.2	2553.9	2631.8	2710.9	7
8	2254.1	2327.9	2402.7	2478.4	2555.2	2633.1	2711.2	8
9	2255.3	2329.1	2403.9	2479.7	2556.5	2634.5	2713.5	9
10	2256.5	2330.4	2405.2	2481.0	2557.8	2635.8	2714.9	10
11	2257.7	2331.6	2406.4	2482.2	2559.1	2637.1	2716.2	11
12	2259.0	2332.8	2407.7	2483.5	2560.4	2638.4	2717.5	12
13	2260.2	2334.1	2408.9	2484.8	2561.7	2639.7	2718.9	13
14	2261.4	2335.3	2410.2	2486.1	2563.0	2641.0	2720.2	14
15	2262.6	2336.6	2411.4	2487.3	2564.3	2642.3	2721.5	15
16	2263.9	2337.8	2412.7	2488.6	2565.6	2643.6	2722.8	16
17	2265.1	2339.0	2414.0	2489.9	2566.8	2644.9	2724.2	17
18	2266.3	2340.3	2415.2	2491.2	2568.1	2646.2	2725.5	18
19	2267.5	2341.5	2416.5	2492.4	2569.4	2647.6	2726.8	19
20	2268.8	2342.8	2417.7	2493.7	2570.7	2648.9	2728.2	20
21	2270.0	2344.0	2419.0	2495.0	2572.0	2650.2	2729.5	21
22	2271.2	2345.2	2420.2	2496.2	2573.3	2651.5	2730.8	22
23	2272.4	2346.5	2421.5	2497.5	2574.6	2652.8	2732.2	23
24	2273.7	2347.7	2422.8	2498.8	2575.9	2654.1	2733.5	24
25	2274.9	2349.0	2424.0	2500.1	2577.2	2655.4	2734.8	25
26	2276.1	2350.2	2425.3	2501.4	2578.5	2656.7	2736.2	26
27	2277.3	2351.5	2426.5	2502.6	2579.8	2658.1	2737.5	27
28	2278.6	2352.7	2427.8	2503.9	2581.1	2659.4	2738.8	28
29	2279.8	2353.9	2429.1	2505.2	2582.4	2660.7	2740.2	29
M	min.	min.	min.	min.	min.	min.	min.	M
L	35	36	37	38	39	40	41	L

L. M	35 min.	36 min.	37 min.	38 min.	39 min.	40 min.	41 min.	L. M
30	2281.0	2355.2	2430.3	2506.5	2583.7	2662.0	2741.5	30
31	2282.3	2356.4	2431.6	2507.7	2585.0	2663.3	2742.8	31
32	2283.5	2357.7	2432 8	2509.0	2586.3	2664.6	2744.2	32
33	22847	2358.9	2434.1	2510.3	2587.6	2665.9	27455	33
34	2285.9	2360.2	24354	2511.6	2588.9	26673	2746.8	34
35	2287.2	2361.4	2436.6	2512.9	2590.2	2668.6	2748.2	35
36	2288.4	2362.7	2437.9	25141	2591.5	26699	2749.5	36
37	22896	23639	2439.1	25154	25927	2671.2	2750.9	37
38	2290.9	2365.1	24404	2516.7	2594.0	2672.5	27522	38
39	2292.1	23664	2441.7	2518.0	2595.3	26738	2753.5	39
40	2293.3	2367.6	2442.9	2519.3	2596.6	2675.2	2754.9	40
41	2294.5	2368.9	24442	2520.5	2597.9	2676.5	2756.2	41
42	2295.8	2370.1	24455	2521.8	25992	2677.8	2757.5	42
43	2297.0	2371.4	2446.7	2523.1	2600.5	2679.1	2758.9	43
44	2298.2	2372.6	2448.0	2524.4	2601.8	26804	2760.2	44
45	2299.5	2373.9	2449.3	25257	2603.1	2681.8	2761.7	45
46	2300.7	2375.1	2450.5	2526.9	26044	2683.1	2762.9	46
47	2301.9	2376.4	2451.8	25282	2605.7	2684.4	2764.2	47
48	2303.2	2377.6	2453.1	2529.5	2607.0	2685.7	2765.6	48
49	2304.4	2378.9	2454.3	25308	2608.3	2687.0	2766.9	49
50	2305.6	2380.1	2455.6	2532.1	2609.7	26884	2768.3	50
51	2306.9	2381.4	2456.9	2533.4	2611.0	2689.7	2769.6	51
52	2308.1	2382.6	2458.1	2534.6	2612.3	2691.0	2771.0	52
53	2309.3	2383.9	2459.4	2535.9	2613.6	2692.3	2772.3	53
54	2310.6	2385.1	2460.7	2537.2	26149	2693.7	2773.6	54
55	2311.8	23864	2461.9	2538.5	2616.2	2695.0	27750	55
56	2313.0	23876	2463.2	2539.8	2617.5	2696.3	2776.3	56
57	2314.3	2388.9	2464.5	2541.1	2618.8	2697.6	27777	57
58	2315.5	2390.1	2465.7	25424	2620.1	2698.9	27790	58
59	2316.8	2391.4	2467.0	2543.6	2621.4	2700.3	2780.4	59
M	min.	min.	min.	min.	min.	min.	min.	M
L.	35	36	37	38	39	40	41	L.

L M	42 min.	43 min.	44 min.	45 min.	46 min.	47 min.	48 min.	L M
0	2781.7	2863 1	2945.8	3029.9	31155	3202.7	3291.5	0
1	2783.1	2864.5	2947.2	3031.4	3117.0	3204.2	3293.0	1
2	2784 4	2865.8	2948.6	3032.8	3118 4	3205.6	3294.5	2
3	2785.8	2867.2	2950.0	3034.2	31199	3207.1	3296.0	3
4	2787.1	2868 6	2951 4	3035 6	3121.3	3208 6	3297.5	4
5	2788.4	2869.9	2952.8	3037.0	3122.7	3210.1	3299 0	5
6	2789.8	2871.3	2954.2	3038.4	3124.2	3211.5	3300.5	6
7	2791.1	2872.7	2955.6	3039.8	31256	3213.0	3302.0	7
8	2792.5	2874 0	2956.9	3041.3	3127.1	3214.5	3303.5	8
9	2793.8	2875.4	2958.3	3042.7	3128.5	32159	3305.0	9
10	2795.2	2876.8	2959.7	3044.1	3130.0	3217 4	3306.5	10
11	2796.5	2878.2	2961.1	3045.5	3131.4	3218.9	3308.0	11
12	2797.9	2879.5	2962.5	3046.9	3132.9	3220.3	3309.5	12
13	2799.2	2880.9	2963.9	3048.4	3134.3	3221.8	3311.0	13
14	2800.6	2882.3	2965.3	3049.8	31357	3223.3	3312.5	14
15	2801.9	2883.6	2966.7	3051 3	3137 2	3224.8	3314.0	15
16	2803 3	2885.0	2968.1	3052.6	31386	3226.2	3315.5	16
17	28046	2886 4	2969.5	3054.0	3140 1	3227.7	3317.0	17
18	2806.0	2887.8	2970.9	30555	3141.5	3229.2	3318.5	18
19	2807.3	2889.1	2972.3	3056.9	3143.0	3230.7	3320.0	19
20	2808.7	2890.5	2973.7	3058.3	3144.4	3232 1	3321.5	20
21	2810.0	2891.9	2975.1	3059.7	3145.9	3233.6	3323.0	21
22	2811.4	28933	2976.5	3061.2	3147.3	3235.1	3324.5	22
23	2812.6	28946	2977.9	3062.6	3148.8	3236.6	3326.0	23
24	2814 1	2896.0	2979.3	3064.0	3150.2	3238.0	3327.5	24
25	2815.5	2897.4	2980.7	3065.4	3151.7	3239.5	3329.0	25
26	2816.8	2898.8	2982.1	3066.8	3153.1	3241.0	3330.5	26
27	2818.2	2900.1	2983.5	3068.3	3154.6	3242.5	3332.1	27
28	2819.5	2901.5	29849	3069.7	3156.0	3244.0	3333.6	28
29	28209	2902.9	2986.3	3071.1	3157.5	3245.4	3335.1	29
M L	min. 42	min. 43	min. 44	min. 45	min. 46	min. 47	min. 48	M L

L. M	42 min.	43 min.	44 min.	45 min.	46 min.	47 min.	48 min	L. M
30	2822.2	2904.3	2987.7	3072.6	3158.9	3246.9	3336.6	30
31	2823.6	29057	2989.1	3074.0	3160.4	3248.4	3338.1	31
32	2825.0	2907.0	2990.5	30754	3161.8	3249.9	33396	32
33	2826.3	2908.4	2991.9	3076.8	3163.3	3251.4	3341.1	33
34	2827.7	2909.8	2993.3	3078.3	31647	3252.8	3342.6	34
35	2829.0	2911.2	2994.7	3079.7	3166.2	32543	3344.1	35
36	2830.4	2912.6	2996.1	3081.1	3167.7	3255.8	3345.6	36
37	2831.7	2913.9	2997.5	3082.5	3169.1	3257.3	3347.2	37
38	2833.1	2915.3	2998.9	3084.0	3170.6	3258.8	3348.7	38
39	2834.5	2916.7	3000.3	30854	3172.0	3260.3	33502	39
40	2835.8	2918.1	3001.7	3086.8	3173.5	3261.7	3351.7	40
41	2837.2	2919.5	30031	3088.3	3174.9	3263.2	33532	41
42	2838.5	2920.9	30045	3089.7	3176.4	3264.7	3354.7	42
43	2839.9	2922.2	3006.0	3091.1	31779	3266.2	3356.2	43
44	2841.3	2923.6	3007.4	3092.6	3179.3	3267.7	3357.8	44
45	2842.6	2925.0	3008.8	3094.0	3180.8	3269.2	3359.3	45
46	2844.0	2926.4	3010.2	3095.4	3182.2	3270.7	3360.8	46
47	2845.4	2927.8	3011.6	3096.9	3183.6	3272.1	3362.3	47
48	2846.7	2929.2	3013.0	3098.3	3185.2	3273.6	3363.8	48
49	2848 1	2930.5	3014.4	3099.7	31866	3275.1	3365.3	49
50	2849.4	2931.9	3015.8	3101.2	3188.1	3276.6	3366.9	50
51	2850.8	2933.3	3017.2	31026	3189.5	3278.1	33684	51
52	2852.2	2934.7	3018.6	3104.0	3191.0	3279.6	3369.9	52
53	2853.5	2936.1	3020.0	3105.5	3192.5	3281.1	3371.4	53
54	2854.9	2937.5	3021.5	3106.9	3193.9	32826	3372.9	54
55	2856.3	2938.9	3022.9	3108.4	3195.4	3284.1	3374.5	55
56	2857.6	2940.3	3024.3	3109.8	3196.9	3285.6	3376.0	56
57	2859.0	2941.6	30257	3111.2	3198.3	3287.0	3377.5	57
58	2860.4	2943.0	3027.1	3112.7	3199.8	3288.5	3379.0	58
59	2861.7	29444	3028.5	3114.1	3201.2	3290.0	3380.6	59
M	min	min.	min.	min.	min.	min.	min.	M
L.	42	43	44	45	46	47	48	L.

L.	49	50	51	52	53	54	55	L.
M	min.	min.	min.	min.	min.	min.	min.	M
0	3382.1	3474.5	3568.8	3665.2	3763.8	3864 6	3968.0	0
1	3383.6	3476.0	3570.4	3666 8	3765.4	3866.3	3969.7	1
2	3385.1	3477.6	3571.0	3668.4	3767.1	3868.0	3971.5	2
3	3386.7	3479.1	3573.6	3670.1	3768.7	3869.7	3973 2	3
4	3388.2	3480.7	3575.2	3671.7	3770 4	3871.4	3974 9	4
5	3389.7	3482.3	3576.8	3673.3	3772.1	3873.2	3976.7	5
6	3391 2	3483.8	3578 4	3675.0	3773.7	3874.9	3978.4	6
7	3392.8	3485.4	3579.9	3676.6	3775.4	3876.6	3980.2	7
8	3394.3	3486.9	3581.5	3678.2	3777.1	3878.3	3981.9	8
9	3395.8	3488.5	3583.1	3679.8	3778.7	3880.0	3983.7	9
10	3397.4	3490.1	35847	3681.5	3780.4	3881.7	3985.4	10
11	3398.9	3491.6	3586.3	3683.1	3782.1	3883.4	3987.2	11
12	3400.4	3493.2	3587.9	36847	3783.7	3885.1	3988.9	12
13	3401.9	3494.7	3589.5	3686.4	37854	3886.8	3990.7	13
14	3403.5	3496.3	3591.1	3688.0	3787.1	3888.5	3992.4	14
15	3405 0	3497.9	3592.7	3689.6	3788.8	3890.2	3994.2	15
16	3406.5	3499.4	3594.3	3691.3	3790.4	3891.9	3996.0	16
17	3408.1	3501.0	3595.9	3692 9	3792.1	38937	3997.7	17
18	3409.6	3502.6	3597.5	3694.5	3793.8	3895.4	3999.5	18
19	3411.1	3504.1	3599.1	3696 2	3795.4	3897.1	4001.2	19
20	3412.7	3505.7	3600.7	3697.8	3797.1	3898.8	4003.0	20
21	3414.2	3507.3	3602.3	3699.4	3798 8	3900.5	4004.7	21
22	3415.7	3508.8	3603.9	3701.1	3800 5	3902.2	4006.5	22
23	3417.3	3510.4	3605.5	3702.7	3802.1	3903.9	4008.3	23
24	3418.8	3512.0	3607.1	3704.4	3803.8	3905.7	4010 0	24
25	3420.3	3513.5	3608.7	3706.0	3805.5	3907.4	4011.8	25
26	3421.9	3515.1	3610.3	3707.6	3807.2	39091	4013.5	26
27	3423.4	3516.7	3611.9	3709.3	3808.9	3910.8	4015.3	27
28	3425.0	3518.2	3613.5	3710.9	3810.5	3912.5	4017.1	28
29	3426.5	35198	3615.1	3712.6	3812 2	3914.3	4018.8	29
M	min.	min	min.	min	min.	min.	min.	M
L.	49	50	51	52	53	54	55	L.

L	49	50	51	52	53	54	55	L
M	min.	min.	min	min.	min.	min.	min.	M
30	3428.0	3521.4	3616.7	3714.2	3813.9	3916.0	4020.6	30
31	3429.6	3523.0	3618.3	3715.8	3815.6	3917.7	4022.4	31
32	3431.1	3524.5	3619.9	3717.5	3817.3	3919.4	4024.1	32
33	3432.7	3526.1	3621.6	3719.1	3818.9	3921.2	4025.9	33
34	3434.2	3527.7	3623.2	3720.8	3820.6	3922.9	4027.7	34
35	3435.7	3529.3	3624.8	3722.4	3822.3	3924.6	4029.4	35
36	3437.3	3530.8	3626.4	3724.1	3824.0	3926.3	4031.2	36
37	3438.8	3532.4	3628.0	3726.7	3825.7	3928.1	4033.0	37
38	3440.4	3534.0	3629.6	3727.4	3827.4	3929.8	4034.7	38
39	3441.9	3535.6	3631.2	3729.0	3829.1	3931.5	4036.5	39
40	3443.5	3537.1	3632.8	3730.7	3830.7	3933.2	4038.3	40
41	3445.0	3538.7	3634.4	3732.3	3832.4	3934.0	4040.1	41
42	3446.6	3540.3	3636.1	3734.0	3834.1	3936.7	4041.8	42
43	3448.1	3541.9	3637.7	3735.6	3835.8	3938.4	4043.6	43
44	3449.7	3543.5	3639.3	3737.3	3837.5	3940.2	4045.4	44
45	3451.2	3545.0	3640.9	3738.9	3839.2	3941.9	4047.2	45
46	3452.7	3546.6	3642.5	3740.6	3840.9	3943.6	4048.9	46
47	3454.3	3548.2	3644.1	3742.2	3842.6	3945.4	4050.7	47
48	3455.8	3549.8	3645.7	3743.9	3844.3	3947.1	4052.5	48
49	3457.4	3551.4	3647.4	4745.5	3846.0	3948.8	4054.3	49
50	3458.9	3552.9	3649.0	3747.2	3847.7	3950.6	4055.1	50
51	3460.5	3554.5	3650.6	3748.8	3849.4	3952.3	4057.8	51
52	3462.0	3556.1	3652.2	3750.5	3851.0	3953.0	4059.6	52
53	3463.6	3557.7	3653.8	3752.1	3852.7	3955.8	4061.4	53
54	3465.1	3559.3	3655.5	3753.8	3854.4	3957.5	4063.2	54
55	3466.7	3560.5	3657.1	3755.5	3856.1	3959.3	4065.0	55
56	3468.3	3562.0	3658.7	3757.1	3857.8	3961.0	4066.8	56
57	3469.8	3564.6	3660.3	3758.8	3859.5	3962.7	4068.5	57
58	3471.4	3565.2	3661.9	3760.4	3861.2	3964.5	4070.3	58
59	3472.9	3567.8	3663.6	3762.1	3862.9	3966.2	4072.1	59
M	min.	min.	min.	min.	min.	min.	min.	M
L	49	50	51	52	53	54	55	L

L. M	56 min.	57 min.	58 min.	59 min.	60 min.	61 min.	62 min.	L. M
0	4073.9	4182.6	4294.3	4409.1	4527.4	4649.2	4775.0	0
1	4075.7	4184.5	4296.2	4411.1	4529.4	4651.3	4777.1	1
2	4077.5	4186.3	4298.1	4413.0	4531.4	4653.4	4779.2	2
3	4079.3	4188.1	4300.0	4415.0	4533.4	4655.4	4781.4	3
4	4081.1	4190.0	4301.9	4416.9	4535.4	4657.5	4783.5	4
5	4082.9	4191.8	4303.7	4418.9	4537.4	4659.6	4785.6	5
6	4084.6	4193.7	4305.6	4420.8	4539.4	4661.6	4787.8	6
7	4086.4	4195.5	4307.5	4422.8	4541.4	4663.7	4789.9	7
8	4088.2	4197.3	4309.4	4424.7	4543.4	4665.8	4792.1	8
9	4090.0	4199.2	4311.3	4426.7	4545.4	4667.8	4794.2	9
10	4091.8	4201.0	4313.2	4428.6	4547.4	4669.9	4796.3	10
11	4093.6	4202.9	4315.1	4430.6	4549.4	4672.0	4798.5	11
12	4095.4	4204.7	4317.0	4432.5	4551.4	4674.1	4800.6	12
13	4097.2	4206.6	4318.9	4434.5	4553.5	4676.1	4802.8	13
14	4099.0	4208.4	4320.8	4436.4	4555.5	4678.2	4804.9	14
15	4100.8	4210.3	4322.7	4438.4	4557.5	4680.3	4807.1	15
16	4102.6	4212.1	4324.6	4440.3	4559.5	4682.4	4809.2	16
17	4104.4	4214.0	4326.5	4442.3	4561.5	4684.4	4811.4	17
18	4106.2	4215.8	4328.4	4444.2	4563.5	4686.5	4813.5	18
19	4108.2	4217.7	4330.3	4446.2	4565.6	4688.6	4815.7	19
20	4109.8	4219.5	4332.2	4448.2	4567.6	4690.7	4817.8	20
21	4111.6	4221.4	4334.1	4450.1	4569.6	4692.8	4820.0	21
22	4113.4	4223.2	4336.0	4452.1	4571.6	4694.9	4822.1	22
23	4115.2	4225.1	4337.9	4454.0	4573.6	4697.0	4824.3	23
24	4117.0	4226.9	4339.8	4456.0	4575.7	4699.0	4826.4	24
25	4118.9	4228.8	4341.8	4458.0	4577.7	4701.1	4828.6	25
26	4120.7	4230.6	4343.7	4459.9	4579.7	4703.2	4830.8	26
27	4122.5	4232.5	4345.6	4461.9	4581.7	4705.3	4832.9	27
28	4124.3	4234.4	4347.5	4463.9	4583.8	4707.4	4835.1	28
29	4126.1	4236.2	4349.4	4465.8	4585.8	4709.5	4837.2	29
M	min.	min.	min.	min.	min.	min.	min.	M
L.	56	57	58	59	60	61	62	L.

L.	56	57	58	59	60	61	62	L.
M	min.	min.	min.	min.	min.	min.	min.	M
30	4127.9	4238.1	4351.3	4467 8	4587.8	4711.6	48394	30
31	41297	4239.9	4353.2	4469.8	4589.9	4713.7	4841.6	31
32	4131.5	4241.8	4355.1	4471.8	4591.9	47158	4843.7	32
33	4133.3	4243.7	4357.1	4473.7	4593.9	4718.9	48459	33
34	4135.2	4245.5	4359.0	4475.7	4596.0	4720.0	4848.1	34
35	4137.0	4247.4	4360.9	4477.7	4598.0	4722.1	4850.3	35
36	4138.8	4249.3	4362.8	4479.7	4600 0	4724.2	4852.4	36
37	4140.6	4251.1	4364.7	4481.6	4602.1	4726.3	4854.6	37
38	4142.4	4253.0	4366.7	4483.6	4604.1	47284	4856.8	38
39	4144.2	4254.9	4368.6	4485.6	4606.1	4730.5	4859.0	39
40	4146.1	4256.7	4370.5	4487.6	46082	4732.6	4861.1	40
41	4147.9	4258.6	4372.4	4489.5	4610.2	4734.7	4863.3	41
42	4149.7	4260.5	4374.3	4491.5	4612.3	4736.8	4865.5	42
43	4151.5	4262.3	4376.3	4493.5	4614.3	4738.9	4867.7	43
44	4153.3	4264.2	4378.2	4495 5	4616.4	4741.0	4869.9	44
45	4155.2	4266.1	4380.1	4497.5	4618.4	4743.2	4872.0	45
46	4157.0	4268.0	4382.0	4499.5	4620.5	4745.3	4874.2	46
47	4158.8	42698	4384.0	4501.5	4622.5	4747.4	4876.4	47
48	4160.6	4271.7	43859	4503.4	4624.6	4749.5	4878.6	48
49	4162.5	4273.6	4387.8	4505.4	4626.6	4751.6	4880.8	49
50	4164.3	4275.5	4389.8	4507.4	4628.7	4753.7	4883.0	50
51	4166.1	4277.3	4391.7	4509.4	46307	4755.9	4885.2	51
52	4168.0	4279.2	43936	4511.4	4632.8	4758.0	4887.4	52
53	4169.8	4281.1	4395.6	4513.4	4634.8	4760.1	48895	53
54	4171.6	4283.0	4397.5	45154	4636.9	4762.2	4891.7	54
55	41734	4284.9	43994	4517.4	4638.9	47643	4893.9	55
56	4175.3	4286.8	4401.4	45194	4641.0	4766.5	4896.1	56
57	4177.1	4288.6	4403.3	4521.4	4643.0	4768.6	4898.3	57
58	4178.9	4290.5	4405.3	4523.4	4645.1	4770.7	4900.5	58
59	4180.8	4292.4	4407.2	4525.4	4647.2	4772.9	4902.7	59
M	min.	min.	min.	min.	min.	min.	min.	M
L.	56	57	58	59	60	61	62	L.

L. M	63 min.	64 min.	65 min.	66 min.	67 min.	68 min.	69 min.	L. M
0	4904 9	5039.4	5178 8	5323.5	5474 0	5630.8	5794 5	0
1	4907.1	5041.7	5181.2	5326 0	5476.6	5633.5	5797.3	1
2	4909.3	5044.0	5183.5	5328.4	5479.1	5636.2	5800.1	2
3	4911.6	5046 3	5185 9	5331.0	5481.7	56388	5802.9	3
4	4913.8	5048.6	5188.3	5333.4	54843	5641.5	5805.7	4
5	4916.0	5050.8	5190.7	5335.8	5486.8	5644.2	5808.5	5
6	4918.2	5053.1	5193.0	5338.3	54894	5646.9	5811.3	6
7	4920.4	5055.4	5195.4	5340.8	5492.0	5649.6	5814.1	7
8	4922 6	5057.7	5197.8	5343.2	5494.5	5652.2	5816.9	8
9	4924 8	5060.0	5200.2	5345.7	5497 1	5654 9	5819 8	9
10	4927.0	5062.3	5202 5	5348.2	5499.7	5657.6	5822 6	10
11	4929 2	5064.6	52049	5350.7	5502.3	5660.3	5825.4	11
12	4931.5	5066 9	5207.3	5353.1	5504.8	5663.0	58282	12
13	4933.7	5069.2	5209.7	5355.6	5507.4	5665.7	5831.0	13
14	4935.9	5071.5	5212.1	5358.1	5510.0	5668.4	5833.8	14
15	4938.1	5073.8	5214.7	5360.6	55126	5671.1	5836.7	15
16	4940.3	5076.1	52169	53631	5515.2	5673.8	5839.5	16
17	4942.6	5078.4	5219.2	5365.5	5517.8	5676.5	5842.3	17
18	4944.8	5080.7	5221.6	5368.0	5520.4	5679.2	5845.1	18
19	4947.0	5083.0	5224.0	5370.5	5523.0	5681.9	5848.0	19
20	4949.2	5085.3	5226.4	5373 0	5525.5	56846	5850.8	20
21	4951.5	5087.6	5228.8	5375.5	5528.1	5687.3	5853.6	21
22	4953.7	5089.9	5231.2	5378.0	5530.7	5690.0	5856.5	22
23	4955.9	5092.3	5233.6	5380.5	55333	5692.7	5859.3	23
24	4958.2	5094.6	5236.0	5383.0	55359	56954	5862.1	24
25	4960.4	5096.9	5238.4	53855	5538.5	5698.2	5865.0	25
26	4962.6	5099.2	5240.8	5388.0	5541.1	5700.9	5867.8	26
27	49649	5101.5	5243.2	5390.5	55438	5703.6	5870.7	27
28	4967.1	5103.8	5245.6	5393.0	5546.4	5706 3	5873.5	28
29	4969.4	5106.2	5248.1	5395.5	5549.0	5709.1	5876.4	29
M L.	min. 63	min. 64	min. 65	min. 66	min. 67	min. 68	min. 69	M L.

L	63	64	65	66	67	68	69	L.
M	min.	min.	min.	min.	min.	min.	min.	M
30	4971.6	5108.5	5250.5	5398.0	5551.6	5711.8	5879.2	30
31	4973.8	5110.8	5252.9	5400.5	5554.2	5714.5	5882.1	31
32	4976.1	5113.1	5255.3	5403.0	5556.8	5717.2	5885.0	32
33	4978.3	5115.5	5257.7	5405.5	5559.4	5720.0	5887.8	33
34	4980.6	5117.8	5260.1	5408.0	5562.1	5722.7	5890.7	34
35	4982.8	5120.1	5262.5	5410.6	5564.7	5725.4	5893.5	35
36	4985.1	5122.4	5265.0	5413.1	5567.3	5728.2	5896.4	36
37	4987.3	5124.8	5267.4	5415.6	5569.9	5730.9	5899.3	37
38	4989.6	5127.1	5269.8	5418.1	5572.6	5733.7	5902.2	38
39	4991.8	5129.4	5272.2	5420.6	5575.2	5736.4	5905.0	39
40	4994.1	5131.8	5274.7	5423.2	5577.8	5739.2	5907.9	40
41	4996.3	5134.1	5277.1	5425.7	5580.4	5741.9	5910.8	41
42	4998.6	5136.5	5279.5	5428.2	5583.1	5744.7	5913.7	42
43	5000.8	5138.8	5281.9	5430.7	5585.7	5747.4	5916.5	43
44	5003.1	5141.1	5284.4	5433.3	5588.4	5750.2	5919.4	44
45	5005.4	5143.5	5286.8	5435.8	5591.0	5752.9	5922.3	45
46	5007.6	5145.8	5289.2	5438.3	5593.6	5755.7	5925.2	46
47	5009.9	5148.2	5291.7	5440.9	5596.3	5758.5	5928.1	47
48	5012.1	5150.5	5294.1	5443.4	5598.9	5761.2	5931.0	48
49	5014.4	5152.9	5296.6	5446.0	5601.6	5764.0	5933.9	49
50	5016.7	5155.2	5299.0	5448.5	5604.2	5766.8	5936.8	50
51	5018.9	5157.6	5301.5	5451.0	5606.9	5769.5	5939.7	51
52	5021.2	5159.9	5304.0	5453.6	5609.5	5772.3	5942.6	52
53	5023.5	5162.3	5306.3	5456.1	5612.2	5775.1	5945.5	53
54	5025.8	5164.6	5308.8	5458.7	5614.8	5777.9	5948.4	54
55	5028.0	5167.0	5311.2	5461.2	5617.5	5780.6	5951.3	55
56	5030.3	5169.4	5313.7	5463.8	5620.2	5783.4	5954.2	56
57	5032.6	5171.7	5316.1	5466.3	5622.8	5786.2	5957.2	57
58	5034.9	5174.1	5318.6	5468.9	5625.5	5789.0	5960.1	58
59	5037.1	5176.4	5321.1	5471.4	5628.1	5791.8	5963.0	59
M	min.	min.	min.	min.	min.	min.	min.	M
L.	63	64	65	66	67	68	69	L.

L.	70°	71	72	73	74	75	76	L.
M	min.	min.	min.	min.	min.	min.	min.	M
0	5965.9	6145.7	6334.8	6534.4	6745.7	6970.3	7210.1	0
1	5968.8	6148.8	6338.1	6537.8	6749.4	6974 2	7214 2	1
2	5971.8	61 51.8	6341.3	6541.3	6753.0	6978.1	7218.3	2
3	5974.7	6154.9	6344.6	6544.7	6756.6	6981.9	7222.5	3
4	5977.6	61 58.0	6347.8	6548.1	6760.3	6985.8	7226.6	4
5	5980.6	6161.1	6351.1	6551.6	6763.9	6989.7	7230.8	5
6	5983.5	6164.2	6354.3	6555.0	6767.6	6993 6	7235.0	6
7	5986.4	6167.3	6357.6	6558.4	6771.2	6997.5	7239 1	7
8	5989.4	6170.4	6360.8	6561.9	6774.9	7001.4	7243.3	8
9	5992.3	6173.5	6364.1	6565.3	6778.5	7005.3	7247.5	9
10	5995.3	6176.5	6367.3	6568.8	6782.2	7009 2	7251.6	10
11	5998.2	6179.6	6370.6	6572.2	67859	7013.1	72558	11
12	6001.2	6182.7	6373.9	6575.7	6789.5	7017.0	7260.0	12
13	6004.1	6185.9	6377.2	6579.2	6793.2	7020.9	7264.2	13
14	6007.1	6189.0	6380 4	6582.6	6796.9	7024.8	7268 4	14
15	6010.0	6192.1	6383.7	6586.1	6800.6	7028.8	7272.6	15
16	6013.0	6195.2	6387.0	6589.6	6804.3	7032.7	7276.8	16
17	6016.0	6198.3	6390.3	6593.0	6808.0	7036.6	7281.0	17
18	6018.9	6201.4	6393.6	6596.5	6811.7	7040.6	72853	18
19	6021.9	6204.5	6396.9	66000	6815.3	7044.5	7289.5	19
20	6024.9	6207.7	6400.1	6603.5	6819.0	7048.5	7293.7	20
21	6027.8	6210.8	6403.4	6607.0	6822.8	7052.4	72980	21
22	6030.8	6213.9	64067	6610.5	6826.5	7056.4	7302.2	22
23	6033.8	6217.0	6410.0	6614.0	6830.2	7060.3	7306.4	23
24	6036.8	6220.2	6413.4	6617.5	6833.9	7064.3	7310.7	24
25	6039.8	6223.3	6416.7	6621.0	6837.6	7068.3	73149	25
26	6042.7	6226.5	6420.0	6624.5	6841.3	7072.2	7319.2	26
27	6045.7	6229.6	6423.3	6628.0	6845.1	7076.2	7323.5	27
28	6048.7	6232.7	6426.6	6631.5	6848.8	7080.2	7327.7	28
29	6051.7	6235.9	6429.9	6635.0	6852.5	7084.2	7332.0	29
M	min.	min.	min	min.	min.	min.	min.	M
L.	70	71	72	73	74	75	76	L.

L	70	71	72	73	74	75	76	L
M	min.	min.	min	min.	min.	min.	min.	M
30	6054.7	6239.0	6433.3	6638.5	6856 3	7088 2	7336.3	30
31	6057 7	6242.2	6436 6	6642.0	6860.0	7092.2	7340 6	31
32	6060.7	6245.3	6439 9	6645 6	6863.8	7096.2	7344.9	32
33	6063.7	6248.5	6443.2	6649.1	6867.5	7100.2	7349 2	33
34	6066.7	6251.7	6446 6	6652.6	6871.3	7104.2	7353.5	34
35	6069.7	6254.8	6449.9	6656 2	6875 0	7108 2	7357 8	35
36	6072 7	6258 0	6453.3	6659.7	6878 8	7112.2	7362.1	36
37	6075 7	6261.2	6456.6	6663.3	6882.6	7116.2	7366.4	37
38	6078.7	6264.3	6460.0	6666.8	6886.3	7120.3	7370.7	38
39	6081.8	6267.5	6463.3	6670.4	6890.1	7124.3	7375.1	39
40	6084.8	6270.7	6466.7	6673.9	6893 9	7128.3	7379 4	40
41	6087.8	6273.9	6470.0	6677.5	6897.7	7132.4	7383.7	41
42	6090.8	6277.1	6473.4	6681.0	6901.5	7136.4	7388 1	42
43	6093.9	6280.2	6476.7	6684.6	6905.3	7140.5	7392.4	43
44	6096.9	6283.4	6480.1	6688.2	6909.1	7144.5	7396.8	44
45	6099 9	6286.6	6483.5	6691.7	6912.9	7148.6	7401.1	45
46	6103.0	6289.8	6486.9	6695.3	6916.7	7152.6	7405.5	46
47	6106.0	6293.0	6490.2	6698.9	6920.5	7156.7	7409.9	47
48	6109.0	6296.2	6493 6	6702.5	6924 3	7160.8	7414 3	48
49	6112.1	6299.4	6497.0	6706.1	6928.1	7164.9	7418.6	49
50	6115.1	6302.6	6500.4	6709.6	6931.9	7169.0	7423.0	50
51	6118.2	6305.8	6503.8	6713.2	6935.7	7173.1	7427.4	51
52	6121.2	6309 0	6507.2	6716.8	6939.6	7177.2	7431 8	52
53	6124.3	6312.3	6510.6	6720 4	6943.4	7181.3	7436.2	53
54	6127.3	6315 5	6514.0	6724.0	6947.2	71854	7440.6	54
55	61304	63187	6517.4	6727.6	6951.1	7189.5	7445.0	55
56	6133.4	6321.9	6520.8	6731.3	6954.9	7193.6	7449.5	56
57	6136.5	6325.1	6524.2	6734.9	69588	7197.7	7453.9	57
58	6139.6	6328.4	6527.6	6738.5	6962.6	7201.8	7458.3	58
59	6142.6	6331.6	6531.0	6742.1	6966.5	7205.9	7462.8	59
M	min.	min.	min.	min.	min.	min.	min.	M
L.	70	71	72	73	74	75	76	L

L.	77	78	79	80	81	82	83	L.
M	min.	min.	min.	min.	min.	min.	min.	M
0	7467.2	7744.6	8045.7	8375.2	8739.1	9145.5	9605 8	0
1	7471.7	7749.4	8050.9	8381.0	8745.5	9152.7	9614.0	1
2	7476.1	7754.2	8056.2	8386.7	8751.9	9159.9	9622.3	2
3	7480.6	7759.0	8061.5	8392.5	8758.3	9167.1	9630.5	3
4	7485.0	7763.9	8066.7	8398.3	8764 7	9174.3	9638.8	4
5	7489.5	7768.7	8072.0	8404.1	8771.2	9181.6	9647.1	5
6	7494.0	7773.5	8077.3	8409 9	8777.6	9188.8	9655 4	6
7	7498.5	7778.4	8082.6	8415.7	8784 1	9196.1	9663.7	7
8	7502.9	7783.3	8087.9	8421.6	8790.6	9203.4	9672.1	8
9	7507.4	7788.1	8093.2	8427.4	8797.1	9210.7	9680.5	9
10	751 1.9	7793.0	8098.5	8433.3	8803.6	9218.1	9688.9	10
11	7516.4	7797.9	8103.8	8439.1	8810.1	9225.4	9697.3	11
12	7521.0	7802.8	8109.2	8445.0	8816.6	9232.8	9705.7	12
13	7525.5	7807.7	8114.5	8450.9	8823.2	9240.1	9714 2	13
14	7530.0	7812.6	8119.9	8456.8	8829.7	9247.5	9722.6	14
15	7534.5	7817.5	8125.2	8462.7	8836.3	9254.9	9731.1	15
16	7539.1	7822.4	8130.6	8468.6	8842.9	9262.4	9739.7	16
17	7543.6	7827.3	8136 0	8474.5	8849.5	92698	9748.2	17
18	7548.1	7832.2	8141.3	8480.4	8856.1	9277.3	9756.8	18
19	7552.7	7837.2	8146.7	8486.4	8862.7	9284.7	9765 3	19
20	7557.3	7842.1	8152.1	8492.3	8869.3	9292.2	9773.9	20
21	7561.8	7847.1	8157.5	8498.3	8876.0	9299.7	9782.6	21
22	7566.4	7852.0	8162.9	8504.2	8882.6	9307.2	9791.2	22
23	7570.0	7857.0	8168 4	8510.2	8889.3	93148	9799.9	23
24	7575.5	7861.9	8173.8	8162 2	8896.0	9322 3	9808.6	24
25	7580.1	7866.9	8179.2	8522.2	8902.7	93299	9817.3	25
26	7584.7	7871.9	8184.7	8528.2	8909.4	9337.5	9826.0	26
27	7589.3	7876.9	8190.1	8534.3	8916.1	9345.1	9834.8	27
28	7593.9	7881.9	8195.6	8540.3	8922.8	9352.7	9843.5	28
29	7598.5	7886.9	8201.1	8546.3	8929.6	9360.4	9852.3	29
M	min.	min.	min.	min.	min.	min.	min.	M
L.	77	78	79	80	81	82	83	L.

M	77 min.	78 min.	79 min.	80 min.	81 min.	82 min.	83 min.	M
30	7603.2	7891.9	8206.6	8552.4	8936.3	9368.0	9861.2	30
31	7607.8	7896.9	8212.1	8558 4	8943.1	9375.7	9870.0	31
32	7612.4	7902.0	8217.6	8564.5	8949.9	9383.4	9878.9	32
33	7617.0	7907.0	8223.1	8570 6	8956.7	9391.1	9878.8	33
34	7621.7	7912.0	8228.6	8576.7	8963.5	9398.8	9896.7	34
35	7626.3	7917.1	8234.1	8582.8	8970.3	9406.5	9905.6	35
36	7631.0	7922.1	8239.7	8588.9	8977.2	9414.3	9914.6	36
37	7635.6	7927.2	8245.2	8595.1	8984.0	9422.1	9923.6	37
38	7640.3	7932.3	8250.7	8601.2	8990.9	9429.8	9932.6	38
39	7645.0	7937.3	8256.3	8607.3	8997.8	9437 7	9941.6	39
40	7649.7	7942.4	8261.9	8613.5	9004 6	9445.5	9950 7	40
41	7654.3	7947.5	8267.5	8619.7	9011.6	9453.3	9956.7	41
42	7659.0	7952.6	8273.0	8625.9	9018.5	9461.2	9968 8	42
43	7663.7	7957.7	8278.6	8632.1	9025.4	9469.1	9978 c	43
44	7668.4	7962.8	8284 3	8638.3	9032.4	9477.0	9987.1	44
45	7673.1	7968.0	8289.9	8644 5	9039.3	9484.9	9996.3	45
46	7677.9	7973.1	8295.5	8650.7	9046.3	9492.8	10005.5	46
47	7682.6	7978.2	8301 1	8656.9	9053 3	9500.8	100147	47
48	7687.3	7983 4	8306.8	8663.2	9060.3	9508.7	10023.9	48
49	7692.5	7988.5	8312.4	8669.5	9067.3	9516.7	10033.2	49
50	7696.8	7993.7	8318.1	8675.7	9074.3	95247	10042.5	50
51	7701.5	7998 9	83237	8682.0	9081.4	9532.7	10051.8	51
52	7706.3	8004.0	8329.4	8688.3	9088 4	9540 8	10061.2	52
53	7711.1	8009.2	8335.1	8694.6	9095.5	9548.8	10070.6	53
54	7715.8	80144	8340.8	8700.9	9102.6	9556.9	10080.0	54
55	7720.6	8019 6	8346.5	8707.2	9109.7	9565.0	10089.4	55
56	7725.4	8024.8	8352.2	8713 6	9116.8	9573.2	10098.8	56
57	7730.2	8030.0	8358.0	8719 9	9124.0	9581.3	10108.3	57
58	7735.0	8035.2	8363.7	8726.3	9131.1	9589.4	10117.8	58
59	7739.8	8040.5	8369.4	8732.7	9138.3	9597.6	10127.3	59
M	min.	min.	min.	min.	min.	min.	min	M
L	77	78	79	80	81	82	83	L

L. M	84 min.	85 min.	86 min.	87 min.	88 min.	89 min.	L. M
0	10136.9	10764.6	11532.5	12521.1	13916.4	16299.6	0
1	10146.5	10776.1	11546 9	12541.3	13945.2	16357.3	1
2	10156.1	10787.6	11561.3	12560 5	13974.2	16416.1	2
3	10165.7	10799.2	11575.8	125799	14003 5	16475.9	3
4	101754	10810.8	11590.3	125994	14033.0	16536.7	4
5	10185.1	10822.5	11605.0	12619.0	14062.8	16598.7	5
6	10194.8	10834.2	11619.6	12638.7	14092.8	16661.8	6
7	10204.5	10845.9	11634.4	12658.5	14123.1	16726.0	7
8	10214.3	10857.7	11649.2	12678.5	14153.7	16791.5	8
9	10224.1	10869.5	11664.0	12698.5	14184.5	16858.3	9
10	10233.9	10881.3	11678.9	12718.7	14215.6	16926.4	10
11	10243.8	10893.2	11693.9	12739.0	14247.0	16995.8	11
12	10253.6	10905.1	11709.0	12759.4	14278.7	17066.7	12
13	10263.5	10917.1	11724.1	12779.9	14310.7	17139.1	13
14	10273.5	10929.1	11739.3	12800.6	14343.0	17213.0	14
15	10283.5	10941.2	11754.6	12821.4	14375 6	172886	15
16	10293.4	10953.3	11769 9	12842.3	14408 5	17365.9	16
17	10303.5	10965.4	11785.3	12863.3	14441.7	17444.9	17
18	10313.5	10977.6	11800.7	12884.5	14475.2	17525.8	18
19	10323.6	10989.8	11816.3	12905.8	14509 1	17508.6	19
20	10333.7	11002.1	11831.9	12927.2	14543.3	17673 5	20
21	10343.9	11014.4	11847.5	12948.7	14577.9	17780.5	21
22	10354.0	11026.8	11863.3	12970.4	14612.8	17869.8	22
23	10364.2	11039.2	11879.1	12992.3	14648.0	17961.5	23
24	10374.5	11051.6	11895 0	13014 2	14683.7	18055.7	24
25	10384.7	11064.1	11910.9	13036.4	14719 7	18152.5	25
26	10395 0	11076.6	11927.0	13058.6	14756.1	18252.2	26
27	104054	11089 2	11959.3	13081.0	14792.8	18354.8	27
28	10415.7	11101.8	11943.1	13103.6	14830.0	18460.6	28
29	10426.1	11114.5	11975.6	13126.3	14867.6	18569.8	29
M L.	min. 84	min. 85	min. 86	min. 87	min. 88	min. 89	M L.

L.	84	85	86	87	88	89	L.
M	min.	min.	min.	min.	min.	min.	M
30	10436.5	11127.2	11995.9	13149.1	14905.6	18682.5	30
31	10447.0	11140.0	12008.3	13172.1	14944.0	18799.0	31
32	10457.4	11152.8	12024.8	13195.3	14982.8	18919.7	32
33	10468.0	11165.7	12041.4	13218.6	15022.1	19044.7	33
34	10478.5	11178.6	12058.0	13242.1	15061.9	19174.1	34
35	10489.1	11191.6	12074.8	13265.7	15102.1	19309.3	35
36	10499.7	11204.6	12091.6	13289.5	15142.8	19449.6	36
37	10510.3	11217.6	12108.5	13313.5	15183.9	19595.9	37
38	10521.0	11230.7	12125.5	13337.6	15225.6	19748.7	38
39	10531.7	11243.9	12142.6	13361.9	15267 8	19908.7	39
40	10542.5	11257.1	12159.7	13386.4	15310.5	20076.4	40
41	10553.2	11270.4	12177.0	13411.0	15353 8	20252.7	41
42	10564.0	11283.7	12194.3	13435.9	15397.6	20438.6	42
43	10574.9	11297.0	12211.7	13460.9	15441.9	20635.1	43
44	10585.8	11310.5	12229.2	13486.1	15486.9	20843.4	44
45	10596.7	11323.9	12246.8	13511.4	15532.4	21065.4	45
46	10607.6	11337.4	12264.5	13537.0	15578.5	21302.5	46
47	10618.6	11351.0	12282.3	13562.8	15625.3	21557.3	47
48	10629.6	11364.6	12300.1	13588.7	15672.7	21832.5	48
49	10640.7	11378.3	12318.1	13614.9	15720.8	22131.6	49
50	10651.7	11392.1	12336.1	13641.2	15769.6	22459.3	50
51	10662.9	11405.9	12354.3	13667.8	15819.1	22821.5	51
52	10674.0	11419.7	12372.5	13694.5	15869.3	23226.4	52
53	10685.2	11433.6	12390.9	13721.5	15920.2	23685.4	53
54	10696.5	11447.6	12409.3	13754.7	15971.9	24215.3	54
55	10707.7	11461.6	12427.9	13776.1	16024.4	24842.1	55
56	10719.0	11475.6	12446.5	13803.7	16077.7	25609.2	56
57	10730.4	11489.8	12465.3	13831.5	16131.8	26598.2	57
58	10741.7	11504.0	12484.1	13859.6	16186.8	27992.1	58
59	10753.2	11518.2	12503.1	13887.9	16242.7	30375.0	59
M	min.	min	min.	min.	min.	min.	M
L.	84	85	86	87	88	89	L.

SECT. IX.

BY the Help of the *Table* of *Meridional Parts*, made according to the Principles laid down and demonſtrated in the former *Section*, it will be eaſy to give Solutions to all the uſual *Problems* propoſed in *Mercator's Sailing*; and thence to find the true Bearing, Diſtance, Difference of Latitude and Longitude, of any two Places propoſed: For in,

PROBLEM I.

If it be required to find the Meridional Difference of Latitude between any two Places; That is,

Caſe I.

If one Place be under the Equator, and the other in North or South Latitude,

Then the Meridional Parts anſwering to the Latitude propoſed, will be the Meridional Difference of Latitude.

Example.

If one Place be under the Equator, and the other in the Latitude of 40° 30′, then the Meridional Difference of Latitude will be found to be 2662.

Caſe II.

If the Places propoſed are on the ſame Side of the Equator; that is, if they are both in North, or both in South Latitude; then the Difference of the Meridional Parts of each Latitude will be the Meridional Difference of Latitude.

Example.

If one Place be in the Latitude of 50° 00′ North, and the other in the Latitude of 13° 30′ North, the Meridional Difference of Latitude will be found to be 2656.9.

For,

For,

From the Meridional Parts answering to the Latitude of 50° 00′	3474.5
Take the Meridional Parts answering to 13° 30′	817.6
There remains the Meridional Difference of Latitude	2656.9

Case III.

If the Places proposed are on contrary Sides of the Equator; that is, if one be in North, the other in South Latitude; then The Sum of the Meridional Parts answering to each Latitude will be the Meridional Difference of Latitude.

Example.

If one Place be in the Latitude of 28° 20′ North, the other in the Latitude of 35° 50′ South, and it be required to find the Meridional Difference of Latitude.

To the Meridional Parts answering to 28° 20′	1773.8
Add the Meridional Parts answering to 35° 50′	2305.6
The Sum will be the Meridional Difference of Latitude	4079.4

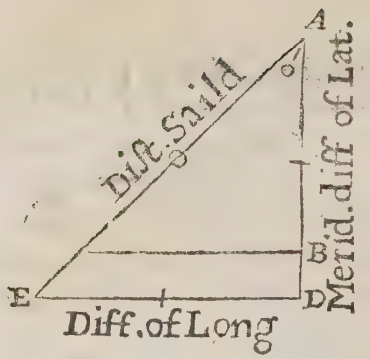
PROBLEM II.

If the Latitudes and Difference of Longitudes of any two Places be given, to find the Course, Distance and Departure.

Example.

If a Ship at the *Lizard*, in the Latitude of 50° 00′ North, be bound to *Barbadoes*, in the Latitude of 13° 30′ North, the Difference of Longitude being 52° 58′, and it be required to know the Course from the *Lizard* to *Barbadoes*, as also the Distance between them.

Geome-



Geometrically.

1. Having found the Meridional Difference of Latitude 2656.9 by the last *Problem*, set it off from A to D.

2. Make DE equal to 3178, the Difference of Longitude, and draw AE.

3. Make AB equal to the Difference of Latitude, in Minutes 2190, and draw BC parallel to DE; then is the Angle at A the Course, and AC the Distance.

To find each of which, *Arithmetically*, it will be,

1*st*. AD : R : : DE : t, $\angle$ at A. That is,

As the Meridional Difference of Latitude 2656.9 —	3.4243752
To the Radius — — — — — — — — — —	10.0000000
So is the Difference of Longitude 3178 — — —	3.5021539
To the Tangent of the Course 50° 06′ — — —	10.0777787

Or SW. 5° 06′ Westerly.

Wherefore,

The direct Course from *Barbadoes* to the *Lizard*, is North East 5° 06′ Easterly.

For the Distance it will be,

2*d*. cs, $\angle$ A : AB : : R : AC. That is,

As the Co-sine of the Course 50° 06′ — — —	9.8071626
To the Difference of Latitude 2190 — — —	3.3404441
So is the Radius — — — — — — — — —	10.0000000
To the Distance 3414.14 — — — — — —	3.5332815

Or, 1138 Leagues.

PRO-

PROBLEM III.

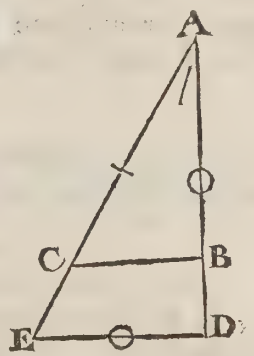
If one Latitude, Course and Distance sail'd were given, to find the other Latitude and Difference of Longitude? That is,

Example.

If a Ship from the *Lizard*, in the Latitude of 50° 00′ North, sails SSW. ½ Westerly 550 Leagues, or 1650 Miles, and it be required to find the Latitude she is in, and how much she has altered her Longitude.

Geometrically.

1. By the Course and Distance given, form the Triangle ABC, according to the Directions given in *Case the 1st* of *Plain Sailing*, and find the Latitude the Ship is in.
2. Find the Meridional Difference of Latitude AD, by *Problem the 1st* of this *Section*, and draw DE parallel to BC, till it meet AC produced in E, and the Thing is done.



Arithmetically.

1*st*. R : AC : : cs, < A : AB. That is,

As the Radius	10.0000000
To the Distance sailed 1650	3.2174839
So is the Co-sine of the Course 28° 07′ ½	9.9454298
To the Difference of Latitude 1455	3.1629137

Hence the Ship will be found to be in the Latitude of 25° 45′, and consequently the Metidional Difference of Lat. will be 1874.7

Wherefore, to find the Difference of Longitude, it will be,

R : AD : : t, DAC : DE. That is,

As

As the Radius ——————————— 10.0000000

To the Meridional Difference of Latitude 1874.7 — 3.2729318
So is the Tangent of the Course 28° 07′ ½ ——— 9.7279567

To the Difference of Longitude 1002.4 West — 3 0008885

Now, If the Ship was bound to *Barbadoes*, and it be required to find upon what Course she must sail, and how far to go directly thither.

Because the Differences of Longitude are both the same Way, take the Difference of Longitude made from the Difference of Longitude between the *Lizard* and *Barbadoes*; and with the Remainder and the two Latitudes, find the Course and Distance by the former *Problem*.

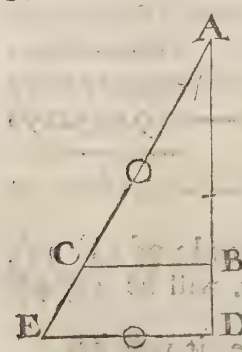
PROBLEM IV.

If both Latitudes and Course were given, to find the Distance, and Difference of Longitude.

Example.

If a Ship from the *Lizard*, in the Latitude of 50° 00′ North, sails SSW. ½ Westerly, and then by Observation is found to be in the Latitude of 24° 30′ North, and it be required to know how far she is from her last Port, and how much she has altered her Longitude.

Geometrically.



1. Having constructed the Triangle ABC, by the Help of the Course and Difference of Latitude, according to the *2d Case* of *Plain Sailing*.

2. Make AD equal to the Meridional Difference of Latitude obtained by the *first Problem*, and draw DE perpendicular to AD, till it meet AC produced; then is AC the Distance sail'd, and DE the Alteration of Longitude.

To find which, *Arithmetically*, it will be;

R : AD : : t, < DAE : DE. That is,

As

As the Radius ——————————— 10.0000000

To the Meridional Difference of Latitude 1957.5 3.2917017
So is the Tangent of the Courſe 28° 07′ —— —— 9.7279567

To the Difference of Longitude 1046.3, or 17° 26′ — 3.0196584

Whence the Diſtance may be found as uſual.

PROBLEM V.

If both Latitudes and Diſtance were given, to find the Courſe and Difference of Longitude?

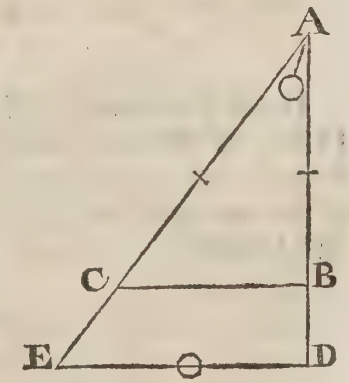
Example.

If a Ship from the *Lizard*, in the Latitude of 50° 00′ North, ſails between the South and the Weſt, until her Diſtance ſailed be 2000 Miles, and then is found to be in the Latitude of 24° 00′ North, and it be required to know upon what Courſe ſhe has ſailed, and how much ſhe has altered her Longitude.

Geometrically.

1. With the Difference of Latitude and Diſtance, conſtruct the Triangle ABC, by the *3d Caſe* of *Plain Sailing*.

2. Let AD be made equal to the Meridional Difference of Latitude, and raiſe the Perpendicular DE, till it meet AC continued in E, then is DE the Difference of Longitude, and BAC the Courſe.



To find which *Arithmetically*,

AC : R : : AB : cs, < A. That is,

As the Distance sailed 2000 —— —— —— —— 3.3010300

To the Radius —— —— —— —— —— —— 10.0000000
So is the Difference of Latitude 1560 —— —— —— 3.1931246

To the Co-sine of the Course 38° 45′ —— —— 9.8920946
Or, S.W. by S. 5° 00′ Westerly.

Wherefore, for the Difference of Longitude, it will be,

R : AD : : t, < A : DE. That is,

As the Radius —— —— —— —— —— —— 10.0000000

To the Meridional Difference of Latitude 1950.4 — 3.2989404
So is the Tangent of the Course 38° 45′ —— —— 9.9044910

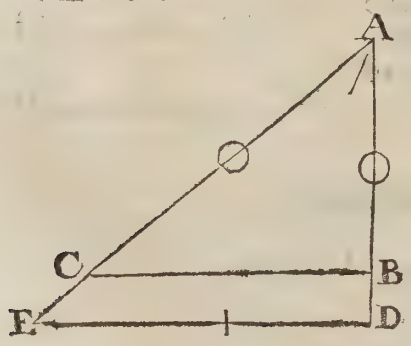
To the Difference of Longitude 1597.5, or 26° 37′ 3.2034314

PROBLEM VI.

If the Latitude of any Place were given, and its Bearing from another, as also their Difference of Longitude, to find the Latitude of the other Place, and their mutual Distance.

Example.

If the Difference of Longitude between the *Lizard*, (in the Latitude of 50° 00′ North) and *Barbadoes* 52° 58′, and the direct Course from the *Lizard* to *Barbadoes* SW. 5° 06′ ½ Westerly, are given to find the Latitude of *Barbadoes*, and its Distance from the *Lizard*.



Geometrically.

1. Make DE equal to the Difference of Longitude, and DEA equal to the Complement of the Course, and it will be,

1*st*. R : ED : : t, < E : AD.

That

That is,

As the Radius —— 10.0000000

To the Difference of Longitude 52° 58′ = 3178 — 3.5021539
So is the Co-tangent of the Course 52° 06′ —— 9.9222737

To the Meridional Difference of Latitude 2657.2 —— 3.4244276

Now, because *Barbadoes* is to the Southward of the *Lizard*,

From the Meridional Parts answering to 50° 00′ —— 3474.5
Take the Meridional Difference of Latitude —— —— 2657.2

There remains the Merid. Parts of the Lat. of *Barbadoes*— 817.3

Which answer to 13° 30′.

Again,

From the Latitude of the *Lizard* —— —— —— 50° 00′
Take the Latitude of *Barbadoes* —— —— —— 13 30

There remains the Difference of Latitude —— —— 36 30

2. Set off the Difference of Latitude from A to B, and draw BC parallel to DE; then will AC be the Distance; and it will be,

s, $\angle$ ACB : AB : : R : AC. That is,

As the Co-sine of the Course 50° 06′ ½ —— —— 9.8070870

To the Difference of Latitude 2190 —— —— 3.3404441
So is the Radius —— —— —— —— —— 10.0000000

To the Distance 3414.74 —— —— —— 3.5333571

PROBLEM VII.

Let us now ſuppoſe a Ship in the Latitude of 50° 00′ North, to ſail firſt SSE. ½ E. 34 Miles, then SE. ½ E. 52 Miles, then ESE. ¼ E. 90 Miles, and let it be required to find the Latitude the Ship is in, and how much ſhe has altered her Longitude?

Becauſe the firſt Courſe is SSE. ½ E. 34 Miles, it will be, by *the 3d Problem* of Mercators Sailing,

As the Radius	10.0000000
To the Co-ſine of the Courſe 28° 07′	9.9454636
So is the Diſtance ſailed 34	1.5314789
To the Difference of Latitude 29° 99′	1.4769425

Whence the Ship will be found to be in the Latitude 49° 30′ N. and conſequently the Meridional Difference of Latitude will be 46.5 Miles.

Whence, to find the Difference of Longitude, it will be,

As the Radius	10.0000000
To the Tangent of the Courſe 28° 07′	9.7278048
So is the Meridional Difference of Latitude 46.5	1.6674530
To the Difference of Longitude 24.85	1.3952578

After the ſame manner, the Difference of Longitude to every ſingle Courſe will be found, as in the following Table.

The Traverſe Table.

Courſes.	Parts.	Diſt.	Diff. of Lat.		Diff. of Long.	
			N.	S.	E.	W.
SSE. ½ E.	2 ½	34	—	29.99	—	24.85
SE. ½ E.	4 ½	52	—	32.99	—	61.78
ESE. ½ E.	6 ½	92	—	30.99	—	131.92
		Diff. of Lat.		93.97	D. Lon.	218.55

Whence

Whence it appears, that the Ship has altered her Longitude 218.55 Miles; that ſhe is got into the Latitude 48° 26′ North: and conſequently her Meridional Difference of Latitude is 144 Miles.

Whence, to find her direct Courſe, it will be, by *Problem the 2d* of *Mercator's Sailing*.

As the Meridional Difference of Latitude 144.0 —— 2.1583625

To the Difference of Longitude 218.55 —— —— 2.3395508
So is the Radius —— 10.0000000

To the Tangent of the Courſe 56° 37′ —— —— 10.1811883

And to find the direct Diſtance, it will be, by the ſame *Problem*,

As the Radius —— 10.0000000

To the Secant of the Courſe 56° 37′ —— —— 10.2594495
So is the proper Difference of Latitude 93.97 —— 1.9729892

To the direct Diſtance ſailed 170.78 —— —— 2.2324387

So that the Ship's direct Courſe is found to be S. 56.37 E. or SE. by E. 0° 22′ Eaſt, and her direct Diſtance from her firſt Port is 170.78 Miles.

As this Method of computing the Difference of Longitude to every ſingle Courſe may be thought irkſome, I have calculated the following Tables, by which the *Difference of Longitude* may be had by Inſpection, in a Manner as eaſy as the *Departure* is obtained in the *Common Traverſe Tables*, as will evidently appear, when I come to ſhew their Uſe.

A TABLE

A

TABLE

OF

DIFFERENCE

OF

Latitude *and* Longitude.

To every

Point, Half Point, and Quarter Point of the COMPASS.

Very useful for the ready and expeditious Working of *Traverses* upon the Principles of *Mercator's Sailing*; whereby the *Difference of Longitude*, as well as of *Latitude*, for every *single Course* and *Distance* may be had by Inspection.

	¼ Point.		½ Point.		¾ Point.		1 Point.		
	D.La	D Lo	D.La.	D.Lo.	D La	D.Lo	D.La.	D Lo.	
1	00 99	00 05	00 99	00 10	00 99	00 15	00 98	00 20	1
2	01 99	00 10	01 99	00 20	01 98	00 30	01 96	00 40	2
3	02 99	00 15	02 99	00 30	02 97	00 45	02 94	00 60	3
4	03 99	00 20	03 98	00 39	03 96	00 59	03 92	00 80	4
5	04 99	00 25	04 98	00 49	04 95	00 74	04 90	00 99	5
6	05 99	00 29	05 97	00 59	05 94	00 89	05 89	01 19	6
7	06 99	00 34	06 97	00 69	06 92	01 04	06 87	01 39	7
8	07 99	00 39	07 96	00 79	07 91	01 19	07 83	01 59	8
9	08 99	00 44	08 96	00 89	08 90	01 34	08 80	01 79	9
10	09 99	00 49	09 95	00 99	09 89	01 48	09 81	01 99	10
11	10 98	00 54	10 95	01 08	10 88	01 63	10 79	02 19	11
12	11 98	00 59	11 94	01 18	11 87	01 78	11 77	02 39	12
13	12 98	00 64	12 94	01 28	12 86	01 93	12 75	02 59	13
14	13 98	00 69	13 93	01 37	13 85	02 07	13 73	02 79	14
15	14 98	00 74	14 93	01 47	14 84	02 22	14 71	02 99	15
16	15 98	00 78	15 92	01 57	15 83	02 37	15 70	03 18	16
17	16 98	00 83	16 92	01 67	16 82	02 52	16 68	03 38	17
18	17 98	00 88	17 91	01 77	17 80	02 67	17 66	03 58	18
19	18 98	00 93	18 91	01 87	18 79	02 82	18 64	03 78	19
20	19 98	00 98	19 90	01 97	19 78	02 97	19 62	03 98	20
21	20 97	01 03	20 90	02 07	20 77	03 11	20 60	04 18	21
22	21 97	01 08	21 89	02 17	21 76	03 26	21 58	04 38	22
23	22 97	01 13	22 89	02 27	22 75	03 41	22 56	04 58	23
24	23 97	01 18	23 89	02 36	23 74	03 55	23 54	04 78	24
25	24 97	01 23	24 89	02 46	24 73	03 70	24 52	04 97	25
26	25 97	01 28	25 87	02 56	25 72	03 85	25 51	05 17	26
27	26 97	01 33	26 87	02 66	26 70	04 00	26 49	05 37	27
28	27 96	01 37	27 86	02 76	27 69	04 15	27 47	05 57	28
29	28 96	01 42	28 86	02 86	28 68	04 30	28 45	05 77	29
30	29 96	01 47	29 85	02 95	29 67	04 45	29 42	05 97	30
31	30 96	01 52	30 85	03 05	30 66	04 59	30 40	06 17	31
32	31 96	01 57	31 84	03 15	31 65	04 74	31 38	06 37	32
33	32 96	01 62	32 84	03 25	32 64	04 89	32 36	06 57	33
34	33 95	01 67	33 83	03 34	33 63	05 03	33 34	06 77	34
35	34 95	01 72	34 83	03 44	34 62	05 18	34 32	06 96	35
36	35 95	01 76	35 82	03 54	35 61	05 33	35 31	07 16	36
37	36 95	01 81	36 82	03 64	36 59	05 48	36 29	07 36	37
38	37 95	01 86	37 81	03 74	37 58	05 63	37 27	07 56	38
39	38 95	01 91	38 81	03 84	38 57	05 78	38 25	07 76	39
40	39 95	01 96	39 81	03 94	39 57	05 93	39 23	07 96	40
41	40 94	02 01	40 80	04 04	40 56	06 07	40 21	08 16	41
42	41 94	02 06	41 80	04 14	41 55	06 22	41 19	08 36	42
43	42 94	02 11	42 80	04 24	42 54	06 37	42 17	08 57	43
44	43 94	02 16	43 79	04 33	43 53	06 51	43 15	08 76	44
45	44 94	02 21	44 79	04 43	44 52	06 66	44 13	08 95	45
46	45 94	02 26	45 78	04 53	45 51	06 81	45 12	09 15	46
47	46 94	02 31	46 78	04 63	46 49	06 96	46 10	09 35	47
48	47 94	02 36	47 77	04 73	47 48	07 11	47 08	09 55	48
49	48 94	02 41	48 77	04 83	48 47	07 26	48 06	09 75	49
50	49 94	02 46	49 76	04 92	49 45	07 42	49 04	09 95	50
	D.La.	D.Lo.	D.La	D.La.	D.La.	D.Lo.	D.La	D.La.	
	¼ Point.		½ Point.		¾ Point.		1 Point.		

	¼ Point.		½ Point.		¾ Point.		1 Point.		
	D.La.	D.Lo.	D.La.	D. Lo.	D La.	D.Lo	D. La.	D Lo	
51	50 93	02 51	50 75	05 02	50 45	07 57	50 02	10 15	51
52	51 93	02 56	51 75	05 12	51 44	07 72	51 00	10 35	52
53	52 93	02 61	52 75	05 22	52 43	07 87	51 98	10 55	53
54	53 93	02 66	53 74	05 31	53 42	08 01	52 96	10 75	54
55	54 93	02 71	54 74	05 41	54 41	08 16	53 94	10 94	55
56	55 93	02 76	55 73	05 51	55 40	08 31	54 93	11 14	56
57	56 93	02 80	56 73	05 61	56 38	08 46	55 91	11 34	57
58	57 93	02 85	57 72	05 71	57 37	08 61	56 89	11 54	58
59	58 93	02 90	58 72	05 81	58 36	08 76	57 87	11 74	59
60	59 93	02 95	59 71	05 91	59 35	08 90	58 85	11 94	60
61	60 92	03 00	60 71	06 00	60 34	09 05	59 83	12 14	61
62	61 92	03 05	61 70	06 10	61 33	09 20	60 81	12 34	62
63	62 92	03 10	62 70	06 20	62 32	09 35	61 79	12 54	63
64	63 92	03 15	63 69	06 29	63 31	09 49	62 77	12 73	64
65	64 92	03 20	64 69	06 39	64 30	09 64	63 75	12 93	65
66	65 92	03 24	65 68	06 49	65 29	09 79	64 74	13 12	66
67	66 92	03 29	66 68	06 59	66 27	09 94	65 72	13 32	67
68	67 92	03 34	67 67	06 69	67 26	10 09	66 70	13 52	68
69	68 92	03 39	68 67	06 79	68 25	10 24	67 68	13 72	69
70	69 92	03 44	69 66	06 89	69 24	10 38	68 66	13 92	70
71	70 91	03 49	70 66	06 99	70 23	10 53	69 64	14 12	71
72	71 91	03 54	71 65	07 09	71 22	10 68	70 62	14 32	72
73	72 91	03 59	72 65	07 19	72 21	10 83	71 60	14 52	73
74	73 91	03 64	73 64	07 28	73 20	10 97	72 58	14 71	74
75	74 91	03 69	74 64	07 38	74 19	11 12	73 56	14 91	75
76	75 91	03 74	75 63	07 48	75 18	11 27	74 55	15 11	76
77	76 90	03 79	76 63	07 58	76 16	11 42	75 53	15 31	77
78	77 90	03 83	77 62	07 68	77 15	11 57	76 51	15 51	78
79	78 90	03 88	78 62	07 78	78 14	11 72	77 49	15 71	79
80	79 90	03 93	79 61	07 88	79 13	11 87	78 46	15 91	80
81	80 90	03 98	80 61	07 97	80 12	12 01	79 44	16 11	81
82	81 90	04 03	81 60	08 07	81 11	12 16	80 42	16 31	82
83	82 89	04 03	82 60	08 17	82 10	12 31	81 40	16 51	83
84	83 89	04 13	83 59	08 26	83 09	12 45	82 38	16 70	84
85	84 89	04 18	84 59	08 36	84 08	12 60	83 36	16 90	85
86	85 89	04 22	85 58	08 46	85 07	12 75	84 35	17 10	86
87	86 89	04 27	86 58	08 56	86 06	12 90	85 33	17 30	87
88	87 89	04 32	87 57	08 66	87 05	13 05	86 31	17 50	88
89	88 89	04 37	88 57	08 76	88 04	13 20	87 29	17 70	89
90	89 89	04 42	89 57	08 86	89 03	13 35	88 27	17 90	90
91	90 88	04 47	90 56	08 96	90 02	13 49	89 25	18 10	91
92	91 88	04 52	91 56	09 06	91 01	13 64	90 23	18 30	92
93	92 88	04 57	92 56	09 16	92 00	13 79	91 21	18 50	93
94	93 88	04 62	93 55	09 25	92 99	13 93	92 19	18 69	94
95	94 88	04 67	94 55	09 35	93 98	14 08	93 17	18 89	95
96	95 88	04 72	95 54	09 45	94 97	14 23	94 16	19 09	96
97	96 88	04 77	96 54	09 55	95 95	14 38	95 14	19 29	97
98	97 88	04 82	97 53	09 65	96 94	14 53	96 12	19 49	98
99	98 88	04 86	98 53	09 75	97 93	14 68	97 10	19 69	99
100	99 88	04 91	99 52	09 85	98 92	14 83	98 08	19 89	100
	D.La.	D.Lo.	D.La.	D. Lo.	D.La.	D. Lo.	D.La.	D Lo.	
	¼ Point.		½ Point.		¾ Point.		1 Point.		

	1 ¼ Point.		1 ½ Point.		1 ¾ Point.		2 Points.		
	D.La.	D. Lo.	D.La.	D. Lo.	D.La.	D. Lo.	D.La.	D.Lo.	
1	00 97	00 25	00 96	00 30	00 94	00 36	00 92	00 41	1
2	01 94	00 50	01 91	00 61	01 88	00 72	01 85	00 83	2
3	02 91	00 75	02 87	00 91	02 83	01 07	02 77	01 24	3
4	03 88	01 00	03 83	01 21	03 77	01 43	03 70	01 66	4
5	04 85	01 25	04 79	01 52	04 71	01 79	04 62	02 07	5
6	05 82	01 50	05 74	01 82	05 65	02 15	05 54	02 49	6
7	06 79	01 75	06 70	02 12	06 59	02 50	06 47	02 90	7
8	07 76	02 00	07 66	02 43	07 53	02 86	07 39	03 31	8
9	08 73	02 25	08 62	02 73	08 47	03 22	08 32	03 73	9
10	09 70	02 50	09 57	03 03	09 42	03 58	09 24	04 14	10
11	10 67	02 76	10 53	03 33	10 36	03 94	10 16	04 55	11
12	11 64	03 01	11 48	03 64	11 30	04 30	11 09	04 97	12
13	12 61	03 26	12 44	03 94	12 25	04 65	12 01	05 38	13
14	13 58	03 51	13 40	04 24	13 19	05 01	12 94	05 80	14
15	14 55	03 76	14 36	04 55	14 13	05 37	13 86	06 21	15
16	15 52	04 01	15 31	04 85	15 07	05 73	14 78	06 63	16
17	16 49	04 26	16 27	05 15	16 01	06 08	15 71	07 04	17
18	17 46	04 51	17 23	05 46	16 95	06 44	16 63	07 45	18
19	18 43	04 76	18 19	05 76	17 89	06 80	17 56	07 87	19
20	19 40	05 01	19 14	06 07	18 83	07 16	18 48	08 28	20
21	20 37	05 26	20 10	06 36	19 77	07 52	19 40	08 69	21
22	21 34	05 51	21 05	06 67	20 71	07 88	20 33	09 11	22
23	22 31	05 76	22 01	06 97	21 66	08 23	21 25	09 52	23
24	23 28	06 01	22 97	07 27	22 60	08 59	22 18	09 94	24
25	24 25	06 26	23 93	07 58	23 54	08 95	23 1-	10 35	25
26	25 22	06 51	24 88	07 88	24 48	09 31	24 02	10 77	26
27	26 19	06 76	25 84	08 18	25 42	09 66	24 95	11 18	27
28	27 16	07 01	26 80	08 49	26 36	10 02	25 87	11 59	28
29	28 13	07 26	27 76	08 79	27 30	10 38	26 80	12 01	29
30	29 10	07 51	28 71	09 10	28 25	10 73	27 72	12 43	30
31	30 07	07 77	29 67	09 40	29 19	11 10	28 64	12 84	31
32	31 04	08 02	30 62	09 71	30 13	11 46	29 57	13 25	32
33	32 01	08 27	31 58	10 01	31 08	11 81	30 49	13 66	33
34	32 98	08 52	32 54	10 31	32 02	12 17	31 42	14 08	34
35	33 95	08 77	33 50	10 62	32 96	12 53	32 34	14 49	35
36	34 92	09 02	34 45	10 92	33 90	12 89	33 26	14 91	36
37	35 89	09 27	35 41	11 22	34 84	13 24	34 19	15 32	37
38	36 86	09 52	36 37	11 53	35 78	13 60	35 11	15 73	38
39	37 83	09 77	37 33	11 83	36 72	13 96	36 03	16 15	39
40	38 80	10 02	38 28	12 13	37 66	14 32	36 95	16 57	40
41	39 77	10 27	39 24	12 43	38 60	14 68	37 87	16 98	41
42	40 74	10 52	40 19	12 74	39 54	15 04	38 80	17 39	42
43	41 71	10 77	41 15	13 04	40 49	15 39	39 72	17 80	43
44	42 68	11 02	42 11	13 34	41 43	15 75	40 65	18 22	44
45	43 65	11 27	43 07	13 65	42 37	16 11	41 57	18 63	45
46	44 62	11 52	44 02	13 95	43 31	16 47	42 49	19 05	46
47	45 59	11 77	44 98	14 25	44 25	16 82	43 42	19 46	47
48	46 56	12 02	45 94	14 56	45 19	17 18	44 34	19 87	48
49	47 53	12 27	46 90	14 87	46 13	17 54	45 26	20 29	49
50	48 50	12 52	47 85	15 17	47 08	17 89	46 19	20 71	50
	D.La.	D.Lo.	D.La	D. Lo.	D.La.	D. Lo.	D.La.	D.Lo.	
	1 ¼ Point.		1 ½ Point.		1 ¾ Point		2 Points.		

	1 ¼ Point.		1 ½ Point.		1 ¾ Point.		1 Point.		
	D.La.	D.Lo.	D.La.	D.Lo.	D.La.	D.Lo.	D.La.	D.Lo.	
51	49 47	12 77	48 81	15 47	48 02	18 25	47 11	21 12	51
52	50 44	13 03	49 76	15 77	48 96	18 61	48 04	21 53	52
53	51 41	13 28	50 72	16 07	49 91	18 96	48 96	21 94	53
54	52 38	13 53	51 68	16 37	50 85	19 32	49 89	22 36	54
55	53 35	13 78	52 64	16 68	51 79	19 68	50 81	22 77	55
56	54 32	14 03	53 59	16 98	52 73	20 04	51 73	23 19	56
57	55 29	14 28	54 55	17 28	53 67	20 39	52 65	23 60	57
58	56 26	14 53	55 51	17 59	54 61	20 75	53 58	24 01	58
59	57 23	14 78	56 47	17 89	55 55	21 11	54 51	24 43	59
60	58 20	15 03	57 42	18 20	56 49	21 47	55 43	24 85	60
61	59 17	15 28	58 38	18 50	57 43	21 83	56 35	25 26	61
62	60 14	15 53	59 33	18 81	58 37	22 19	57 28	25 68	62
63	61 11	15 78	60 29	19 11	59 32	22 54	58 20	26 09	63
64	62 08	16 03	61 25	19 41	60 26	22 90	59 13	26 51	64
65	63 05	16 28	62 21	19 72	61 20	23 26	60 05	26 92	65
66	64 02	16 53	63 16	20 02	62 14	23 62	60 97	27 34	66
67	64 99	16 78	64 12	20 32	63 08	23 97	61 90	27 75	67
68	65 96	17 03	65 08	20 63	64 02	24 33	62 82	28 16	68
69	66 93	17 28	66 04	20 93	64 96	24 69	63 74	28 58	69
70	67 90	17 53	66 99	21 23	65 91	25 05	64 67	28 99	70
71	68 87	17 79	67 94	21 53	66 85	25 41	65 59	29 40	71
72	69 84	18 04	68 89	21 84	67 79	25 77	66 52	29 82	72
73	70 81	18 29	69 85	22 14	68 74	26 12	67 44	30 23	73
74	71 78	18 54	70 81	22 44	69 68	26 48	68 37	30 65	74
75	72 75	18 79	71 77	22 75	70 62	26 84	69 29	31 06	75
76	73 72	19 04	72 72	23 05	71 56	27 20	70 21	31 48	76
77	74 69	19 29	73 68	23 35	72 50	27 55	71 14	31 89	77
78	75 66	19 54	74 64	23 66	73 44	27 91	72 06	32 30	78
79	76 63	19 79	75 60	23 97	74 38	28 27	72 99	32 72	79
80	77 60	20 04	76 56	24 27	75 32	28 62	73 91	33 14	80
81	78 57	20 29	77 52	24 57	76 26	28 99	74 83	33 55	81
82	79 54	20 54	78 47	24 87	77 20	29 35	75 76	33 96	82
83	80 51	20 79	79 43	25 17	78 15	29 70	76 68	34 37	83
84	81 48	21 04	80 39	25 47	79 09	30 06	77 61	34 79	84
85	82 45	21 29	81 35	25 78	80 03	30 42	78 53	35 20	85
86	83 42	21 54	82 30	26 08	80 97	30 78	79 45	35 62	86
87	84 39	21 79	83 26	26 38	81 91	31 13	80 38	36 03	87
88	85 36	22 04	84 22	26 69	82 85	31 49	81 30	36 44	88
89	86 33	22 29	85 18	26 99	83 79	31 85	82 33	36 86	89
90	87 30	22 54	86 13	27 30	84 74	32 20	83 15	37 28	90
91	88 27	22 79	87 09	27 60	85 68	32 57	84 07	37 69	91
92	89 24	23 05	88 04	27 91	86 62	32 93	85 00	38 10	92
93	90 21	23 30	89 00	28 21	87 57	33 28	85 92	38 51	93
94	91 18	23 55	89 96	28 51	88 51	33 66	86 85	38 93	94
95	92 15	23 80	90 92	28 82	89 45	34 00	87 77	39 34	95
96	93 12	24 05	91 87	29 12	90 39	34 36	88 69	39 76	96
97	94 09	24 30	92 83	29 42	91 33	34 72	89 62	40 17	97
98	95 06	24 55	93 79	29 73	92 27	35 07	90 54	40 58	98
99	96 03	24 80	94 75	30 03	93 21	35 43	91 47	41 00	99
100	97 00	25 05	95 70	30 33	94 16	35 78	92 39	41 42	100
	D.La.	D.Lo.	D.La.	D.Lo.	D.La.	D.Lo.	D.La.	D.Lo.	
	1 ¼ Point.		1 ½ Point.		1 ¾ Point.		1 Point.		

	2¼ Points.		2½ Points.		2¾ Points.		3 Points.		
	D.La.	D.Lo.	D.La.	D.Lo.	D.La.	D.Lo.	D.La.	D.Lo.	
1	00 90	00 47	00 88	00 53	00 86	00 60	00 83	00 67	1
2	01 81	00 95	01 76	01 07	01 72	01 20	01 65	01 34	2
3	02 71	01 42	02 65	01 60	02 57	01 80	02 49	02 00	3
4	03 62	01 89	03 53	02 14	03 43	02 40	03 33	02 67	4
5	04 52	02 36	04 41	02 67	04 29	03 00	04 16	03 34	5
6	05 42	02 84	05 29	03 21	05 15	03 60	04 99	04 01	6
7	06 33	03 31	06 17	03 74	06 00	04 20	05 82	04 68	7
8	07 23	03 78	07 06	04 28	06 86	04 80	06 65	05 35	8
9	08 14	04 26	07 94	04 81	07 72	05 39	07 48	06 01	9
10	09 04	04 73	08 82	05 35	08 58	05 99	08 31	06 68	10
11	09 94	05 20	09 70	05 88	09 44	06 59	09 14	07 35	11
12	10 85	05 68	10 58	06 42	10 30	07 19	09 97	08 02	12
13	11 75	06 15	11 47	06 95	11 15	07 79	10 80	08 68	13
14	12 66	06 62	12 35	07 49	12 01	08 39	11 64	09 35	14
15	13 56	07 09	13 23	08 02	12 87	08 99	12 47	10 02	15
16	14 46	07 57	14 11	08 56	13 73	09 59	13 30	10 69	16
17	15 37	08 04	14 99	09 09	14 58	10 19	14 13	11 36	17
18	16 27	08 51	15 88	09 63	15 44	10 79	14 96	12 03	18
19	17 18	08 99	16 76	10 16	16 30	11 39	15 79	12 69	9
20	18 08	09 46	17 64	10 69	17 15	11 99	16 63	13 36	20
21	18 98	09 93	18 52	11 22	18 01	12 58	17 46	14 03	21
22	19 89	10 41	19 40	11 75	18 87	13 18	18 29	14 70	22
23	20 79	10 88	20 29	12 29	19 72	13 78	19 12	15 36	23
24	21 70	11 35	21 17	12 83	20 58	14 38	19 96	16 03	24
25	22 60	11 82	22 05	13 36	21 44	14 98	20 79	16 70	25
26	23 50	12 30	22 93	13 90	22 30	15 58	21 62	17 37	26
27	24 41	12 77	23 81	14 43	23 15	16 18	22 45	18 04	27
28	25 31	13 24	24 70	14 97	24 01	16 78	23 28	18 71	28
29	26 22	13 72	25 58	15 50	24 87	17 38	24 11	19 37	29
30	27 12	14 19	26 46	16 04	25 73	17 98	24 94	20 04	30
31	28 02	14 66	27 34	16 57	26 59	18 57	25 77	20 71	31
32	28 93	15 14	28 22	17 11	27 45	19 17	26 60	21 38	32
33	29 83	15 61	29 11	17 64	28 30	19 77	27 43	22 05	33
34	30 74	16 08	29 99	18 18	29 16	20 37	28 27	22 72	34
35	31 64	16 55	30 87	18 71	30 02	20 97	29 10	23 39	5
36	32 54	17 03	31 75	19 25	30 88	21 57	29 93	24 06	36
37	33 45	17 50	32 63	19 78	31 73	22 17	30 76	24 73	37
38	34 35	17 97	33 52	20 32	32 59	22 77	31 59	25 40	38
39	35 26	18 45	34 40	20 85	33 45	23 37	32 42	26 06	39
40	36 16	18 92	35 28	21 38	34 31	23 97	33 26	26 73	40
41	37 06	19 39	36 16	21 91	35 17	24 57	34 09	27 40	41
42	37 97	19 87	37 04	22 45	36 02	25 17	34 92	28 07	42
43	38 87	20 34	37 92	22 98	36 88	25 77	35 76	28 73	43
44	39 78	20 81	38 80	23 52	37 74	26 37	36 59	29 40	44
45	40 68	21 28	39 68	24 05	38 60	26 97	37 42	30 0	45
46	41 58	21 76	40 56	24 59	39 45	27 57	38 25	30 7[illegible]	46
47	42 49	22 23	41 44	25 12	40 31	28 17	39 08	31 41	47
48	43 39	22 70	42 33	25 66	41 17	28 77	39 91	32 07	48
49	44 30	23 18	43 21	26 19	42 03	29 37	40 74	32 74	49
50	45 20	23 65	44 10	26 73	42 89	29 97	41 57	33 41	50
	D.La.	D.Lo.	D.La.	D.Lo.	D.La.	D.Lo.	D.La.	D. Lo.	
	2¼ Points.		2½ Points.		2¾ Points.		3 Points.		

	2¼ Points.		2½ Points.		2¾ Points.		3 Points.		
	D.La.	D. Lo.	D.La.	D. Lo	D.La.	D. Lo.	D.La.	D.Lo.	
51	46 10	24 12	44 98	27 26	43 75	30 56	42 40	34 08	51
52	47 01	24 60	45 86	27 80	44 61	31 16	43 23	34 74	52
53	47 91	25 07	46 75	28 33	45 46	31 76	44 06	35 41	53
54	48 82	25 54	47 63	28 87	46 32	32 36	44 90	36 08	54
55	49 72	26 01	48 51	29 40	47 18	32 96	45 73	36 75	55
56	50 62	26 49	49 39	29 94	48 04	33 56	46 56	37 42	56
57	51 53	26 96	50 27	30 47	48 89	34 16	47 39	38 09	57
58	52 43	27 44	51 15	31 01	49 75	34 76	48 22	38 76	58
59	53 34	27 91	52 03	31 54	50 61	35 36	49 05	39 42	59
60	54 24	28 38	52 91	32 07	51 46	35 96	49 89	40 09	60
61	55 14	28 85	53 79	32 60	52 32	36 56	50 72	40 75	61
62	56 05	29 33	54 67	33 14	53 18	37 16	51 55	41 42	62
63	56 95	29 80	55 56	33 67	54 04	37 76	52 39	42 09	63
64	57 85	30 27	56 44	34 21	54 89	38 36	53 22	42 76	64
65	58 76	30 74	57 32	34 74	55 75	38 96	54 05	43 43	65
66	59 66	31 22	58 20	35 28	56 61	39 56	54 88	44 10	66
67	60 57	31 69	59 08	35 81	57 46	40 16	55 71	44 77	67
68	61 47	32 16	59 97	36 35	58 32	40 76	56 54	45 44	68
69	62 38	32 64	60 85	36 88	59 18	41 36	57 37	46 10	69
70	63 28	33 11	61 73	37 42	60 04	41 96	58 20	46 77	70
71	64 18	33 58	62 61	37 95	60 90	42 55	59 03	47 43	71
72	65 09	34 06	63 49	38 49	61 76	43 15	59 86	48 10	72
73	65 99	34 53	64 38	39 02	62 61	43 75	60 69	48 77	73
74	66 90	35 00	65 26	39 56	63 47	44 35	61 53	49 44	74
75	67 80	35 47	66 14	40 09	64 33	44 95	62 36	50 11	75
76	68 70	35 95	67 02	40 63	65 19	45 55	63 19	50 78	76
77	69 61	36 42	67 90	41 16	66 04	46 15	64 02	51 45	77
78	70 51	36 89	68 79	41 70	66 90	46 75	64 85	52 12	78
79	71 42	37 37	69 67	42 23	67 76	47 35	65 68	52 78	79
80	72 32	37 84	70 55	42 76	68 62	47 94	66 52	53 45	80
81	73 22	38 31	71 43	43 29	69 48	48 54	67 35	54 11	81
82	74 13	38 79	72 31	43 83	70 34	49 14	68 18	54 78	82
83	75 03	39 26	73 20	44 36	71 19	49 74	69 01	55 45	83
84	75 94	39 73	74 08	44 90	72 05	50 34	69 85	56 12	84
85	76 84	40 20	74 96	45 43	72 91	50 94	70 68	56 79	85
86	77 74	40 68	75 81	45 97	73 77	51 54	71 51	57 46	86
87	78 65	41 15	76 72	46 50	74 62	52 14	72 34	58 13	87
88	79 56	41 62	77 61	46 04	75 48	52 74	73 17	58 80	88
89	80 46	42 10	78 49	47 57	76 34	53 34	74 00	59 47	89
90	81 36	42 57	79 37	48 11	77 20	53 94	74 83	60 14	90
91	82 26	43 04	80 25	48 64	78 06	54 54	75 66	60 80	91
92	83 17	43 52	81 13	49 18	78 92	55 14	76 49	61 47	92
93	84 07	43 99	82 02	49 71	79 77	55 74	77 32	62 14	93
94	84 98	44 47	82 90	50 25	80 63	56 34	78 16	62 81	94
95	85 88	44 93	83 78	50 78	81 49	56 94	78 99	63 48	95
96	86 78	45 41	84 66	51 32	82 35	57 54	79 82	64 15	96
97	87 69	45 88	85 54	51 85	83 20	58 14	80 65	64 81	97
98	88 59	46 35	86 42	52 39	84 06	58 74	81 48	65 48	98
99	89 50	46 83	87 30	52 92	84 92	59 34	82 31	66 15	99
100	90 40	47 30	88 18	53 4	85 77	59 94	83 15	66 82	100
	D.La.	D Lo	D.La.	D Lo.	D.La.	D. Lo	D.La.	D. Lo.	
	2¼ Points.		2½ Points.		2¾ Points.		3 Points.		

	3¼ Points.		3½ Points.		3¾ Points.		4 Points.		
	D.La	D.Lo	D.La	D.Lo	D.La.	D. Lo.	D.La.	D Lo.	
1	00 80	00 74	00 77	00 82	00 74	00 91	0 [illegible]	01 00	1
2	01 61	01 48	01 55	01 64	01 48	01 81	01 41	02 00	2
3	02 41	02 22	02 32	02 46	02 22	02 72	02 12	03 00	3
4	03 21	02 97	03 09	03 28	02 96	03 63	02 83	04 00	4
5	04 02	03 71	03 86	04 10	03 70	04 53	03 53	05 00	5
6	04 82	04 45	04 64	04 92	04 45	05 44	04 24	06 00	6
7	05 62	05 19	05 41	05 74	05 19	06 34	04 95	07 00	7
8	06 43	05 93	06 18	06 57	05 93	07 25	05 66	08 00	8
9	07 23	06 67	06 96	07 39	06 67	08 16	06 36	09 00	9
10	08 03	07 42	07 73	08 21	07 41	09 06	07 07	0 00	10
11	08 83	08 16	08 50	09 02	08 15	09 97	07 78	11 00	11
12	09 64	09 90	09 28	09 84	08 89	10 87	08 48	12 00	12
13	10 44	10 64	10 05	10 66	09 63	11 78	09 19	13 00	13
14	11 24	10 39	10 82	11 48	10 37	12 69	09 90	14 00	14
15	12 05	11 13	11 59	12 30	11 11	13 59	10 60	15 00	15
16	12 85	11 87	12 37	13 12	11 86	14 50	11 31	16 00	16
17	13 65	12 61	13 14	13 94	12 60	15 40	12 02	17 00	17
18	14 46	13 35	13 91	14 77	13 34	16 31	12 73	18 00	18
19	15 26	14 09	14 69	15 59	14 08	17 22	13 43	19 00	19
20	16 06	14 83	15 46	16 41	14 82	18 13	14 14	20 00	20
21	16 86	15 58	16 23	17 22	15 56	19 04	14 85	21 00	21
22	17 67	16 32	17 01	18 04	16 30	19 94	15 55	22 00	22
23	18 47	17 06	17 78	18 86	17 04	20 85	16 26	23 00	23
24	19 27	17 81	18 55	19 68	17 78	21 76	16 97	24 00	24
25	20 08	18 55	19 32	20 50	18 52	22 66	17 67	25 00	25
26	20 88	19 29	20 10	21 32	19 27	23 57	18 38	26 00	26
27	21 68	20 03	20 87	22 14	20 01	24 47	19 09	27 00	27
28	22 48	20 77	21 64	23 97	20 75	25 38	19 80	28 00	28
29	23 28	21 51	22 42	23 80	21 49	26 29	20 50	29 00	29
30	24 08	22 25	23 19	24 62	22 23	27 19	21 21	30 00	30
31	24 89	22 99	23 96	25 44	22 97	28 10	21 92	31 00	31
32	25 69	23 73	24 74	26 26	23 71	29 00	22 62	32 00	32
33	26 50	24 47	25 51	27 08	24 45	29 91	23 33	33 00	33
34	27 30	25 22	26 28	27 90	25 19	30 82	24 04	34 00	34
35	28 11	25 96	27 05	28 72	25 93	31 72	24 74	35 00	35
36	28 91	26 70	27 83	29 54	26 68	32 63	25 45	36 00	36
37	29 71	27 44	28 60	30 36	27 42	33 53	26 16	37 00	37
38	30 51	28 18	29 37	31 19	28 16	34 44	26 87	38 00	38
39	31 31	28 92	30 15	32 01	28 90	35 35	27 57	39 00	39
40	32 12	29 67	30 92	32 82	29 64	36 25	28 28	40 00	40
41	32 92	30 41	31 69	33 65	30 38	37 16	28 99	41 00	41
42	33 73	31 15	32 47	34 47	31 12	38 06	29 69	42 00	42
43	34 53	31 89	33 24	35 29	31 86	38 97	30 40	43 00	43
44	35 33	32 64	34 01	36 11	32 60	39 88	31 11	44 00	44
45	36 14	33 38	34 78	36 93	33 34	40 78	31 81	45 00	45
46	36 94	34 12	35 56	37 75	34 09	41 69	32 52	46 00	46
47	37 74	34 86	36 33	38 57	34 83	42 59	33 23	47 00	47
48	38 54	35 60	37 10	39 40	35 57	43 50	33 94	48 00	48
49	39 35	35 34	37 88	40 22	36 31	44 4	34 64	49 00	49
50	40 16	37 08	38 65	41 03	37 05	45 32	35 35	50 00	50
	D.La.	D.Lo.	D.La	D.Lo.	D La	D Lo.	D.La	D. Lo.	
	3¼ Points.		3½ Points.		3¾ Points.		4 Points.		

	3¼ Points.		3½ Points.		3¾ Points.		4 Points.		
	D.La.	D. Lo.	D.La.	D. Lo	D.La.	D. Lo.	D.La.	D.Lo.	
51	40 96	37 82	39 42	41 85	37 79	46 23	36 06	51 00	51
52	41 77	38 57	40 20	42 67	38 53	47 13	36 76	52 00	52
53	42 57	39 31	40 97	43 49	39 27	48 04	37 46	53 00	53
54	43 37	40 06	41 74	44 31	40 01	48 95	38 17	54 00	54
55	44 18	40 80	42 51	45 13	40 75	49 85	38 88	55 00	55
56	44 98	41 54	43 29	45 95	41 50	50 76	39 59	56 00	56
57	45 78	42 28	44 06	46 77	42 24	51 66	40 30	57 00	57
58	46 58	43 02	44 83	47 60	43 97	52 57	41 01	58 00	58
59	47 39	43 76	45 61	48 42	43 72	53 48	41 71	59 00	59
60	48 19	44 50	46 38	49 24	44 46	54 38	42 43	60 00	60
61	48 99	45 24	47 15	50 06	45 20	55 29	43 14	61 00	61
62	49 80	45 98	47 93	50 88	45 94	56 19	43 84	62 00	62
63	50 60	46 72	48 70	51 70	46 68	57 10	44 55	63 00	63
64	51 40	47 47	49 47	52 52	47 42	58 01	45 26	64 00	64
65	52 21	48 21	50 25	53 34	48 16	58 91	45 96	65 00	65
66	53 01	48 95	51 02	54 16	48 90	59 82	46 67	66 00	66
67	53 81	49 69	51 79	54 98	49 64	60 72	47 38	67 00	67
68	54 62	50 43	52 56	55 81	50 38	61 63	48 09	68 00	68
69	55 42	51 17	53 34	56 63	51 12	62 54	48 79	69 00	69
70	56 23	51 92	54 11	57 45	51 86	63 44	49 50	70 00	70
71	57 03	52 66	54 88	58 27	52 61	64 35	50 21	71 00	71
72	57 84	53 40	55 66	59 09	53 35	65 25	50 92	72 00	72
73	58 64	54 14	56 43	59 91	54 09	66 16	51 62	73 00	73
74	59 44	54 9	57 20	60 73	54 83	67 07	52 33	74 00	74
75	60 25	55 63	57 97	61 55	55 57	67 97	53 03	75 00	75
76	61 05	56 37	58 75	62 37	56 32	68 88	53 74	76 00	76
77	61 85	57 11	59 52	63 19	57 06	69 78	54 45	77 00	77
78	62 66	57 85	60 29	64 02	57 80	70 69	55 16	78 00	78
79	63 46	58 59	61 07	64 84	58 54	71 60	55 86	79 00	79
80	64 26	59 33	61 84	65 65	59 28	72 50	56 57	80 00	80
81	65 06	60 07	62 61	66 47	60 02	73 41	57 28	81 00	81
82	65 87	60 82	63 39	67 29	60 76	74 32	57 98	82 00	82
83	66 67	61 56	64 16	68 11	61 50	75 23	58 69	83 00	83
84	67 47	62 31	64 93	68 93	62 24	76 14	59 40	84 00	84
85	68 28	63 05	65 70	69 75	62 98	77 04	60 10	85 00	85
86	69 08	63 79	66 48	70 57	63 73	77 95	60 81	86 00	86
87	69 88	64 53	67 25	71 39	64 47	78 85	61 52	87 00	87
88	70 69	65 27	68 02	72 22	65 21	79 76	62 23	88 00	88
89	71 49	66 01	68 80	73 04	65 95	80 67	62 93	89 00	89
90	72 29	66 75	69 57	73 86	66 69	81 57	63 64	90 00	90
91	73 09	67 49	70 34	74 68	67 43	82 48	64 35	91 00	91
92	73 90	68 23	71 12	75 50	68 17	83 38	65 05	92 00	92
93	74 70	68 97	71 89	76 32	68 91	84 29	65 76	93 00	93
94	75 50	69 72	72 66	77 14	69 65	85 20	66 47	94 00	94
95	76 31	70 46	73 43	77 96	70 39	86 11	67 17	95 00	95
96	77 11	71 20	74 21	78 78	71 14	87 01	67 88	96 00	96
97	77 91	71 94	74 98	79 60	71 88	87 91	68 59	97 00	97
98	78 72	72 68	75 75	80 43	72 62	88 82	69 30	98 00	98
99	79 52	73 42	76 73	81 25	73 37	89 73	70 00	99 00	99
100	80 32	74 17	77 30	82 07	74 10	90 63	70 71	100 00	100
	D.La.	D Lo.	D.La.	D Lo.	D La.	D. Lo	D.La.	D Lo.	
	3¼ Points.		3½ Points.		3¾ Points.		4 Points.		

	4 ¼ Point.		4 ½ Point.		4 ¾ Point.		5 Points.		
	D.La.	D. Lo	D.La.	D. Lo.	D.La.	D. Lo.	D.La.	D.Lo.	
1	00 67	01 10	00 63	01 22	00 60	01 35	00 56	01 50	1
2	01 34	02 21	01 27	02 44	01 19	02 70	01 11	02 99	2
3	02 01	03 31	01 90	03 66	01 79	04 05	01 67	04 49	3
4	02 69	04 41	02 54	04 87	02 38	05 39	02 22	05 99	4
5	03 36	05 52	03 17	06 09	02 98	06 7	02 78	07 48	5
6	04 03	06 62	03 81	07 31	03 57	08 09	03 33	08 98	6
7	04 70	07 72	04 44	08 53	04 17	09 44	03 88	10 48	7
8	05 37	08 83	05 07	09 75	04 77	10 79	04 44	11 97	8
9	06 04	09 93	05 71	10 97	05 36	12 14	05 00	13 47	9
10	06 72	11 03	06 34	12 19	05 96	13 48	05 56	14 97	10
11	07 39	12 13	06 97	13 41	06 56	14 83	06 12	16 47	11
12	08 06	13 24	07 61	14 63	07 15	16 18	06 67	17 96	12
13	08 73	14 34	08 24	15 85	07 75	17 53	07 23	19 46	13
14	09 41	15 44	08 88	17 06	08 34	18 87	07 78	20 96	14
15	10 08	16 55	09 51	18 28	08 94	20 22	08 34	22 45	15
16	10 75	17 65	10 15	19 50	09 53	21 57	08 89	23 95	16
17	11 42	18 75	10 78	20 72	10 13	22 92	09 45	25 45	17
18	12 09	19 86	11 41	21 94	10 73	24 27	10 00	26 94	18
19	12 76	20 96	12 05	23 16	11 32	25 62	10 56	28 44	19
20	13 43	22 07	12 69	24 37	11 91	26 97	11 11	29 94	20
21	14 10	23 17	13 32	25 59	12 51	28 32	11 67	31 44	21
22	14 77	24 28	13 96	26 81	13 10	29 67	12 22	32 93	22
23	15 44	25 38	14 59	28 03	13 70	31 02	12 78	34 42	23
24	16 12	26 48	15 23	29 24	14 29	32 36	13 33	35 92	24
25	16 79	27 59	15 86	30 46	14 89	33 71	13 89	37 41	25
26	17 46	28 69	16 50	31 68	15 48	35 06	14 44	38 91	26
27	18 13	29 79	17 13	32 90	16 08	36 41	15 00	40 41	27
28	18 80	30 90	17 76	34 12	16 68	37 76	15 55	41 91	28
29	19 47	32 00	18 40	35 34	17 27	39 11	16 11	43 41	29
30	20 15	33 10	19 03	36 56	17 87	40 45	16 67	44 90	30
31	20 82	34 20	19 66	37 78	18 47	41 80	17 23	46 40	31
32	21 49	35 31	20 30	39 00	19 06	43 15	17 78	47 89	32
33	22 16	36 41	20 93	40 22	19 66	44 50	18 34	49 39	33
34	22 84	37 51	21 57	41 43	20 25	45 84	18 89	50 89	34
35	23 51	38 62	22 20	42 65	20 85	47 19	19 45	52 38	35
36	24 18	39 72	22 83	43 87	21 44	48 54	20 00	53 88	36
37	24 85	40 82	23 46	45 09	22 04	49 89	20 56	55 38	37
38	25 52	41 93	24 10	46 31	22 64	50 24	21 11	56 87	38
39	26 19	43 03	24 74	47 53	23 23	51 59	21 67	58 37	39
40	26 86	44 13	25 38	48 74	23 83	53 93	22 22	59 86	40
41	27 53	45 23	26 01	49 96	24 43	55 28	22 78	61 36	41
42	28 20	46 34	26 65	51 18	25 02	56 63	23 33	62 85	42
43	28 87	47 44	27 28	52 40	25 62	57 98	23 89	64 35	43
44	29 55	48 54	27 92	53 61	26 21	59 32	24 45	65 85	44
45	30 22	49 65	28 55	54 83	26 81	60 67	25 00	67 34	45
46	30 88	50 75	29 19	56 05	27 40	62 02	25 55	68 84	46
47	31 56	51 85	29 82	57 27	28 00	63 37	26 11	70 34	47
48	32 23	52 96	30 45	58 49	28 60	64 72	26 66	71 83	48
49	32 90	54 06	31 09	59 71	29 19	66 07	27 22	73 33	49
50	33 58	55 17	31 72	60 93	29 78	67 42	27 78	74 83	50
	D.La.	D.Lo.	D.La.	D. Lo.	D.La.	D. Lo.	D.La.	D.Lo.	
	4 ¼ Point.		4 ½ Point.		4 ¾ Point		5 Points.		

	4¼ Points.		4½ Points.		4¾ Points.		5 Points.		
	D.La.	D. Lo	D.La.	D. Lo	D.La.	D.Lo.	D.La.	D.Lo	
51	34 25	56 27	32 35	62 15	30 38	68 77	28 34	76 33	51
52	34 92	57 38	32 99	63 37	30 97	70 12	28 89	77 82	52
53	35 59	58 48	33 62	64 59	31 57	71 47	29 45	79 32	53
54	36 27	59 58	34 26	65 80	32 16	72 81	30 00	80 82	54
55	36 94	60 69	34 89	67 02	32 76	74 16	30 56	82 31	55
56	37 61	61 79	35 53	68 24	33 35	75 51	31 11	83 81	56
57	38 28	62 89	36 16	69 46	33 95	76 86	31 67	85 31	57
58	38 95	64 00	36 79	70 68	34 55	78 21	32 22	86 80	58
59	39 62	65 10	37 43	71 90	35 14	79 56	32 78	88 30	59
60	40 29	66 20	38 06	73 11	35 74	80 90	33 33	89 80	60
61	40 96	67 30	38 69	74 33	36 34	82 25	33 89	91 30	61
62	41 63	68 41	39 33	75 55	36 93	83 60	34 44	92 79	62
63	42 30	69 51	39 96	76 77	37 53	84 95	35 00	94 29	63
64	42 98	70 61	40 60	77 98	38 12	86 29	35 55	95 79	64
65	43 65	71 72	41 23	79 20	38 72	87 64	36 11	97 28	65
66	44 32	72 82	41 87	80 42	39 31	88 99	36 66	98 78	66
67	44 99	73 92	42 50	81 64	39 91	90 34	37 22	100 28	67
68	45 66	75 03	43 13	82 86	40 51	91 69	37 77	101 77	68
69	46 33	76 13	43 77	84 08	41 10	93 04	38 33	103 27	69
70	47 01	77 23	44 41	85 30	41 70	94 38	38 89	104 77	70
71	47 68	78 33	45 04	86 52	42 30	95 73	39 45	106 25	71
72	48 35	79 44	45 68	87 74	42 89	97 08	40 00	107 75	72
73	49 02	80 54	46 31	88 96	43 49	98 43	40 56	109 25	73
74	49 70	81 64	46 95	90 17	44 08	99 77	41 11	110 75	74
75	50 37	82 75	47 58	91 39	44 68	101 12	41 67	112 24	75
76	51 04	83 85	48 22	92 61	45 27	102 47	42 22	113 74	76
77	51 71	84 95	48 85	93 83	45 87	103 82	42 78	115 24	77
78	52 38	86 06	49 48	95 05	46 47	105 17	43 33	116 73	78
79	53 05	87 16	50 12	96 27	47 06	106 52	43 89	118 23	79
80	53 72	88 27	50 75	97 48	47 66	107 87	44 45	119 73	80
81	54 39	89 37	51 38	98 70	48 26	109 22	45 00	121 23	81
82	55 06	90 48	52 02	99 92	48 85	110 57	45 55	122 72	82
83	55 73	91 58	52 65	101 14	49 45	111 92	46 11	124 22	83
84	56 41	92 68	53 29	102 35	50 04	113 27	46 66	125 72	84
85	57 08	93 79	53 92	103 57	50 64	114 61	47 22	127 21	85
86	57 75	94 89	54 56	104 79	51 23	115 96	47 77	128 71	86
87	58 42	95 99	55 19	106 01	51 83	117 31	48 33	130 21	87
88	59 09	97 10	55 82	107 23	52 43	118 66	48 88	131 71	88
89	59 76	98 20	56 46	108 45	53 02	120 01	49 44	133 20	89
90	60 44	99 30	57 10	109 67	53 61	121 35	50 00	134 69	90
91	61 11	100 40	57 73	110 89	54 21	122 70	50 56	136 19	91
92	61 78	101 51	58 37	112 11	54 80	124 05	51 11	137 68	92
93	62 45	102 61	59 00	113 33	55 40	125 40	51 67	139 18	93
94	63 13	103 72	59 64	114 54	55 99	126 74	52 22	140 68	94
95	63 80	104 82	60 27	115 76	56 59	128 09	52 78	142 17	95
96	64 47	105 92	60 91	116 98	57 18	129 44	53 33	143 67	96
97	65 14	107 02	61 54	118 20	57 78	130 79	53 89	145 17	97
98	65 81	108 13	62 17	119 42	58 38	132 14	54 44	146 66	98
99	66 48	109 23	62 81	120 64	58 97	133 49	55 00	138 16	99
100	67 16	110 33	63 44	121 85	59 57	134 83	55 56	149 66	100
	D.La.	D Lo.	D.La.	D Lo.	D.La.	D.Lo	D.La.	D. Lo.	
	4¼ Points.		4½ Points.		4¾ Points.		5 Points.		

	5 ¼ Points.		5 ½ Points.		5 ¾ Points.		6 Points.		
	D.La.	D.Lo.	D.La.	D Lo	D.La.	D.Lo.	D.La.	D.Lo.	
1	00 51	01 67	00 47	01 87	00 43	02 11	00 38	02 41	1
2	01 03	03 34	00 94	03 74	00 86	04 23	00 77	04 83	2
3	01 54	05 01	01 41	05 61	01 28	06 34	01 15	07 24	3
4	02 06	06 67	01 89	07 48	01 71	08 46	01 53	09 66	4
5	02 57	08 34	02 36	09 35	02 14	10 57	01 91	12 07	5
6	03 08	10 01	02 83	11 23	02 57	12 69	02 30	14 49	6
7	03 60	11 68	03 30	13 10	02 99	14 80	02 68	16 90	7
8	04 11	13 35	03 77	14 97	03 42	16 91	03 06	19 31	8
9	04 63	15 02	04 24	16 84	03 85	19 03	03 44	21 73	9
10	05 14	16 68	04 71	18 71	04 28	21 14	03 83	24 14	10
11	05 65	18 35	05 18	20 58	04 71	23 25	04 21	26 55	11
12	06 17	20 02	05 65	22 45	05 14	25 37	04 60	28 97	12
13	06 68	21 69	06 12	24 32	05 56	27 48	04 98	31 38	13
14	07 20	23 35	06 60	26 19	05 99	29 60	05 36	33 80	14
15	07 71	25 02	07 07	28 06	06 42	31 71	05 74	36 21	15
16	08 22	26 69	07 54	29 94	06 85	33 83	06 13	38 63	16
17	08 74	28 36	08 01	31 81	07 27	35 95	06 51	41 04	17
18	09 25	30 03	08 48	33 68	07 70	38 06	06 89	43 45	18
19	09 77	31 70	08 95	35 55	08 13	40 17	07 27	45 87	19
20	10 28	33 37	09 43	37 42	08 55	42 29	07 65	48 28	20
21	10 79	35 04	09 90	39 29	08 98	44 40	08 03	50 69	21
22	11 31	36 71	10 37	41 16	09 41	46 52	08 42	53 11	22
23	11 82	38 38	10 84	43 03	09 83	48 63	08 80	55 52	23
24	12 34	40 04	11 32	44 90	10 26	50 75	09 18	57 94	24
25	12 85	41 71	11 79	46 77	10 69	52 86	09 56	60 35	25
26	3 36	43 38	12 26	48 65	11 12	54 98	09 95	62 77	26
27	13 88	45 05	12 73	50 52	11 54	57 09	10 33	65 18	27
28	14 39	46 72	13 20	52 39	11 97	59 20	10 71	67 59	28
29	14 91	48 39	13 67	54 26	12 40	61 32	11 09	70 01	29
30	15 42	50 05	14 14	56 13	12 83	63 43	11 48	72 43	30
31	15 93	51 72	14 61	58 00	13 26	65 54	11 86	74 84	31
32	16 45	53 39	15 08	59 87	13 69	67 66	12 25	77 26	32
33	16 96	55 06	15 55	61 74	14 11	69 77	12 63	79 67	33
34	17 48	56 72	16 03	63 61	14 54	71 89	13 01	82 09	34
35	17 99	58 39	16 50	65 48	14 97	74 00	13 39	84 50	35
36	18 50	60 06	16 97	67 35	15 40	76 12	13 78	86 92	36
37	19 02	61 73	17 44	69 22	15 82	78 23	14 16	89 33	37
38	19 53	63 40	17 91	71 09	16 25	80 34	14 54	91 74	38
39	20 05	65 07	18 38	72 96	16 68	82 46	14 92	94 16	39
40	20 56	66 74	18 86	74 83	17 10	84 57	15 31	96 57	40
41	21 07	68 41	19 33	76 70	17 53	86 68	15 69	98 98	41
42	21 59	70 08	19 80	78 57	17 96	88 80	16 08	101 40	42
43	22 10	71 75	20 27	80 44	18 28	90 91	16 46	103 81	43
44	22 62	73 41	20 75	82 31	18 81	93 03	16 84	106 23	44
45	23 13	75 08	21 22	84 18	19 24	95 14	17 22	108 64	45
46	23 64	76 75	21 69	86 06	19 67	97 26	17 61	111 06	46
47	24 16	78 42	22 16	87 93	20 09	99 37	17 99	113 47	47
48	24 67	80 09	22 63	89 80	20 52	101 48	18 37	115 88	48
49	25 19	81 76	23 10	91 67	20 95	103 60	18 75	118 30	49
50	25 71	83 42	23 57	93 54	21 38	105 71	19 14	120 71	50
	D.La	D.Lo.	D.La	D. Lo	D La	D. Lo.	D.La.	D.Lo.	
	5¼ Points.		5½ Points.		5 ¾ Points		6 Points.		

	5¼ Points.		5½ Points.		5¾ Points.		6 Points.		
	D.La.	D. Lo.	D.La.	D. Lo.	D La.	D. Lo.	D.La.	D.Lo	
51	26 23	85 09	24 04	95 41	21 81	107 82	19 52	123 12	51
52	26 74	86 76	24 51	97 28	22 2	109 94	19 91	125 54	52
53	27 25	88 43	24 98	99 15	22 67	112 05	20 29	127 95	53
54	27 77	90 09	25 46	101 02	23 09	114 17	20 67	130 37	54
55	28 28	91 76	25 93	102 89	23 52	116 28	21 05	132 78	55
56	28 79	93 43	26 40	104 77	23 95	118 40	21 44	135 20	56
57	29 30	95 10	26 87	106 64	24 37	120 51	21 82	137 61	57
58	29 82	96 77	27 34	108 51	24 80	122 62	22 20	140 02	58
59	30 34	98 44	27 81	110 38	25 23	124 74	22 57	142 44	59
60	30 85	100 10	28 28	112 25	25 65	126 86	22 96	144 85	60
61	31 3	101 77	28 75	114 12	26 07	128 97	23 34	147 26	61
62	31 88	103 44	29 22	115 99	26 51	131 09	23 73	149 68	62
63	32 39	105 11	29 69	117 86	26 93	133 20	24 11	152 09	63
64	32 91	106 77	30 17	119 73	27 36	135 32	24 49	154 51	64
65	33 42	108 44	30 64	121 60	27 79	137 43	24 87	156 92	65
66	33 93	110 11	31 11	123 48	28 22	139 55	25 26	159 34	66
67	34 45	111 78	31 58	125 35	28 64	141 66	25 64	161 75	67
68	34 96	113 45	32 05	127 23	29 07	143 77	26 02	164 16	68
69	35 48	115 12	32 52	129 10	29 50	145 89	26 40	166 58	69
70	35 99	116 79	33 00	130 96	29 93	148 00	26 79	168 99	70
71	36 50	118 46	33 47	132 83	30 36	150 11	27 17	171 48	71
72	37 02	120 13	33 94	134 70	30 79	152 23	27 56	173 82	72
73	37 53	121 80	34 41	136 57	31 22	154 34	27 94	176 23	73
74	38 05	123 46	34 89	138 44	31 64	156 46	28 32	178 65	74
75	38 56	125 13	35 36	140 31	32 07	158 57	28 70	181 06	75
76	39 07	126 80	35 83	142 19	32 49	160 69	29 09	183 48	76
77	39 59	128 47	36 30	144 06	32 92	162 80	29 47	185 89	77
78	40 10	130 14	36 77	145 93	33 35	164 91	29 85	188 30	78
79	40 62	131 81	37 24	147 80	33 78	167 03	30 23	190 72	79
80	41 13	133 47	37 71	149 67	34 2	169 15	30 62	193 14	80
81	41 64	135 14	38 18	151 54	34 6-	171 27	31 00	195 55	81
82	42 16	136 81	38 65	153 41	35 07	173 38	31 39	197 97	82
83	42 67	138 48	39 12	155 28	35 49	175 49	31 77	200 38	83
84	43 19	140 14	39 60	157 15	35 92	177 61	32 15	202 80	84
85	43 70	141 81	40 07	159 02	36 35	179 72	32 53	205 21	85
86	44 21	143 48	40 54	160 90	36 78	181 84	32 92	207 63	86
87	44 71	145 15	41 02	162 77	37 20	183 95	33 30	210 04	87
88	45 24	146 82	41 48	164 64	37 63	186 06	33 68	212 45	88
89	45 76	148 49	41 95	166 51	38 06	188 18	34 06	214 87	89
90	46 27	150 16	42 42	168 38	38 48	190 29	34 44	217 28	90
91	46 78	151 83	42 89	170 25	38 91	192 40	34 82	219 69	91
92	47 30	153 50	43 36	172 12	39 34	194 52	35 21	222 11	92
93	47 81	155 17	43 83	173 99	39 76	196 63	35 59	224 52	93
94	48 33	156 83	44 31	175 86	40 19	198 75	35 97	226 94	94
95	48 84	158 50	44 78	177 73	40 62	200 86	36 35	229 35	95
96	49 35	160 17	45 25	179 61	41 05	202 98	36 74	231 78	96
97	49 87	161 84	45 72	181 48	41 47	205 09	37 12	234 18	97
98	50 38	163 51	46 19	183 35	41 90	207 20	37 50	236 59	98
99	50 90	165 18	46 66	185 22	42 33	209 32	37 88	239 01	99
100	51 41	166 84	47 14	187 09	42 76	211 43	38 27	241 42	100
	D.La.	D Lo	D.La.	D. Lo.	D.La.	D. Lo	D.La.	D Lo.	
	5¼ Points.		5½ Points.		5¾ Points.		6 Points.		

	6 ¼ Points.		6 ½ Points.		6 ¾ Points.		7 Points.		
	D.La.	D. Lo.	D.La.	D. Lo	D.La.	D. Lo.	D.La.	D.Lo.	
1	00 35	02 79	00 29	03 30	00 24	03 99	00 20	05 03	1
2	00 68	05 59	00 58	06 59	00 49	07 98	00 39	10 05	2
3	01 01	08 38	00 87	09 89	00 73	11 98	00 59	15 08	3
4	01 35	11 18	01 16	13 19	00 97	15 97	00 78	20 11	4
5	01 69	13 97	01 45	16 48	01 22	19 96	00 98	25 14	5
6	02 02	16 77	01 74	19 78	01 46	23 95	01 17	30 16	6
7	02 36	19 56	02 03	23 08	01 70	27 95	01 37	35 19	7
8	02 70	22 36	02 32	26 37	01 94	31 94	01 56	40 22	8
9	03 03	25 15	02 61	29 67	02 19	35 93	01 76	45 25	9
10	03 37	27 95	02 90	32 97	02 43	39 92	01 95	50 27	10
11	03 71	30 74	03 19	36 27	02 67	43 91	02 15	55 30	11
12	04 04	33 54	03 48	39 56	02 92	47 90	02 34	60 32	12
13	04 38	36 33	03 77	42 86	03 16	51 90	02 54	65 35	13
14	04 72	39 13	04 06	46 16	03 40	55 89	02 73	70 38	14
15	05 06	41 92	04 35	49 45	03 65	59 88	02 93	75 41	15
16	05 39	44 72	04 64	52 75	03 89	63 87	03 12	80 43	16
17	05 73	47 51	04 93	56 05	04 13	67 87	03 31	85 46	17
18	06 07	50 31	05 22	59 34	04 37	71 86	03 51	90 49	18
19	06 40	53 10	05 51	62 64	04 62	75 85	03 71	95 52	19
20	06 74	55 90	05 81	65 93	04 85	79 84	03 90	100 55	20
21	07 08	58 69	06 10	69 23	05 10	83 83	04 10	105 58	21
22	07 41	61 49	06 39	72 52	05 35	87 82	04 29	110 60	22
23	07 75	64 28	06 68	75 82	05 59	91 82	04 49	115 63	23
24	08 09	67 08	06 97	79 12	05 83	95 81	04 68	120 66	24
25	08 43	69 87	07 26	82 41	06 08	99 80	04 88	125 69	25
26	08 76	72 67	07 55	85 71	06 32	103 80	05 07	130 71	26
27	09 10	75 46	07 84	89 01	06 56	107 79	05 27	135 74	27
28	09 44	78 26	08 13	92 30	06 80	111 78	05 46	140 77	28
29	09 77	81 05	08 42	95 60	07 05	115 77	05 66	145 80	29
30	10 11	83 84	08 71	98 90	07 26	119 77	05 85	150 82	30
31	10 45	86 63	09 00	102 20	07 53	123 76	06 05	155 85	31
32	10 78	89 43	09 29	105 49	07 78	127 75	06 24	160 87	32
33	11 12	92 22	09 58	108 79	08 02	131 75	06 44	165 90	33
34	11 46	95 02	09 87	112 09	08 26	135 74	06 63	170 93	34
35	11 80	97 82	10 16	115 38	08 51	139 73	06 83	175 96	35
36	12 13	100 61	10 45	118 68	08 75	143 72	07 02	180 98	36
37	12 47	103 40	10 74	121 98	08 99	147 72	07 22	186 01	37
38	12 81	106 20	11 03	125 27	09 23	151 71	07 41	191 04	38
39	13 14	108 99	11 32	128 57	09 48	155 70	07 61	196 07	39
40	13 48	111 79	11 61	131 86	09 72	159 69	07 80	201 09	40
41	13 82	114 58	11 90	135 16	09 96	163 68	08 00	206 12	41
42	14 15	117 38	12 19	138 45	10 21	167 67	08 19	211 14	42
43	14 49	120 17	12 48	141 75	10 45	171 67	08 39	216 17	43
44	14 84	122 97	12 77	145 04	10 69	175 66	08 58	221 20	44
45	15 17	125 76	13 06	148 34	10 94	179 65	08 78	226 23	45
46	15 50	128 56	13 35	151 64	11 18	183 64	08 97	231 25	46
47	15 84	131 35	13 64	154 94	11 42	187 64	09 17	236 28	47
48	16 18	134 15	13 93	158 23	11 66	191 63	09 36	241 31	48
49	16 51	136 94	14 22	161 53	11 91	195 62	09 56	246 34	49
50	16 85	139 74	14 51	164 83	12 15	199 61	09 76	251 37	50
	D.La.	D.Lo.	D.La	D. L o	D.La.	D. Lo.	D.La.	D.Lo.	
	6 ¼ Points.		6 ½ Points.		6 ¾ Points		7 Points.		

	6 ¼ Points.		6 ½ Points.		6 ¾ Points.		7 Points.		
	D.La.	D.Lo.	D.La.	D. Lo.	D La.	D.Lo.	D.La.	D. Lo	
51	17 19	142 53	14 80	168 13	12 39	203 60	09 96	256 40	51
52	17 52	145 33	15 09	171 42	12 64	207 59	10 15	261 42	52
53	17 86	148 12	15 38	174 72	12 88	211 59	10 35	266 45	53
54	18 20	150 92	15 67	178 02	13 12	215 58	10 55	271 48	54
55	18 54	153 71	15 96	181 31	13 37	219 57	10 74	276 51	55
56	18 87	156 51	16 25	184 61	13 61	223 56	10 94	281 53	56
57	19 21	159 30	16 54	287 91	13 85	227 56	11 13	286 56	57
58	19 55	162 10	16 83	191 20	14 09	231 55	11 33	291 59	58
59	19 88	164 89	17 12	194 50	14 34	235 54	11 52	296 62	59
60	20 21	167 69	17 42	197 79	14 58	239 53	11 71	301 64	60
61	20 55	170 48	17 71	201 09	14 82	243 52	11 91	306 67	61
62	20 88	173 28	18 00	204 38	15 07	247 51	12 10	311 69	62
63	21 22	176 07	18 29	207 68	15 31	251 51	12 30	316 72	63
64	21 56	178 87	18 58	210 98	15 55	255 50	12 49	321 75	64
65	21 90	181 67	18 87	214 27	15 80	259 49	12 69	326 78	65
66	22 23	184 46	19 16	217 57	16 04	263 48	12 88	331 80	66
67	22 57	187 25	19 45	220 87	16 28	267 48	13 08	336 83	67
68	22 91	190 05	19 74	224 16	16 52	271 47	13 27	341 86	68
69	23 24	192 84	20 03	227 46	16 77	275 46	13 47	346 89	69
70	23 58	195 64	20 32	230 76	17 01	279 46	13 66	351 91	70
71	23 92	198 43	20 61	234 06	17 25	283 45	13 86	356 94	71
72	24 25	201 23	21 90	237 35	17 50	287 44	14 05	361 96	72
73	24 59	204 02	21 19	240 65	17 74	291 44	14 25	366 99	73
74	24 93	206 82	21 48	243 95	17 98	295 43	14 45	372 02	74
75	25 27	209 61	21 77	247 24	18 23	299 42	14 64	377 05	75
76	25 60	212 41	22 06	250 54	18 47	303 41	14 84	382 07	76
77	25 94	215 20	22 35	253 84	18 71	307 41	15 03	387 10	77
78	26 28	218 00	22 64	257 13	18 95	311 40	15 23	392 13	78
79	26 61	220 79	22 93	260 43	19 20	315 39	15 42	397 16	79
80	26 95	223 58	23 22	263 72	19 44	319 38	15 61	402 19	80
81	27 29	226 37	23 51	267 02	19 68	323 37	15 81	407 22	81
82	27 62	229 17	23 80	270 31	19 93	327 36	16 00	412 24	82
83	27 96	231 96	24 09	273 61	20 17	331 36	16 20	417 27	83
84	28 30	234 76	24 38	276 91	20 41	335 35	16 39	422 30	84
85	28 64	237 55	24 67	280 20	20 66	339 34	16 59	427 33	85
86	28 97	240 35	24 96	283 50	20 90	343 33	16 78	432 35	86
87	29 31	243 14	25 25	286 80	21 14	347 33	16 98	437 38	87
88	29 65	245 94	25 54	290 09	21 38	351 32	17 17	442 41	88
89	29 98	248 73	25 83	293 39	21 63	355 31	17 37	447 44	89
90	30 32	251 53	26 13	296 69	21 87	359 30	17 56	452 46	90
91	30 66	254 32	26 42	299 99	22 11	363 29	17 76	457 49	91
92	30 99	257 12	26 71	303 28	22 36	367 28	17 95	462 51	92
93	31 33	259 91	27 00	306 58	22 60	371 28	18 15	467 54	93
94	31 67	252 71	27 29	309 88	22 84	375 27	18 35	472 57	94
95	32 01	265 50	27 58	313 17	23 09	379 26	18 54	477 60	95
96	32 34	268 30	27 87	316 47	23 33	383 25	18 74	482 62	96
97	32 68	271 10	28 16	319 77	23 57	387 25	18 93	487 65	97
98	33 02	273 89	28 45	323 06	23 81	391 24	19 13	492 68	98
99	33 35	276 68	28 74	326 36	24 06	395 23	19 32	497 71	99
100	33 69	279 48	29 03	329 66	24 30	399 22	19 51	502 73	100
	D.La.	D.Lo.	D.La.	D. Lo.	D.La.	D. Lo.	D.La.	D.Lo.	
	6 ¼ Points.		6 ½ Points.		6 ¾ Points.		7 Points.		

	7 ¼ Points.		7 ½ Points.		7 ¾ Points.		
	D.La	D. Lo	D.La.	D.Lo.	D La.	D. Lo.	
1	00 15	06 74	00 10	10 15	00 05	20 36	1
2	00 29	13 48	00 20	20 30	00 10	40 71	2
3	00 44	20 22	00 29	30 46	00 15	61 07	3
4	00 59	26 96	00 39	40 61	00 20	81 42	4
5	00 73	33 70	00 49	50 76	00 24	101 78	5
6	00 88	40 44	00 59	60 92	00 29	122 13	6
7	01 03	47 18	00 68	71 07	00 34	142 49	7
8	01 17	53 92	00 78	81 23	00 39	162 84	8
9	01 32	60 66	00 88	91 38	00 44	183 20	9
10	01 47	67 40	00 98	101 53	00 49	203 56	10
11	01 62	74 14	01 08	111 68	00 54	223 92	11
12	01 76	80 88	01 18	121 83	00 59	244 27	12
13	01 91	87 62	01 27	131 99	00 64	264 63	13
14	02 06	94 36	01 37	142 14	00 69	284 98	14
15	02 20	101 10	01 47	152 29	00 73	305 34	15
16	02 35	107 84	01 57	162 45	00 78	325 69	16
17	02 50	114 58	01 67	172 60	00 83	346 05	17
18	02 64	121 32	01 76	182 76	00 88	366 40	18
19	02 79	128 06	01 86	192 91	00 93	386 76	19
20	02 93	134 79	01 96	203 06	00 98	407 11	20
21	03 08	141 53	02 06	213 21	01 03	427 47	21
22	03 22	148 27	02 16	223 37	01 08	447 82	22
23	03 37	155 01	02 26	233 52	01 13	468 18	23
24	03 52	161 75	02 35	243 67	01 18	488 53	24
25	03 66	168 49	02 45	253 82	01 22	508 89	25
26	03 81	175 23	02 55	263 98	01 27	529 24	26
27	03 96	181 97	02 65	274 13	01 32	549 60	27
28	04 10	188 71	02 74	284 29	01 37	569 95	28
29	04 25	195 45	02 84	294 44	01 42	590 31	29
30	04 40	202 19	02 94	304 60	01 47	610 67	30
31	04 55	208 93	03 04	314 75	01 52	631 03	31
32	04 69	215 67	03 14	324 90	01 57	651 38	32
33	04 84	222 41	03 24	335 06	01 62	671 74	33
34	04 99	229 15	03 33	345 21	01 67	692 09	34
35	05 13	235 89	03 43	355 36	01 71	712 45	35
36	05 28	242 63	03 53	365 52	01 76	732 80	36
37	05 43	249 37	03 63	375 67	01 81	753 16	37
38	05 57	256 11	03 72	385 83	01 86	773 51	38
39	05 72	262 85	03 82	395 98	01 91	793 86	39
40	05 87	269 58	03 92	406 13	01 96	814 22	40
41	06 02	276 32	04 02	416 28	02 01	834 58	41
42	06 16	283 06	04 12	426 44	02 06	854 93	42
43	06 31	289 80	04 21	436 59	02 11	875 29	43
44	06 46	296 54	04 31	446 74	02 16	895 64	44
45	06 60	293 28	04 41	456 89	02 20	916 00	45
46	06 75	310 02	04 51	467 05	02 25	936 35	46
47	06 90	316 76	04 61	477 20	02 30	956 71	47
48	07 04	323 50	04 70	487 36	02 35	977 06	48
49	07 19	330 24	04 80	497 51	02 40	997 42	49
50	07 24	336 98	04 90	507 66	02 45	1017 78	50
	D.La.	D Lo	D.La.	D. Lo.	D.La.	D. Lo.	
	7 ¼ Points.		7 ½ Points.		7 ¾ Points.		

	7 ¼ Points.		7 ½ Points.		7 ¾ Points.		
	D.La.	D. Lo.	D.La.	D. Lo.	D La.	D. Lo	
51	07 49	343 72	05 00	517 81	02 50	1038 14	51
52	07 63	350 46	05 10	527 96	02 55	1058 49	52
53	07 78	357 20	05 19	538 12	02 60	1078 85	53
54	07 93	363 94	05 29	548 27	02 65	1099 20	54
55	08 07	370 68	05 39	558 42	02 69	1119 56	55
56	08 22	377 42	05 49	568 58	02 74	1139 91	56
57	08 37	384 16	05 59	578 73	02 79	1160 27	57
58	08 51	390 90	05 68	588 89	02 84	1180 62	58
59	08 66	397 64	05 78	599 04	02 89	1200 98	59
60	08 80	404 37	05 88	609 19	02 94	1221 34	60
61	08 95	411 11	05 98	619 34	02 99	1241 70	61
62	09 09	417 85	06 08	629 50	03 04	1262 05	62
63	09 24	424 59	06 17	639 65	03 09	1282 41	63
64	09 39	431 33	06 27	649 80	03 14	1302 76	64
65	09 53	438 07	05 37	659 95	03 18	1323 12	65
66	09 68	444 81	06 47	670 11	03 23	1343 47	66
67	09 83	451 55	06 57	680 26	03 28	1363 83	67
68	09 97	458 29	06 66	690 42	03 33	1384 18	68
69	10 12	465 03	06 76	700 57	03 38	1404 54	69
70	10 27	471 77	06 86	710 72	03 43	1424 89	70
71	10 42	478 51	06 96	720 87	03 48	1445 25	71
72	10 56	485 25	07 06	731 02	03 53	1465 60	72
73	10 71	491 99	07 15	741 18	03 58	1485 96	73
74	10 86	498 73	07 25	751 33	03 63	1506 31	74
75	11 00	505 47	07 35	761 48	03 67	1526 67	75
76	11 15	512 21	07 45	771 64	03 72	1547 02	76
77	11 30	518 95	07 55	781 79	03 77	1567 38	77
78	11 44	525 69	07 64	791 95	03 82	1587 73	78
79	11 59	532 43	07 74	802 10	03 87	1608 09	79
80	11 74	539 16	07 84	812 26	03 92	1628 45	80
81	11 89	545 90	07 94	822 41	03 97	1648 81	81
82	12 03	552 64	08 04	832 56	04 02	1669 16	82
83	12 18	559 38	08 13	842 72	04 07	1689 52	83
84	12 33	566 12	08 23	852 87	04 12	1709 87	84
85	12 47	572 86	08 33	863 02	04 16	1730 23	85
86	12 62	579 60	08 43	873 18	04 21	1750 58	86
87	12 77	586 34	08 53	883 33	04 26	1770 94	87
88	12 91	593 08	08 62	893 49	04 31	1791 29	88
89	13 06	599 82	08 72	903 64	04 36	1811 65	89
90	13 21	606 56	08 82	913 79	04 42	1832 00	90
91	13 36	613 30	08 92	923 94	04 47	1852 36	91
92	13 50	620 04	09 02	934 09	04 52	1872 71	92
93	13 65	626 78	09 11	944 25	04 57	1893 07	93
94	13 80	633 52	09 21	954 40	04 62	1913 42	94
95	13 94	640 26	09 31	964 55	04 66	1933 78	95
96	14 09	647 00	09 41	974 71	04 71	1954 13	96
97	14 24	653 74	09 51	984 86	04 76	1974 49	97
98	14 38	660 48	09 60	995 02	04 81	1994 84	98
99	14 53	667 22	09 70	1005 17	04 86	2015 20	99
100	14 67	673 96	09 80	1015 32	04 91	2035 56	100
	D.La.	D Lo.	D.La	D. Lo.	D.La	D. Lo.	
	7 ¼ Points.		7 ½ Points.		7 ¾ Points.		

The Uſe of the TABLE OF *Difference* of *Latitude* and *Longitude.*

BY the Help of this *Table* and the *Table* of *Meridional Parts*, may all the *Problems*, which come properly under the Head of *Mercator's Sailing*, be readily ſolved by Inſpection.

For *Example.*

Suppoſe a Ship in the Latitude of 60° 36′ North, ſails NE. by E. 63 Miles, and it be required to find the Latitude the Ship is in, and how much ſhe has altered her Longitude.

Becauſe NE. by E. is 5 Points from the North on the Right-hand *Page* (inaſmuch as the Diſtance ſailed is more than 50) in the firſt Column find the Diſtance ſailed 63, and at the Top look for 5 Points, (as in the *common Table* of *Difference of Latitude* and *Departure*) and right under it, againſt 63, you will find 35.00 Miles for the proper *Difference of Latitude.*

This, becauſe the Ship ſails from a North Latitude Northerly, added to the Latitude the Ship ſailed from, will give 61° 11′ for the Latitude the Ship is in. The *Meridional Parts* anſwering to 61° 11′ are 4672.0, and the *Meridional Parts* anſwering to 60° 36′, are 4600.0; whence the *Meridional Difference* of *Latitude* will be found to be 72 Miles; entering again the ſame firſt Column of Diſtances with this *Meridional Difference* of *Latitude* 72, in the Column intitled *D. Lo.* under 5 Points, you will find 107.75 Miles for the true *Difference* of *Longitude.*

Let us now ſuppoſe the Ship in the Latitude of 60° 36′ North, to ſail SE. by E. 63 Miles, and let it be required to find the Latitude the Ship is in, and how much ſhe has altered her Longitude.

Becauſe

Becauſe the Courſe the Ship has made forms the ſame Angle with the Meridian, as in the former *Example*, and the Diſtance is ſtill the ſame, the proper *Difference of Latitude* will be found the ſame as before, 35.00 Miles; this therefore ſubtracted from 60° 36′, the Latitude the Ship ſailed from (inaſmuch as the Latitude is North and the Courſe Southerly) will leave 60° 01′ for the Latitude the Ship is in. The Meridional Parts anſwering to 60° 36′ are 46.000; the Meridional Parts anſwering to 60° 01′ are 4529.4; whence the Meridional *Difference of Latitude* will be found to be 70.6 Miles: Entering then the ſame Column of Diſtances with the Number 70, right under 5 Points, you will find (making a proportional Allowance for $\frac{6}{10}$) 105.66 Miles for the *Difference of Longitude* in this Caſe; more by 2.09 Miles than in the former *Example*, although the Courſe and Diſtance are the ſame as to Quantity.

For a *third Example*, let the *Traverſe* in *Page* 204 be propoſed, where the Latitude the Ship ſailed from was 50° 00′ North, the firſt Courſe SSE. $\frac{1}{2}$ E. 34 Miles, the ſecond Courſe SE. $\frac{1}{2}$ E. 52 Miles, the third Courſe ESE. $\frac{1}{2}$ E. 92 Miles; and let it be required to find the Latitude the Ship is in, how much ſhe has altered her Longitude, and her Bearing and Diſtance from her firſt Port?

Becauſe the firſt Courſe is SSE. $\frac{1}{2}$ E. Diſtance 34 Miles, enter the Table in *Page* 212, with the Diſtance 34 in the firſt or laſt Column, and right againſt it, under 2 $\frac{1}{2}$ Points in the Column intitled *D. La.* you will find the Difference of Latitude made to be 29.99 Miles; this being ſubtracted from 50° 00′ N. the Latitude departed from becauſe the Courſe is Southerly, will leave 49° 30′ for the Latitude the Ship is in; whence the Meridional Difference of Latitude will be found to be 46.5 Miles: Entering again the firſt or laſt Column of the ſame Table with 46.5, you will find under 2 $\frac{1}{2}$ Points in the Column intitled *D. Lo.* 24.85 Miles for the Difference of Longitude.

Again, becauſe the ſecond Courſe SE. $\frac{1}{2}$ E. and the Diſtance 52 Miles, entering the *Table*, Page 217, as before, with the Diſtance ſailed, in the Side Column under 4 $\frac{1}{2}$ Points, the Courſe you will find the Alteration of Latitude to be 32.99 Miles; whence the Latitude the Ship is in will be found to be 48° 57′, and conſequently the Meridional Difference of Latitude will be 50.5 Miles: Entering therefore again the ſame Table with the Meridional Diſ-

ference of Latitude 50.5, in the Side Column right under 4 $\frac{1}{4}$ Points, you will find the Difference of Longitude to be 61.78 Miles.

And laſtly, becauſe the *third Courſe* is ESE. $\frac{1}{4}$ E. and the Diſtance 92 Miles, enter the Table in *Page* 220, with the Diſtance in the Side Column as before, and right under 6 $\frac{1}{2}$ Points the Courſe, you will find the Difference of Latitude to be 30.99 Miles; whence the Latitude will be found to be 48° 26′, and the Meridional Difference of Latitude 47 Miles; with which entering the ſame Table under 6 $\frac{1}{2}$ Points the Courſe, you will find the Difference of Longitude made in this Run to be 131.92 Miles.

Now, if the ſeveral Differences of Latitude and Differences of Longitude belonging to each Courſe and Diſtance be placed in a *Traverſe Table*, after the manner in *Page* 204, each in its proper Column, we ſhall find that the Ship has made 218.55 Miles of Eaſt Longitude, 93.97 Miles of Difference of Latitude; whence ſhe will be found to be in the Latitude of 48° 26′ North, agreeing exactly with the *Logarithmic Calculation* in *Pages* 204 and 205.

In order to find the Bearing and Diſtance of the Ship from her firſt Port, we have given her Difference of Longitude made 218.55 Miles, her proper Difference of Latitude 93.97 Miles, and her Meridional Difference of Latitude 144 Miles. Now, becauſe this laſt Number exceeds the Limits of the Table which goes no farther 100 Miles, therefore I take any proportional Part of the whole; as for Example, the one 4th (for like Parts in this Caſe are in the ſame Proportion as their Wholes) then we ſhall have 54.64 for the Difference of Longitude, 23.49 for the proper Difference of Latitude, and 36. for the Meridional Difference of Latitude. This being done, I enter the left-hand Column with the Meridional Difference of Latitude 36, and carrying my Eye along the ſame horizontal Rank of Differences of Longitude, I find under 5 Points 53.88, which being neareſt to the given Difference of Longitude 54.64, but ſomething leſs, I conclude, that the direct Bearing is 5 Points, or NW. by W. a little Weſterly: But by making a proportional Allowance for .76, the Exceſs of the given Difference of Longitude 54.64, above the Tabular Difference of Longitude 53.88, we ſhall find it amounts to 24 Minutes: If this be added to 56° 15′, equal to 5 Points, we ſhall have

have North 56° 39′ West for the true Bearing, which shews that the direct Course the Ship has made is South 56° 39′ E. or SE. by E. 24 Minutes Easterly, differing but two Minutes from the Course obtained by *Calculation*, in *Page* 205.

Again, with 36, the fourth Part of the Meridional Difference of Latitude, entering the Column *D. La.* under 5 Points, I find that the neareſt Number to 23.49, the fourth Part of the proper Difference of Latitude is 23.33, which ſtands againſt 42 in the laſt Column, and by making a proportional Allowance for 0.16 its Defect, it will anſwer to 42.286, which being quadrupled will give 169.14 for the direct Diſtance, differing but 1.36 Mile from the direct Diſtance found by *Calculation*, in *Page* 205.

The Diſtances run have been taken large, and the Differences of Latitude and Longitude found to the one hundredth Part of a Mile, which has occaſioned the finding the direct Courſe and Bearing, as well as the direct Diſtance, a little perplexed; but in Runnings where the direct Diſtances are not large, nor ſo great a Degree of Exactneſs required, the whole that is wanted may be found by Inſpection, without the Trouble of making proportional Parts; tho', in my Opinion, no Pains, how great ſoever, ought to be ſpared to keep Reckonings to the greateſt Degree of Exactneſs, conſidering what great Conſequences depend upon them.

This facile Way of finding the Difference of Longitude to every ſingle Courſe and Diſtance, the Trouble being very little more than that of finding the Departure in the *common Traverſe Table*, is I think ſufficient, to excite every Perſon who is a Lover of Truth and of his own Safety, to make uſe of no other Way, ſince he may be ſure it will never fail him, let his Voyage be never ſo long, or never ſo far to the Northward or Southward; beſides he will get rid of all thoſe little Difficulties that uſually attend the common Way of keeping Reckonings, which ſo often puzzle and confound the Account, that Mariners are ſometimes at a Loſs to know what to do; and tho' to Perſons who have been accuſtomed to keep their Reckonings the common Way it may ſeem a little irkſome, yet if they will but once ſet about this Way, they will ſoon overcome the imaginary Trouble that attends it, and it will become as familiar to them as the Method they have hitherto made uſe of.

Before I conclude this *Section*, I shall take the Liberty, for the Sake of the *Young Navigator*, to transcribe some few Paragraphs out of the *First Volume* of my *System* published in the Year 1723, in order to set his Notions right concerning *Difference of Longitude*, *Meridian Distance* and *Departure*; and let him see, that tho' these are synonymous Terms in *Plain Sailing*, constantly signifying the same Thing, and in every Question are represented by the same Right Line, yet, in the *True Sailing*, they are essentially different one from another; and in the same *Problem* are, as they really should be, represented or expressed by different Lines, and are of different Values; a Thing that now and then is apt to puzzle and confound young Beginners, and Persons who are not so well grounded in the first Principles of this Science as they ought to be.

By *Difference of Longitude*, is meant the Arch of the Equator, contain'd or intercepted, between the Meridians passing through any two Places: Thus the Length of the Arch of the Equator contain'd, or intercepted, between the Meridians passing through the *Lizard* and *Barbadoes*, being found by Observation to contain 3178 Nautical Miles, or 52 Degrees 58 Minutes, the Difference of Longitude between the *Lizard* and *Barbadoes* is said to be 52 Degrees 58 Minutes; and all Places that lie under the Meridian of the *Lizard* are said to differ in Longitude from *Barbadoes*, or any other Place under the Meridian of *Barbadoes*, 52 Deg. 58 Min.

By *Meridian Distance* (in the Nautical Sense) is meant the Distance of one Place from the Meridian of another, measured in the Arch of the Parallel of Latitude passing through them, and intercepted between them; so that supposing the Difference of Longitude, or Arch of the Equator, contained or intercepted between the Meridian of the *Lizard* and the Meridian of *Barbadoes*, to be 52 deg. 58 min. or 3178 Nautical Miles, and that the *Lizard* lies in the 50th Degree of Latitude, the Meridian Distance, or Length of the Arch of the Parallel of Latitude passing through the *Lizard*, and intercepted between it and the Meridian of *Barbadoes*, will be found by the *1st Case* of *Parallel Sailing*, to be 2042.8 Nautical Miles; and a Ship in sailing from *Barbadoes*, or any other Point under the Meridian of *Barbadoes* till she arrive at the *Lizard*, is said to have made 2042.8 Miles of Meridian Distance.

Again,

Again, ſuppoſing *Barbadoes* to lie in the 13th $\frac{1}{2}$ Degree of Latitude from the Equator, the Length of the Arch of the Parallel of Latitude paſſing through *Barbadoes*, and intercepted between *Barbadoes* and the Meridian of the *Lizard*, will be found by the former Method of *Calculation*, to be 3090.1 Miles; ſo that a Ship in ſailing from the *Lizard*, or from any other Place under the Meridian of the *Lizard* till ſhe arrive at *Barbadoes*, is ſaid to have made 3090.1 Miles of Meridian Diſtance, each of which Meridian Diſtances is leſs than the true Difference of Longitude, or Arch of the Equator, intercepted between the Meridian's paſſing thro' the *Lizard* and *Barbadoes*, and are to each other, as the Sines of the Diſtances of the Places from the Pole or Co-ſine of their Latitudes, or Diſtances from the Equator, (the Difference of Longitude being equal to the Sine of 90 Degrees) and may therefore be all expreſs'd by different Right-Lines, and are of different Values; as the Reader will readily perceive, if he conſtructs theſe two Caſes in the ſame Figure, according to the Directions given in the *firſt Caſe* of *Parallel Sailing*.

By the *Departure*, is meant the Diſtance between the Meridians paſſing through two Places upon the *Plain Chart*, which ſuppoſes the Earth's Superficies to be a Plain Superficies, the Meridians parallel to each other, and the Degrees of Latitude every where equal to each other, and equal to the Degrees of Longitude, and is therefore the ſame with the Difference of Longitude: So that ſtrictly ſpeaking, the Departure is the Difference of Longitude between two Places, upon the Principles of *Plain Sailing*, and between the *Lizard* and *Barbadoes* will be found to be 2119.2 Miles leſs than the true Meridian Diſtance between *Barbadoes* and the *Lizard* 470.9 Miles, and greater than the true Meridian Diſtance between the *Lizard* and *Barbadoes* by 576.4 Miles; and ſhort of the true Difference of Longitude between the *Lizard* and *Barbadoes* by 578.8 Miles. And hence ariſes the Error of *Plain Sailing*; and to remedy this very Thing was the *Mercator's Sailing* invented; which it does to the utmoſt Degree of Exactneſs, as every one muſt allow that rightly underſtands what has been ſaid upon this Head.

And indeed, properly ſpeaking, *Departure* has nothing to do with *Mercator's Sailing*, nor has it any Share in it; and 'tis for this Reaſon that I have omitted the *7th*, *8th*, and *9th Problems*, and

and taken no Notice of it in the *first six Problems*, which, together with the *Problems* of *Parallel Sailing*, are the only *Problems* belonging to the *True Sailing*.

Again, whosoever thoroughly considers the Nature of *Departure* and *Difference of Longitude*, must readily grant, that even the Method made use of by our most expert Navigators, and generally taught in Books for finding out the Difference of Longitude, by increasing the Absolute Departure made in one Day's Runnings, in proportion as the Meridional Difference of Latitude is to the proper or common Difference of Latitude, cannot be strictly true.

For though to the same Course and Distance, there will always be the same Departure and proper Difference of Latitude, and to the same Latitude, proper Difference of Latitude and Departure, there will always be the same Difference of Longitude, yet, when several Departures come to be compounded in one, as is the Case of almost every Day's Work, the Thing is altered, as an easy Instance will shew.

Suppose (for *Example*) a Ship in the Latitude of 60 deg. North, sails 100 Leagues due East, and after that North East, till she be got into the Latitude of 70 deg. In this Case she will have made but 300 Leagues of Departure, and 2038.6 Miles, or 33.58 $\frac{6}{10}$ of Difference of Longitude.

Let us now suppose the same Ship to sail 100 Leagues due West, in the Parallel of 70 Degrees, then South West till she be got into the Latitude of 60 Degrees; 'tis plain, that in this Case she has made but 300 Leagues of Departure, as before; but 2315.8 Miles, or 38 Degrees 35 Minutes $\frac{8}{10}$ of Difference of Longitude, greater in proportion than the former Difference of Longitude, as the Co-sine of 60 Degrees is to the Co-sine of 70 Degrees, notwithstanding the Meridional and Proper Difference of Latitude in both Cases are the same.

I have chosen this easy Instance, because every one that is but moderately skill'd may understand it; but the same Absurdity will follow in compounding of any other Courses together, as the sagacious Reader will readily apprehend.

For these Reasons, in this Edition, I have left out *Middle Latitude Sailing*, as being dependant upon *Departure*, and, in my Opinion, it ought entirely to be laid aside: For what Man in his Senses will make use of an approximating Method, when he can

come

come at a true Solution of his *Problem* with the ſame, if not leſs Trouble.

How the ſeveral Problems in Mercator's Sailing *may be ſolved by the* Logarithmic Tangents, *inſtead of* Meridional Parts, *may be ſeen from Page* 303 *to Page* 311 *of the* 1*ſt Volume of my* Syſtem. *And how the* ſecond *and* third Problems *which moſt of all occur in Uſe, may be ſolved by the Simple Rules of* Multiplication *and* Diviſion, *without the Aſſiſtance of the* Tables of Logarithms, Sines, Tangents, *and* Secants, *or any* Artificial Numbers *whatſoever, may be ſeen in Pages* 350, &c. *of the ſame Book.*

SECT. X.

HAVING ſufficiently ſhewn the Defects of the *Plain Chart*, and in the former Part of the 8*th Section*, exhibited the eſſential Properties of a *Projection*, in which all Places whatſoever (except ſuch as lie under the very Pole) may be laid down according to their *Latitude*, *Longitude* and *Courſe*; we ſhall now proceed to ſhew how the *CHART* (commonly call'd *Mercator's*) may be conſtructed; either for the whole or any particular Part of the *Terraqueous Globe*.

Let it be required to deſcribe a particular *Chart* for a Voyage between the Parallels of 35 and 50 Degrees of Latitude, which may ſerve very well to prick down Runnings between the Land's End and the *Streights* Mouth.

Having provided a Line of equal Parts of a convenient Length,

Draw the North and South Line AB, and croſs it at Right-angles in the Point A, with another Right Line AD, which will repreſent the Parallel of 35 Degrees. Then

From the Meridional Parts anſwering to 50 Degrees —	3475
Take the Meridional Parts anſwering to 35 Degrees —	2244
	1231

Set

Set off the Exceſs from A to B, and draw BC parallel to AD, which will be the Parallel of 50 Degrees in this *Chart*.

For the intermediate Parallels between 35 and 50, take the Difference between the Meridional Parts of any of the given Parallels of Latitude, and the Parallel of Latitude to be drawn; and ſet it off, as the Caſe directs, in the Meridian; that is, from A upwards towards B: If the Meridional Parts of 35 had been taken from the Meridional Parts of the Parallel to be drawn, or downwards from B to A. If the Meridional Parts of the Parallel of Latitude to be drawn, had been taken from the Meridional Parts anſwering to 50 Degrees; and through the Point laſt found, draw a Line parallel to AD, or BC, and it will be the Parallel required.

Suppoſe it were required to draw the Parallel of 40 Degrees.

From the Meridional Parts anſwering to 40 Degrees —— 2623
Take the Meridional Parts anſwering to 35 Degrees —— 2244

There remains —— —— —— 379.

Set off the Remainder 379 from A to 40, draw the Line 40.40 parallel to AD, and it will be the Parallel required.

Or,

From the Meridional Parts anſwering to 50 Degrees — 3475.
Take the Meridional Parts anſwering to 40 Degrees — 2623.

There remains —— —— —— 852.

Set off the Remainder 852 from B downwards, towards A to 40 Degrees, draw the Line 40.40, and it will be the Parallel required.

After the ſame Manner may any other Parallel be drawn, or the Meridional Line be divided.

The Meridian Line being thus divided, and the Parallels drawn, ſet off the equal Degrees of Longitude from A towards D, and draw the Meridian Line DC parallel to AB. Now, if through the ſeveral Degrees of Longitude, Lines parallel to AB be drawn, they will repreſent ſo many ſeveral Meridians.

After

After this manner may a *Chart* be made to ſerve for any Voyage whatſoever, provided the Latitudes of the Places the Ship ſailed from, and to which ſhe is bound, be known.

But if it be requiſite that the *Chart* ſhould commence from the Equator, or that the Equator ſhould run through any Part of it, there is nothing elſe to be done, but to ſet off the Meridional Parts themſelves from the Equator, on the Meridian Line extended, and it will be divided accordingly.

In like manner may a *Chart* be made as large or as ſmall as you pleaſe, either for the whole World, or any particular Part thereof.

But before it can be fit for Uſe, it will be very neceſſary to deſcribe the Rumb-Lines in ſome Part thereof. I have choſe for Conveniency-ſake to place them in each Corner, as you may ſee in the *Chart:* And, for the more ready pricking down of Places, I have diſtinguiſhed the 10*th* and 5*th* Lines from the reſt, which I would adviſe the *Practitioner* to do, ſince it will render his counting upon the *Chart* more certain and expeditious.

Upon the *Chart* thus made, may any particular Country be deſcribed, or the Sea-Coaſts thereof; and by the Help of it, the Ingenious Mariner may at all times find the Place he is in, and readily know upon what Courſe, and how far he muſt ſail to gain his deſired Port.

And ſince this Treatiſe is written purpoſely for his Uſe and Benefit, I ſhall not only ſhew him how to diſcover where he is, and find how he muſt alter his Courſe, if need be, but readily to trace out upon it any Part of the Sea-Coaſt. And,

1. If it were required to lay down a Place, and find a Prick in the *Chart* by its Latitude and Longitude, or rather by its Latitude and Difference of Longitude, from any Place given.

Having fix'd upon a firſt Meridian, which I would always cauſe to paſs through the Place from whence I begun my Voyage, ſince the Places prick'd down may eaſily enough be transferred into a *General Chart.*

1. Count the Longitude in the graduated Parallel from the firſt Meridian in the *Chart*, or the Difference from the Meridian paſſing through the given Place, and through the Point found, draw a new Meridian.

2. Draw a Parallel of Latitude thro' the given Degree of Latitude found out in the Graduated Meridian; where this interſects the Meridian before drawn, gives the Place or Prick to be found.

Example.

Admit the Place to be prick'd down was in the Latitude of 47° 30′, and lay 2° 30′ in Longitude to the Eaſtward of the firſt Meridian in the *Chart*.

Having found out the Points of 47° 30′ in the Graduated Meridians, draw the Line *a b*, and through the Point of 2° 30′ in the Graduated Parallels AD and BC, draw the Line *c d*, the Point of Interſection S of this Meridian; and the Parallel laſt drawn will be the Prick or Place required.

After the ſame manner may the Induſtrious Mariner, by the Help of his Latitude and Longitude, find the Place he is in at all Times, and prick down any Place whereſoever he arrives; provided his Latitude and Difference of Longitude from ſome known Place be had, and conſequently be able to deſcribe the Sea-Coaſts of any Country or Kingdom whatſoever.

By the Inverſe Method of this, may the Latitude of any Place in a *Mercator's Projection*, and its Difference of Longitude from any Place be found; *viz.* by drawing a Parallel through it, or laying a Ruler over it parallel to any Eaſt or Weſt Line, and obſerving where it cuts the Graduated Meridian for the Latitude: And likewiſe by drawing a Meridian through it, or laying a Ruler directly North or South, to diſcover what Degree of the Graduated Parallel is cut by it, in order to know its Longitude.

Hence we are taught how to find the Difference of Latitude of any two Places upon the *Chart*; *viz.* by finding the Latitude of each, according to the former Directions, and taking the Leſſer from the Greater, to diſcover the Exceſs or Difference.

Hence we are taught how to find the Difference of Longitude between any two Places on the *Chart*; *viz.* by finding firſt the Longitudes of each ſeparately, and then ſubtracting the Leſſer from the Greater.

Hence we are taught how to find the Diſtance of any two, or more Places, in the *Chart* that lies under the ſame Meridian, *viz.* by finding the Difference of Latitude between them, and reducing of it into Minutes, in order to diſcover the Miles of Diſtance.

Hence we are taught how to find the Miles of Diſtance between any two or more Places, that lye under the ſame Parallel, *viz.* by finding the Difference of Longitude between them, and reducing it into Minutes. And then,

With

With 90 Degrees of a Line of Sines, upon the Segment of the Parallel connecting the two Places together as a Baſe, deſcribe an Equicrural Triangle; and from the Vertex towards the Baſe, ſet off on each Side the Sine of the Complement of the Latitude of the Parallel; and the Diſtance of theſe two Pricks applied to the Graduated Parallel, will ſhew the Degrees and Minutes of Diſtance; which muſt be reduced into Miles by multiplying by 60. All which is abundantly manifeſt from what has been ſaid in the former Section.

An Example will make it eaſy.

Admit it was required to find the Miles of Diſtance between the Places Q and R, lying both under the Parallel of 39° 30′.

On Q R as a Baſe, deſcribe the Equicrural Triangle Q R T, and make T Q or T R the Radius of a Line of Sines: Set off the Sine of the Complement of the Latitude 50° 30′, from T to X and W, then the Diſtance X W applied to the Graduated Parallel, will ſhew the Degrees and Minutes of Diſtance.

Or,

Make the Diſtance Q R equal to the Radius of a Line of Sines upon the Sector, and the Co-ſine of the Latitude upon the ſame Sector, will be the Diſtance; which muſt be applied to the Graduated Parallel, in order to diſcover the Miles of Diſtance.

Tho' the Method generally taught in Books for finding the Diſtance of two Places in the *Mercators Projection*, lying under the ſame Parallel, be not ſtrictly true, yet it is ſufficiently exact in many Caſes, and is as follows;

Take half the Diſtance between the Points of the Compaſſes, and ſetting one Foot in the Graduated Meridian, where the Parallel cuts it, turn the other Foot about, and obſerve what Degrees it cuts both above and below: Subtract the Leſſer Number from the Greater, and convert the Reſidue into Minutes, and you have the Miles of Diſtance.

Or,

Take the Diſtance from half a Degree in the Meridian, above the Parallel to half a Degree below; and ſee how many of thoſe Parts is contained between the two Places, convert the Sum into Minutes, by multiplying by 60, and you have the Miles of Diſtance.

From what has been ſaid, it will not be difficult, if the Difference of Latitude and Longitude of any two Places be known, from the Place of the one, to find the Place of the other.

Hence, If two Places lye under the ſame Meridian, and the Latitude of one and its Diſtance from the other be given, it will be eaſy from the Place of the one, to find the Place of the other.

Hence, If two Places lye under the ſame Parallel, and the Longitude of the one be known, and its Diſtance from the other, it will be eaſy from the Place of the one, to find the Place of the other.

Theſe three laſt *Corollaries* are of great Uſe to Sailors, to prick down any New Land, dangerous Shoal or Rock.

2. If it were required to lay down a Place by its Latitude, and Bearing from ſome other; or, *which is the ſame Thing*, having the Difference of Latitude and Courſe given, to make a Prick upon the *Chart*, and diſcover the Place of the Ship.

Example.

Admit from *S* a Place in the Latitude of 47° 30′ North, I ſail away SE. by S. 5° 29′ Eaſterly, until I am in the Latitude of 39° 30′ North, and would prick the Place of the Ship.

Through the Points *m* and *n* at 39° 30′, the Latitude ſailed into, draw the Parallel *m n*, and from *S*, the Place the Ship departed from, draw S Q parallel to *q r*, the SE. by S. ½ Eaſt Line; where this interſects the Parallel *m n* as at Q, it gives the Place of the Ship.

After the ſame manner may any other Point be found from the ſame *Data*, and conſequently the ſeveral Runnings of the Ship be deſcribed.

By the Inverſe Method of this, *viz.* by ſeeing to which of the Rumb-Lines, the Line connecting any two Places together is parallel, may the Bearing of two Places be found, which is of great Uſe to the Sailors for directing their Courſe.

Hence, without any more to do, may the induſtrious Sailor know how to alter his Courſe continually, in order to gain his deſired Port.

3. To lay down a Place by its Longitude, and Bearing from ſome other: Or, *which is the ſame*, Having the Courſe and Difference of Longitude, to make a Prick upon the *Chart*, and diſcover the Place of the Ship.

Example.

Admit from ♌ I ſail away NW. by N. 5° 25′ Weſterly, till I have altered my Longitude 9° 00′, and would make a Prick upon the *Chart*, and diſcover the Place of the Ship. Having

Having counted 9° 00′ in the Graduated Parallel, from *o* the Interſection of the *Meridian*, paſſing through the Place ſailed from Weſtwards towards *A*: Or, *which is the ſame Thing*, Set off 9° 00′ of the Graduated Parallel from Q to R in the Parallel *m n*; draw the Meridian *d c*, and through Q draw a Parallel to the Rumb-Line *q r*, till it cut the Meridian *d c* in S, the Point ſought.

By the Inverſe Method of this, may the Bearing and Difference of Longitude between any two Places be eaſily found, *viz.* for the Difference of Longitude, by counting the Portion of the Graduated Parallel, cut off by the Meridians paſſing through the Places, and for the Bearing, by examining to which of the Rumb-Lines the Line connecting the Centers of the two Places is parallel to.

4. To lay down a Place by its Latitude, and Diſtance from ſome known Place: Or, *which is the ſame Thing*, By the Diſtance run, and Difference of Latitude, to make a Prick in the *Chart*, and diſcover the Place of the Ship.

Example.

Admit from *S* a Place in the Latitude of 47° 30′. I ſail between the South and the Eaſt 619 Miles, till I am got into the Latitude of 39° 30′ North, and would find the Place of the Ship in the *Chart*.

Having drawn the Meridian *c d* paſſing thro' the given Place, take off the Degrees of Difference of Latitude from the Graduated Parallel AD, and ſet them from *S* downwards in the Meridian to K; through K draw the Parallel KL; then with the Diſtance between the Points of the Compaſſes, ſetting one Foot in *S*, with the other turn'd about croſs the Parallel laſt drawn in L. Laſtly, through the Point laſt found L, and the Point firſt given S, draw the Line SL, and continue it till it meet the Parallel *m n*, into which the Ship ſail'd in Q, which will be the Place of the Ship required.

Hence we are taught how to find the Diſtance between any two Places on the *Chart*, which are not ſituated under the ſame Meridian or Parallel, *viz.* by taking the Degrees of Difference of Latitude off from the Graduated Parallel, and transferring them in the Meridian, from *S* downwards to K, and drawing the Parallel K L

till

till it meet the Line SQ connecting the two Places together; for then the Distance SL between the Place first given, and the Point last found, applied to the Graduated Parallel, shews the Degrees and Minutes of Distance which are to be reduced into Miles, as we have formerly shewn.

5. To lay down a Place by its Bearing and Distance from another; or, *which is the same Thing*, From the Course and Distance given, to make a Prick, and find the Place of the Ship.

Example.

Admit a Ship in the Latitude of 47° 30′ North, sails SE. by S. 5° 29′, East 619 Miles, and it be required to find the Place of the Ship.

Having drawn a Meridian Line through *S*, the Place of the Ship departed from, draw the Rumb-Line S Q parallel to the Rumb-Line *q r*, and set off upon it the Distance in Miles from S to L, and draw the Perpendicular L K; then apply the common Difference of Latitude S K, to the Graduated Parallel, and observe the Number of Degrees. Count the same Number of Degrees in the Graduated Meridian from *a* to *m*, and draw the Parallel *m n*, and produce the Rumb-Line SL, till it intersect the Parallel *m n* in Q; which will be the Place of the Ship sought.

By the Inverse Method of this, may the Bearing and Distance of any two or more Places in the *Chart* be found, and that so easily to any who understands but the last Paragraph, that it would be needless to give an Example.

Thus I have shewn, from Principles first laid down, and then demonstrated, not only how a general or particular *Chart* may be made, but how any Place, and consequently any Country, or the Sea-Coasts thereof, may be prick'd down from any possible *Data*: As also how the careful Mariner may prick down his several Runnings, and thence know the Place he is in at all Times; and find how far, or upon what Course, he must steer to arrive at his intended Port.

Upon this *Chart* all Places may be laid down true according to their *Latitudes* and *Longitudes*, and what is of the greatest Benefit, and gives it the Preference before all other Contrivances whatsoever, for *Nautical Uses*, is, that the Rumb-Line or Path that a Ship

Ship deſcribes in Sailing according to the Direction of the Compaſs, (the only Guide that Mariners have at preſent) is repreſented, or laid down, by a ſtrait Line, whence the true Bearing of any two Places, or the true Courſe that a Ship muſt ſail from one Port to another, may be truly and exactly determined, and that with the greateſt Eaſe poſſible.

This Rectilinearity of the Rumb-Line is the Chief and Primary Property of the *Mercator's Projection*, to which all the other Parts are made ſubſervient; and to accompliſh this very Thing after the Invention of the Compaſs, was the great Endeavour of our Fore-fathers for many Years, and never perfected, till undertaken by Mr. *Wright*, which happy Invention of his will make his Name famous to Poſterity.

And hence it is, that all *Nautical Problems* are reſolved by this *Projection* to the greateſt Degree of Exactneſs that Projections are capable of, and infinitely more eaſy than by the Globe itſelf; where, becauſe the Meridians meet in a Point, the Rumb-Lines become Spiral Lines, which render the laying down of Runnings (the *principal Data* that Mariners have to find the Place of the Ship at all Times) and determining the Bearings and Diſtances of Places, (the next Thing wanting to direct the Courſe a-new) eſpecially if the Places are pretty far from the Equator, in a manner impracticable.

As it is impoſſible by any Contrivance whatſoever to repreſent any Part or Portion of a *Spherical Surface* upon a *Plane*, but that it will be diſtorted and appear under a different Form, from what it does upon the *Surface* of the *Sphere*; and conſequently different *Projected Portions*, when compared together, cannot have the ſame Ratio one to another as to *Magnitude*, that the Parts they repreſent have to each other upon the Globe itſelf; yet that Contrivance that repreſents them ſo, that the Dimenſions of the *Projected Parts* (tho' diſtorted, when meaſured by Scales) retain between themſelves the ſame Ratio, that the Dimenſions of the Parts they repreſent upon the Globe have to each other truly and exactly, will undoubtedly give true Solutions to all *Problems* that can ariſe from it; as is the Caſe of all Kinds of *Projections* of the *Sphere* whatſoever.

For *Example*; In this *Projection*, tho' an Iſland in the Latitude of 60 Degrees, when projected, will have its Breadth from North

to

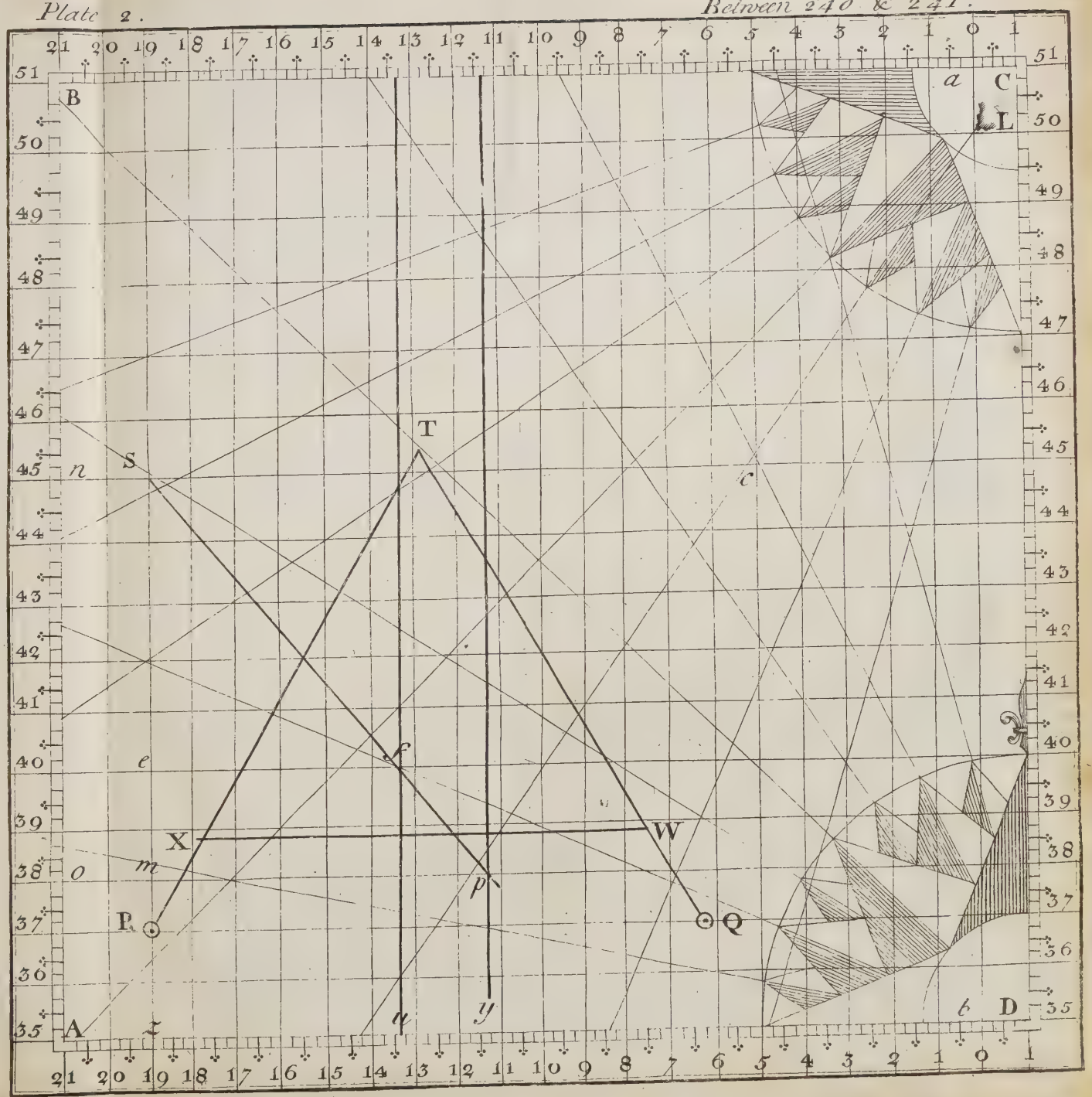
to South repreſented in near a double Ratio of what it would be, if the ſame Iſland were placed under the Equator; and conſequently, theſe two projected Iſlands, when compared together, will not have the ſame Ratio between themſelves as to *Magnitude*, that the Places they repreſent upon the *Surface* of the *Sphere* really have.

Yet, if the Breadth from North to South of the diſtorted Iſland, in the Latitude of 60 Degrees, when compared with the Length from Eaſt to Weſt, each being meaſured by proper Scales, be found to have the ſame Ratio, that the Breadth and Length of the Iſland it repreſents upon the Surface of the Sphere have to each other, it neceſſarily follows, that all Concluſions drawn from this Repreſentation are equally true, and conducive with thoſe that are deduced from the Globe itſelf that it repreſents; or, *which is the ſame Thing*, and comes nearer to our preſent Purpoſe, tho' a Degree of Longitude, in the Latitude of 60 Degrees, be repreſented upon this Projection as large again as it really is upon the Surface of the Globe, yet, inaſmuch as a Degree of Latitude at that Diſtance from the Equator is increaſed in the ſame Ratio, and this Law obſerved at every Point of Diſtance from the Equator, it follows, that the true Courſe and Diſtance between any two Places, howſoever ſituated, will be found to be the ſame upon this Projection, as they are upon the Surface of the Sphere; and conſequently, all the Solutions of the ſeveral Caſes ariſing from changing the Things given and required, will be the ſame upon this Projection as they are upon the Surface of the Globe itſelf; which renders this *Chart* the moſt apt and proper Inſtrument, and the beſt adapted to the Mariner's Uſe, as any that can be thought of, till Mankind ſhall find out a better Contrivance than the *Compaſs*, for guiding or directing a Ship through the wide and pathleſs Ocean.

PART

Plate 2.

Between 240 & 241.



PART III.

Of the Projection of the SPHERE.

SECT. I.

DEFINITIONS.

CIRCLES described upon the Surface of the Globe, are diſtinguiſh'd into great and ſmall Circles.

1. Great Circles are thoſe which divide the Surface of the Sphere into two equal Parts; or thro' which, if the Globe be cut by a Plane, it will be divided equally. But

2. Thoſe are called ſmall Circles which cut the Surface of the Sphere unequally, or through which, if the Globe be cut by a Plane, it will be divided into two unequal Parts.

3. The Pole of every Circle, whether great or ſmall, is that Point upon the Surface of the Sphere which is every way equally diſtant from its Circumference.

Whence it follows,

1. That every Circle whatſoever deſcribed upon the Superficies of the Sphere has two Poles; and ſince every ſmall Circle is equidiſtant from ſome one great Circle; it follows,

2. That the Poles of every ſmall Circle, are the ſame with the Poles of thoſe great Circles to which they are parallel.

3. All ſmall Circles parallel to each other have the ſame common Poles.

General Definition.

Conceive the Eye placed ſomewhere in the Superficies of the Globe, and at the ſame Time a Plane to cut the Globe, and to ſtand at Right Angles to the Line connecting the Centers of the

Globe and Eye. If Infinite Right Lines be imagined to flow from the Eye to every Point in the Circumference of every Circle deſcribed on the Surface thereof, they will trace out upon the *Cutting Plane*, what is called a *Stereographick Projection of the Sphere.*

Whence, and from *Corollary* the 1*ſt* of the *Definitions*, it follows,

1. That the Eye is placed in one of the Poles of that Circle through which the Plane paſſes, and upon which the Projection is to be formed; which, for Diſtinction-ſake, we call the *Plane of Projection:* Alſo the Pole oppoſite to the Eye, *the Remote Pole.*

2. That the Pole when projected falls in the Center of the *Plane of Projection.*

3. Since all great Circles which interſect each other in the Eye Point, lie in the ſame Plane with the Eye; they will, when projected, become Right Lines, interſecting each other in the *Center* or *Pole of Projection.*

4. That the Poles of every ſuch Circle will be found in the Periphery of the *Plane of Projection*, at 90 Degrees Diſtance from either of its Interſection therewith.

5. That the Lines flowing from the Eye, to any other Circle whatſoever, form a Conical Superficies. And in order to enquire into the Nature and Property of theſe Circles when projected, we ſhall premiſe the two following *Lemma's.*

Lemma the 1*ſt.*

1. That if an upright Cone be cut with a Plane parallel to the Baſe, the Section made in the Superficies of the Cone will be the Circumference of a Circle, whoſe Center will be found in the Right Line connecting the *Vertex* of the Cone, and Center of its Baſe.

Whence, and from the *General Definition*, it follows,

1. That all ſmall Circles parallel to the *Plane of Projection*, will become Circles in the *Projection.*

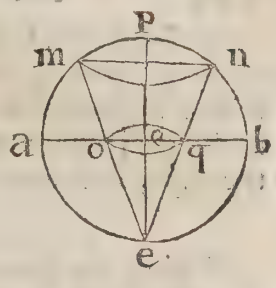
2. That they will all have one common Center, *viz.* the Center and *Pole of Projection.*

3. That their common Center, *viz.* the Center of the *Projection*, will be their common Pole.

4. That their Radij, or Semidiameters, will be equal to the Semi-tangents of their Diſtances from the *remote Pole of Projection.*

For,

For in the adjacent *Figure*,



Suppoſing *e* the Eye, *a b* the *Plane of Projection*, *m n* the Circle to be projected, and *p* its remote Pole: *o c* equal to *c q*, will be equal to the Semi-tangent of *p m* or *p n*, the Diſtance of the Circle *m n*, to be projected from *p* the remote Pole.

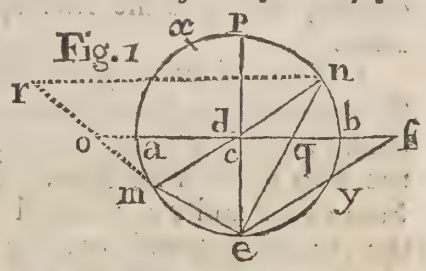
Lemma the 2d.

If an inclined Cone be cut with a Plane, not parallel to the Baſe; yet ſo that it cuts off a Part ſimilar to the whole, the Section made in the Superficies of the Cone, will be a Circle.

Whence it follows,

1. That all Circles whatſoever, (except ſuch as lie in the ſame Plane with the Eye, which have been already conſidered) tho' they are not parallel to the Periphery of the *Plane of Projection*, yet become Circles in the Projection.

Fig. 1



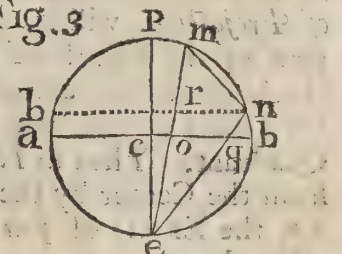
For if, as before, *e* be the Eye, *a b* the *Plane of Projection*, *m n* the Circle to be projected, and *p* its remote Pole.

Through *n* draw *n r* parallel to *a b*; then is the Angle *r n e* equal to *e m n*, (*by Cor. the firſt of Theor. the 9th*) and *m e n* common to the Triangles *e m n* and *e n r*.

Fig. 2

Wherefore the Triangles *e m n*, *e n r*, and conſequently *e o q* are Similar; and *o q* will be a Circle, by the *2d Lemma*.

Fig. 3



2. That of all the Diameters of any one of those great Circles to be projected, their Centers and Poles will be found in that projected Diameter which connects the two Intersections of the Circle to be projected; and that other great Circle which passes thro' the Eye, and cuts the Circle to be projected at Right Angles.

Definition.

Which, for Brevity-sake, I call the *Projected Axis*: Thus *o q* is the *Projected Axis*.

3. That all small Circles will have their *Projected Axes* in that Line in which the *Projected Axis* of the great Circle to which they are parallel lyes.

4. That the Semi-tangents of the greatest and least Distances of any Circle from the remote *Pole of Projection*; set off either on the same or contrary Sides of the Center of rojection, as the *Case* directs, will give the Intersections or Extremities of the *Projected Axis*.

5. That of every great or small Circle within which the *Pole of Projection* lyes, their Diameters or *Projected Axes*, are equal to the Sum of the Semi-tangents of their greatest and least Distances from the *Pole of the Projection*. Thus in *Fig.* 1 and 2, *o q* equal to the Sum of *o c* and *c q*, is equal to the Sum of the Semi-tangents of the Arches *m p* and *p n*, the greatest and least Distances of the Circle from the remote *Pole of Projection*.

6. That of every small Circle without which the remote *Pole of Projection* lyes, its Diameter or *Projected Axis*, is equal to the Difference of its greatest and least Distance from the *Pole of Projection*. Thus in *Fig.* 3. *o q* is equal to the Difference of *c q* and *c o*, the Semi-tangents of *p n* and *p m* the greatest and least Distances of the Circle from the *Pole of Projection*.

7. That of the two Poles of every great Circle, and consequently of all small Circles parallel to it, that which falls within the *Plane of Projection*, will be distant from the Center, by the Semi-tangent of the Excess of the greatest Distance, of the projected great Circle from the remote *Pole of Projection* above a Quadrant; or the Defect of the least Distance from the *Pole of Projection* to a Quadrant. Thus in *Fig.* 1. *d* the Pole of the Circle, *n m* is distant from the Center by the space of *d c*, the Semi-tangent of the Arch *x p*, the Excess of *p m* above a Quadrant, or the Defect of *p n* to a Quadrant.

8. That

8. That its other Pole will be in the *Projected Axis*, on the contrary Side of the Center, and distant therefrom by the Semi-tangent of the projected Circle's nearest Distance from the remote *Pole of Projection* lessen'd by a Quadrant. Thus likewise in *Fig.* 1. the exterior Pole of the Circle, *n m* is distant from the Center *c*, by *c f* the Semi-tangent of *p y*, the nearest Distance of the Circle from the *Pole of Projection*, augmented by a Quadrant.

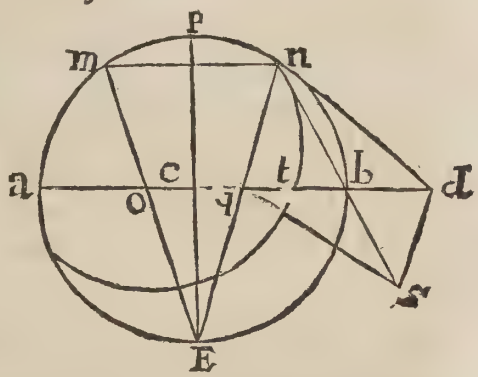
9. Hence we are taught how to find the Interior Pole of any great Circle, and consequently of all small Circles parallel to it, *viz.* by setting off the Semi-tangent of the Complement of its nearest Distance from the remote *Pole of Projection*, in its projected Axis, on the contrary Side of the Center with the Intersection.

10. But for the Exterior Pole, by setting off the Semi-tangent of its nearest Distance, augmented by a Quadrant in the projected *Axis*, from the Center on the same Side with the Intersection.

Proposition I.

In *Stereographic Projection*, the Angles made by the Circles on the Surface of the *Sphere*, are equal to the Angles made by their Representatives on the Plane of the *Projection*.

Suppose the Eye at E, projecting the Spherical Angle *t n b*, upon the Plane *a c b* at Right-Angles to the Plane *a p b* E at the Point *q*. I say, the Angle *s q d* is equal to the Angle *t n b*, equal to the Angle *s n d*, made by the two Planes *p n b* and *n t m*.



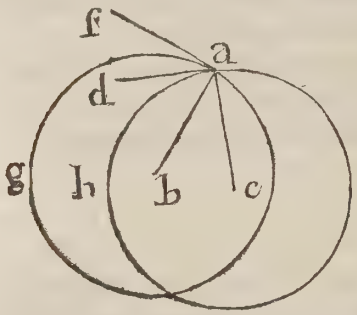
From the Angular Point *n*, draw *n d* and *n s*, Tangents to the two Circles *n b* E and *n t m*.

Because the Planes *a c t b* and *s n d* are both perpendicular to the Plane of the Circle *a p b* E, by *Construction*, their common Intersection *d s*, will be Perpendicular to the Right Line *a c b* continued, and consequently the Angles *q d s* and *n d s* are Right Angles.

From

From *n* draw *m n* parallel to *a b*, and join the Points E and *m*, then will the Angle *d n* E be equal to the Angle *n m* E, equal (*by Prop. the 6th of Part the 1ſt*) to *m n* E equal (*by Prop. 3d of Part the 1ſt*) to *n q d*; therefore is *n d* equal to *d q*. Wherefore, in the Triangles *s n d*, *s q d* are *n d* equal to *q d*; the Angles *n d s* and *q d s* are Right, and the Side *d s* Common; wherefore (*by Propoſition the 5th of Part the 1ſt*) the Angle *s q d* upon the Plane of the Projection, is equal to the Angle *s n d*, made by the two Planes, forming the Angle *t n b* upon the Spherical Surface, which was to be demonſtrated.

Propoſition II.



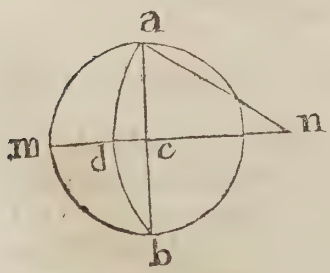
If two Circles *c a h* and *b a g*, interſect each other in the Point *a*, the Angle *g a h* formed by them at their Interſection, ſhall be equal to the Angle *b a c*, made by the Radij *a b* and *a c* drawn to the Point of Interſection *a*.

Conſtruction.

To the Point of Interſection *a*, draw the Line *f a*, a Tangent to the Circle *g a b* and *d a*, a Tangent to the Circle *h a c*.

Demonſtration.

Now, Becauſe the infinitely ſmall Portions of the Circles *g a b* and *h a c*, do coincide with the Tangents *f a* and *d a*, and conſequently have the ſame Direction; therefore the Curve-lined Angle *g a h*, is equal to the Right-lined Angle *f a d*, formed by the Tangents *f a* and *d a*: And becauſe the Angle *f a b* is equal to *d a c*, take away from each the Interjacent Angle *b a d*, and there will remain the Angle *b a c* equal to *f a d*, equal to *g a h*, which was to be proved.



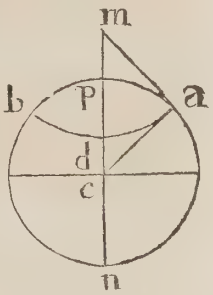
Whence it follows,

1. That of every great Circle to be projected (which does not lie in the Plane of the Eye) the Tangent of the Complement of its Diſtance from the *Pole of Projection*; ſet off in the projected *Axis*, on the contrary Way with its Interſection, will give its Center; for *c n* is equal to the Tangent of *c a n*, the Complement of *d a c*.

2. That

2. That the Secant of the Complement of the ſame Diſtance, will be its Semidiameter; for *a n* equal to *d n*, is equal to the Secant of *c a n*, the Complement of *d a c*.

3. Since *n a* equal to *d n*, is equal to *d c* more *c n*, it follows univerſally, That of any great Circle to be projected (that does not Interſect in the Eye) if the Semi-tangent of its Diſtance from the remote *Pole of Projection*, be ſet off in the projected *Axis*, on one Side of the Center, it will give its Interſection; and the Tangent of the Complement of the ſame Diſtance, ſet off in the ſame projected *Axis* on the contrary Side, will give its Center.



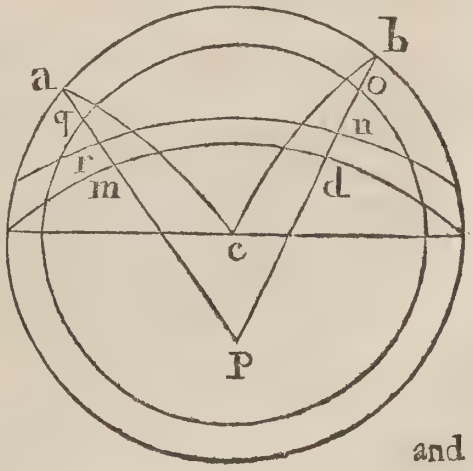
4. In all Circles which cut the Periphery of the *Plane of Projection* at Right Angles: Or, *which is the ſame Thing*, whoſe Poles lie in the Circumference of the Plane of Projection, that the Secant of the Complement of their Diſtances from the remote Pole of Projection, ſet off from the Center in the projected *Axis*, will give their Centers; for *c m* is equal to the Secant of *p c a*.

5. That the Tangents of the ſame Diſtances will be equal to their Semidiameters; for *m a* equal, *m d* is equal to the Tangent of *p c a*, or *p a* the Complement of *d c*, its Diſtance from the *Pole of Projection*.

6. Since *c m* is equal to *c d*, more *d m*, it follows, that the Semi-tangent of its Diſtance from the *Pole of Projection*, ſet off in the projected *Axis*, gives one Interſection; and the Co-tangent of the ſame Diſtance, ſet off from the Interſection the ſame Way, gives the Center.

Propoſition III.

If thro' the remote Poles of two Circles, two other Circles be deſcribed upon the Surface of the Sphere, they will cut off equal Arches in thoſe great Circles; and likewiſe in the ſmall Circles equidiſtant from thoſe Poles thro' which they paſs.



Let *p* be the Pole of the Circles *m d* and *r n*, *c* the Pole of the Circles *a b* and *q o*; *p d n o b*

and

and *p m r q a*, the Repreſentatives of the two Circles paſſing thro' the Poles *p* and *c*.

I ſay,

1. Of the great Circles *m d* and *a b*, the Arches, *m d* and *a b* cut off, are equal.

For the Arch *p d b* is equal to *c b d*, and *p m a* equal to *c a m*, and the Angle *a p b* equal to the Angle *a c b*: Wherefore, the Triangles *a p b* and *m a c b d* are equal, and conſequently *a b* equal to *m d*. Q. E. D.

2. Of the leſſer Circles *q o* and *r n* equally diſtant from the Poles *c* and *p*. I ſay, the Arches *q o* and *r n* are equal, for the Arch *c a q* is equal to *p m r*, and *c b o* equal to *p d n*, and the Angle *q a c b o* equal to *r p n*, as before; wherefore the Triangles *q a c b o* and *r p n* are equal, and conſequently the Arches *q o* and *r n* are equal. Q. E. D.

Whence it follows,

1. That if through the Pole of any projected great Circle, and any Part or Segment thereof, Lines be drawn to the Periphery of the *Plane of Projection*, they will cut off an Arch equivalent to the Segment of the projected Arch.

2. That if through the Pole of any ſmall Circle, Lines be drawn to the Extremities of any Segment or Portion thereof (and continued if need be) they will cut off an Arch equivalent to it in that ſmall Circle which is parallel to the *Plane of Projection*; and as far diſtant from the Under-Pole, as the given Circle is diſtant from its Upper-Pole.

Whoever is well acquainted with what has been ſaid of Projections in this *Section*, will find it no difficult Task to deſcribe any Circle great or ſmall in any Projection, or meaſure any Part or Segment of it when projected: And therefore I ſhall, in the following *Section*, conſider the Affections and Properties of the ſeveral Triangles made upon the Surface of the Globe, by the mutual Interſection of the great Circles; and draw ſuch Conſequences as will moſt eaſily conduce to the Solution of the ſeveral Caſes.

SECT.

SECT. II.

Of Spherical Trigonometry.

DEFINITIONS.

1. AS a *Plain Triangle*, the Subject of *Plain Trigonometry*, is formed by the mutual Interſection of three ſtrait Lines upon a *Plain Surface*: So a *Spherical Triangle*, the Subject of *Spherical Trigonometry*, is made by the meeting together, or Interſection of the Arches of three great Circles on the Surface of the *Sphere*.

2. So that as each Angle of a Plain Triangle is made by the meeting together of two Right Lines on a Plain Surface, each Angle of a *Spherical Triangle* is formed by the meeting together of two great Circles on a Spherical Surface.

3. *Again.* As a Plain Angle is meaſured by the Arch of a Circle, deſcribed from the Angular Point, and intercepted between the Lines which form the Angle: So a *Spherical Angle* is meaſured by the Arch of a great Circle, deſcribed about the Angular Point on the Surface of the Sphere, and intercepted between the Arches which conſtitute the Angle.

1. Hence it is, that if the Sides that contain the *Spherical Angle* are continued till they are Quadrants, the Arch of the great Circle (of which the Angular Point is the Pole) contained between them, is the meaſure of the Angle.

2. Hence it is, that a *Spherical Angle* is the ſame with the Angle of Inclination of the two Planes which paſs through the Circles conſtituting the Angle.

3. Hence it is, that the oppoſite Angles A and B, at the Interſections of any two great Circles are equal.

And ſince the Arch of a great Circle is the ſhorteſt Tract between two Points on the Surface of the Sphere, as a ſtrait Line is the ſhorteſt Tract between two Points on a Plain Surface, it follows, from the *Gemetrical Theorems* in the *Firſt Part*.

4. That the Arch of a great Circle falling upon another, makes Angles equal to two Right Angles.

5. That the oppoſite Angles, made by the Interſection of two great Circles are equal.

6. That of two *Spherical Triangles*, If one Angle in each, and the two containing Sides be reſpectively equal; or if one Side and the two adjacent Angles be ſeverally equal, then the other Parts, and conſequently the two Triangles are equal.

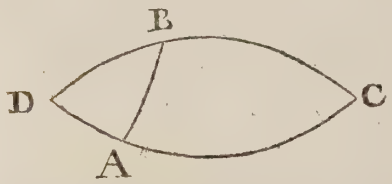
7. That an Equicrural *Spherical Triangle* hath its two Angles at the Baſe mutually equal; and, *on the contrary*, If the Angles at the Baſe are equal, the oppoſite Sides will be alſo equal.

8. Hence *Spherical Triangles* mutually equilateral, are alſo mutually equiangular.

9. That the greateſt Angle of every *Spherical Triangle*, is oppoſite to the greateſt Side; and, *on the contrary*, the greateſt Side ſubtends the greateſt Angle.

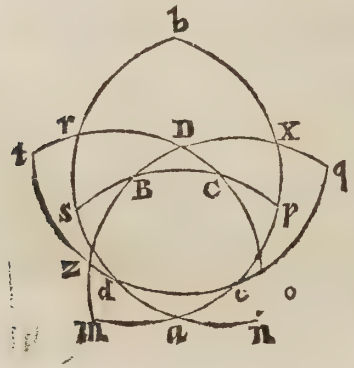
10. Hence it is, That any two Sides of a *Spherical Triangle* taken together, are greater than the third that remains.

Again. Since all great Circles cut each other into two equal Parts, it follows,



11. That every Side of a *Spherical Triangle*, is leſs than a Semicircle; for D B is leſs than D B C, and D A leſs than D A C.

12. That the Sum of the three Sides of any *Spherical Triangle*, is leſs than a Circle; for B A is leſs than B C and A C taken together; and conſequently B A more B D more D A, is leſs than D B C, more DAC; the Sum of which two laſt make a Circle.



From the *firſt Corollary* of the 3*d Definition*, it follows,

That the Poles *b*, *d*, *c* of the Triangle B D C, when connected together by Arches of great Circles; will with thoſe Arches conſtitute a Triangle *b d c*, the Supplement of whoſe { Angles / Sides } are equal to the { Sides / Angles } of the Triangle B D C. About

About the Points B, D, and C, as Poles, deſcribe the great Circles *b p c a*, *q c d t* and *b s d a*.

Now, *c* being the Pole of B D, *c m* is a Quadrant, and equal to *p b*; *b* being the Pole of B C: Add therefore to each Part the interjacent Arch *p c*, and the whole *c b* will be equal to *p m*, the Supplement of *p x* the Meaſure of the Angle at B.

Again: *d* being the Pole of D C, *d n* equal to a Quadrant, is equal to *s b*: Add *s d* to each, and it will make *b d* equal to *s n*, the Supplement of *r s*, the Meaſure of the Angle at C.

Laſtly, From *d o* equal to a Quadrant, equal to *c q*, take *c o*; and there remains *d c* equal to *o q*, the Meaſure of the Supplement of the Angle at D. *q. e. d.*

1. Hence it is manifeſt, that the Triangle *d a c*, conſtituted between the three nearer Poles, hath its $\left\{\begin{matrix}\text{Sides}\\\text{Angles}\end{matrix}\right\}$ equal to the $\left\{\begin{matrix}\text{Angles}\\\text{Sides}\end{matrix}\right\}$ of the Triangle B D C, (except that the greateſt Side *d c*, is the Supplement of the greateſt Angle at D; and the Angle at *a*, the Supplement of the greateſt Side B C.

2. Wherefore, Since the three Angles of any *Spherical Triangle* being given, the three Sides of another *Spherical Triangle* are known, whoſe Angles are equal to the Sides of the former Triangle (except that the greateſt Angle of the Triangle given, is equal to the Supplement of the greateſt Side of the Triangle found; and the greateſt Angle of the Triangle found, equal to the greateſt Angle of the Triangle given) it follows, that the three Angles being given, the Sides may be eaſily found; and, *on the contrary*, the Sides being given, the Angles may be eaſily had.

Hence, and from the 12*th*, it is, that the three Angles of any *Spherical Triangle*, are greater than two Right Angles.

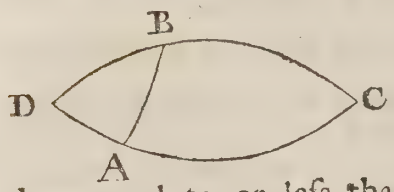
For by the 12*th*, *b d*, *d c*, *b c*, the Supplements of the Angles C, D and B of the Triangle B D C, are leſs than four Right Angles. That is, ſix Right Angles leſſened by the Sum of the Angles C, D, B of the Triangle B D C, are leſs than four Right Angles: That is, two Right Angles leſſened by the Sum of the Angles C, D, and B, is leſs than nothing; wherefore the Sum of the Angles C, D, and B, is greater than two Right Angles.

Again: *d a* and *a c*, the Meaſures of the Angles C and B taken together, are greater than *d c* the Supplement of the Angle at D: And ſince *d c* and its Supplement make but two Right Angles, *d a*, and the Supplement of *d c* taken together, ſhall make more: That is, the Angles B A C, D and C taken together, make more than two Right Angles.

3. And ſince by the *4th*, the Internal and External Angles of any *Spherical Triangle*, are equal to ſix Right Angles; it is manifeſt, that their Sum can never exceed ſix Right Angles.

Whence it is manifeſt, That in any *Spherical Triangle*, any two Angles taken together, are greater than the Supplement of the third that remains; and conſequently the External Angle is leſs than the Sum of the two Inward and Oppoſite.

From the *7th* and *9th Corollary*, it follows,



That in any *Spherical Triangle*, if the Sum of the Legs of the Angle DBA be greater than, equal to, or leſs than a Semicircle; the Internal Angle at the Baſe, ſhall be accordingly greater than, equal to, or leſs than the outward and oppoſite Angle at the Baſe.

1. If D B more B A, be equal to D B C, then B A is equal to B C; and conſequently the Angle B A C is equal to C, which is equal to D by the third. *q. e. d.*

2. If D B more B A, be greater than D B C, then B A is greater than B C; and conſequently the Angle B A C, is leſs than the Angle C, which is equal to the Angle D by the third; and conſequently the Angle at D, is greater than B A C. *q. e. d.*

3. If D B more B A, be leſs than D B C, then B A is leſs than B C; and conſequently the Angle B A C is greater than the Angle C, which is equal to the Angle D; wherefore the Angle D is leſs than B A C. *q. e. d.*

Hence likewiſe is it farther manifeſt, that if any two Sides of a *Spherical Triangle* taken together, be greater than, equal to, or leſs than a Semicircle, the Sum of the Angles oppoſite to them, ſhall be accordingly greater than, equal to, or leſs than a Semicircle, or two Right Angles.

1. When

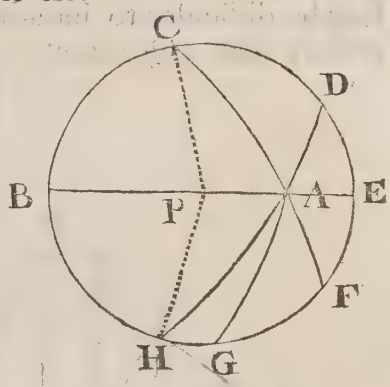
1. When D B more B A, is equal to D B C, then the Angles at D and D A B, are equal to D A B, more B A C; equal to two Right Angles by the *4th.*

2. When D B, more B A, is greater than D B C; then the Angles D, and D A B, are greater than the Sum of D A B and B A C; for D is greater than B A C.

3. When D B, more B A, is leſs than D B C, the Angle at D equal to C, is leſs than B A C; and conſequently the Angles D, and D A B taken together, are leſs than D A B and B A C taken together; which are equal to two Right Angles by the *4th.*

Hence it is manifeſt, that in an Equicrural Triangle, if one of the equal Legs be equal to, greater than, or leſs than a Quadrant, then the oppoſite Angles at the Baſe, are each accordingly equal to, greater than, or leſs than a Right Angle.

Of ſeveral Arches of great Circles, falling from the ſame Point of the Surface of the Sphere on another Circle, the greateſt is that which paſſes through the Pole of the Circle; and the next to this, is greater than that which is farther off.



For if P be the Pole of the Circle B H F E, and G the Pole of B P A E; then is A B equal A P more P B, equal A P more P H, greater than A H, which is greater than A F, which is therefore greater than A E; likewiſe the Arch H F E, greater than H P, greater than B H. Hence it is,

1. That the Legs of a Right-angled *Spherical Triangle*, are of the ſame Kind with the oppoſite Angles: For in the Triangle B H A, becauſe B A is greater than the Quadrant B P, the Angle B H A, is greater than the Right Angle B H P, and in the Triangle H A E, becauſe A E is leſs than the Quadrant P E; the Angle E H A is leſs than the Right Angle E H P.

2. If the Legs (and conſequently the Angles) are of the ſame Kind; *that is*, both greater, or both leſs than Quadrants, the Hypothenuſe ſhall be leſs than a Quadrant. Thus in the Triangles F A B and F A E, the Hypothenuſe F A is leſs than a Quadrant:

But

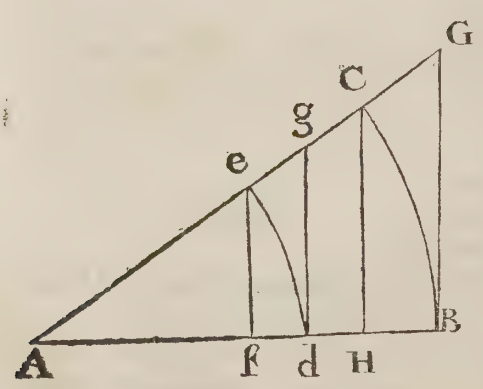
But if the Legs are of a different Kind, *that is*, one greater and the other leſs than a Quadrant, the Hypothenuſe ſhall be accordingly greater than a Quadrant: So in the Triangle A B H, the Hypothenuſe A H is greater than a Quadrant.

3. Wherefore, If the Hypothenuſeof a Right-angled Triangle is leſs than 90 Degrees, the Legs, and conſequently the Angles, are ſimilar; *that is*, both are either greater or leſs than a Quadrant: And, *on the contrary*, If the Hypothenuſe be greater than a Quadrant, the Legs, and conſequently their oppoſite Angles, ſhall be diſſimilar, one greater, the other leſs than 90 Degrees.

4. If the Hypothenuſe be leſs than 90 Degrees, each of the Legs ſhall be ſimilar to its adjacent Angle; *that is*, If the Leg be greater than 90 Degrees, the Angle ſhall be likewiſe greater, and the contrary: Wherefore, *vice verſa*, If the Leg of a Right-angled Triangle be ſimilar, or of the ſame Kind with its adjacent Angle, the Hypothenuſe ſhall be leſs than a Quadrant.

5. If the Hypothenuſe be greater than 90 Degrees, each Leg ſhall be diſſimilar to its adjacent Angle; and, *on the contrary*, If the Leg be diſſimilar to its adjacent Angle, the Hypothenuſe ſhall be greater than a Quadrant.

Propoſition I.



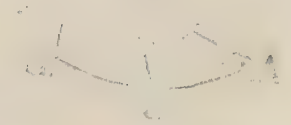
In Right-angled *Spherical Triangles*, having the ſame Acute Angle at the Baſe, the Sines of the Hypotheneſes are proportional to the Sines of the Perpendiculars, and the contrary.

For in the Right-angled *Spherical Triangles* A B C, and A *d e*, Right-angled at B and *d*. Becauſe C H and *e f* are parallel, it will be,

A *e* : A C : : *e f* : C H. That is,

As the Sine of the Hypothenuſe A *e*,
To the Sine of the Hypothenuſe A C;
So is the Sine of the Perpendicular *e d*,
To the Sine of the Perpendicular B C, and the contrary, *q. e. d.*

Propo-



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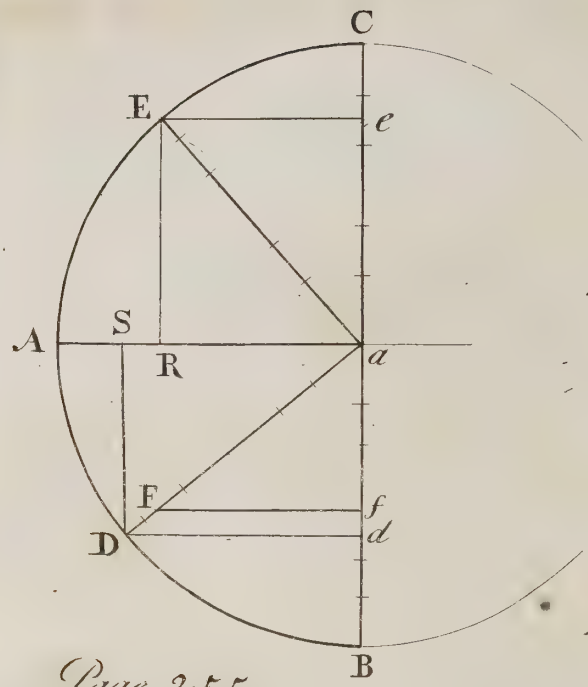

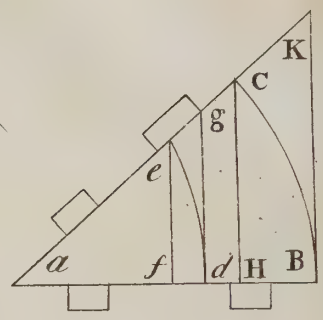

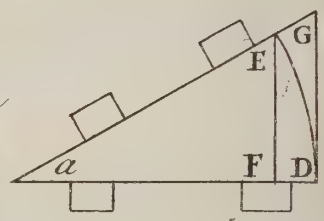

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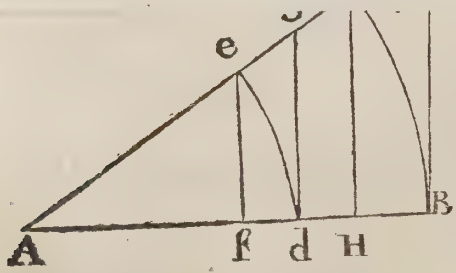


tional to the Sines of the Perpendiculars, and the contrary.

For in the Right-angled *Spherical Triangles* A B C, and A *d e*, Right-angled at B and *d*. Because C H and *e f* are parallel, it will be,

A *e* : A C : : *e f* : C H. That is,

As the Sine of the Hypothenuse A *e*,
To the Sine of the Hypothenuse A C;
So is the Sine of the Perpendicular *e d*,
To the Sine of the Perpendicular B C, and the contrary, *q. e. d.*

Propo-

Propoſition II.

In Right-angled *Spherical Triangles*, having the ſame Acute Angle at the Baſe, the Sines of the Baſes are proportional to the Tangents of the Perpendiculars, and the contrary.

For in the ſame Right-angled *Spherical Triangles* A *d e* and A B C, becauſe *d g* and B G are parallel, it will be,

A *d* : A B : : *d g* : B G. That is,

As the Sine of the Baſe A *d*,
To the Sine of the Baſe A B;
So is the Tangent of the Perpendicular *d e*,
To the Tangent of the Perpendicular B C.

For the Benefit of thoſe who cannot readily conceive how A *e* and A C are the Sines of the Hypothenuſes: Alſo how A *d* and A B are the Sines of the Baſes of two Right-angled *Spherical Triangles*, which have the ſame Acute Angle at the Baſe, the following Contrivance is made uſe of.

The

The upper Quadrant A E C of the Semicircle B A C muſt be turned about upon its Axis A R *a*, till the Sectors of Circles *a* B C and *a* D E, with their Tangents, being erected perpendicularly over the Lines *a* B and *a* D, the Points E and C of the Quadrant, A E C *a* coincide with the Points E and C of the Sectors *a* D E and *a* B C, then will be formed the *Spherical Triangles* A D E and A B C, Right-angled at D and B, and *a* C will be the Sine of the Hypothenuſe *a* C, of the Triangle A B C, alſo *a* B the Sine of its Baſe *a* B.

Again: E F equal to *e f* is the Sine of the Perpendicular D E, and D G equal to *d g*, the Tangent of the ſame Perpendicular: wherefore, *&c.* as was before determined.

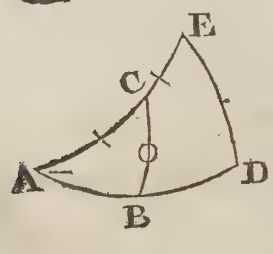
By the Help of theſe two *Propoſitions*, and the preceding Properties, it will not be difficult to give *Solutions* to all the ſeveral Caſes of Right and Oblique-angled *Spherical Triangles.*

SECT. III.

Of the Solution *of the* Sixteen Caſes *of* Right-angled Spherical Triangles.

Caſe I.

GIVEN the Hypothenuſe and one of the Acute Angles, to find the Side oppoſite to the given Angle.



Example.

Given $\left\{\begin{matrix} A\,C \\ \angle A \end{matrix}\right\}$ to find B C.

Produce A C and A B till they are Quadrants, and deſcribe the Arch E D, which will be equal to the Angle A, and it will be by *Propoſition the 1ſt*,

s, A E : s, A C : : s, E D : s, C B. That is,

As the Radius,
To the Sine of the Hypothenuſe A C;
So is the Sine of the Given Angle B A C,
To the Sine of the oppoſite Side B C. *q. e. d.*

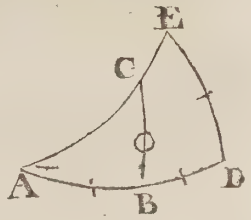
Caſe

Case II.

Given one Side and the adjacent Angle, to find the other Side.

Example.

Given $\left\{\begin{matrix} AB \\ <A \end{matrix}\right\}$ to find BC.



Produce AB and AC till they are Quadrants, and describe the Arch ED, which is equal to the Angle at A, and it will be,

s, AD : s, AB : : t, DE t, BC, by *Prop. the 2d.* That is,

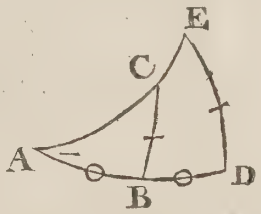
As the Radius,
To the Sine of the Given Side AB;
So is the Tangent of the Given Angle BAC,
To the Tangent of the Side BC. *q. e. i.*

Case III.

Given one Side and the Angle opposite, to find the other Side.

Example.

Given $\left\{\begin{matrix} BC \\ <A \end{matrix}\right\}$ to find AB.



Produce AB to D, and AC to E, till they are Quadrants, and describe the Arch DE, and it will be, *by Prop. the 2d,*

t, ED : t, CB : : s, DA : s, BA. That is,

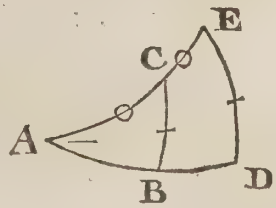
As the Tangent of the Given Angle BAC,
To the Tangent of the Given Side BC;
So is the Radius,
To the Sine of the Side AB. *q. e. i.*

Case IV.

Given one Side and an Angle opposite, to find the Hypothenuse.

Example.

Given $\left\{\begin{matrix} BC \\ <A \end{matrix}\right\}$ to find AC.



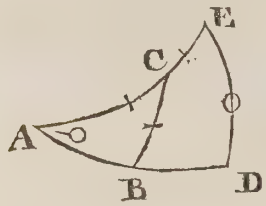
Having produced AB and AC to Quadrants, and connected the Points E and D by an Arch, it will be, by *Prop. the 1st,*

s, DE : s, BC : : s, EA : s, CA. That is,

As the Sine of the Given Angle B A C,
To the Sine of the oppoſite Side B C;
So is the Radius,
To the Sine of the Hypothenuſe. *q. e. i.*

Caſe V.

The Hypothenuſe and one Side, to find the Angle oppoſite to the Given Side.



Example.

Given $\left\{\begin{matrix} A\,C \\ B\,C \end{matrix}\right\}$ to find the Angle at A.

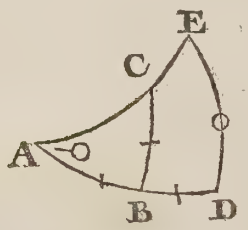
Continue A C and A B to Quadrants, and draw the Arch D E, and it will be, *by Prop. the 1ſt*,

s A C : s A E : : s, C B : s E D. That is,

As the Sine of the Hypothenuſe,
To the Sine of the Perpendicular;
So is the Radius,
To the Sine of the Angle at A. *q. e. i.*

Caſe VI.

The Sides Given, to find either of the Angles.



Example.

Given $\left\{\begin{matrix} A\,B \\ B\,C \end{matrix}\right\}$ to find the Angle at A.

One of the Sides and Hypothenuſe being produced till they are Quadrants, and the Extremities connected by the Arch D E, it will be, *by Prop. the 2d*,

s A B : s A D : : t B C : t, D E. That is,

As the Sine of one of the Sides A B,
To the Radius;
So is the Tangent of the other Side B C,
To the Tangent of its oppoſite Angle B A C. *q. e. i.*

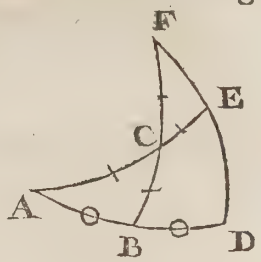
Caſe

Case VII.

Given the Hypothenuse and one of the Legs, to find the other Leg

Example.

Given $\left\{\begin{matrix} A\,C \\ B\,C \end{matrix}\right\}$ to find A B.



Continue A C, A B and B C, till they are Quadrants, and connect the Points F, E, and D, by the Arch F E D, and it will be, F C : F B : : C E : B D. That is, *by Prop. the 1st*,

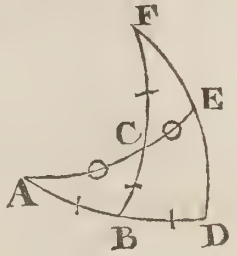
As the Sine of the Complement of C B,
To the Radius;
So is the Sine of the Complement of A C, or the Co-sine of AC,
To the Sine of the Complement or Co-sine of A B. *q. e. i.*

Case VIII.

The Sides being given, to find the Hypothenuse;

Example.

Given $\left\{\begin{matrix} A\,B \\ B\,C \end{matrix}\right\}$ to find A C.



Having continued the Sides A B and BC, also the Hypothenuse to Quadrants, and joined the Points F, E, and D, by the Arch FED, it will be, F B : F C : : B D : C E. That is, *by Prop. the 1st*,

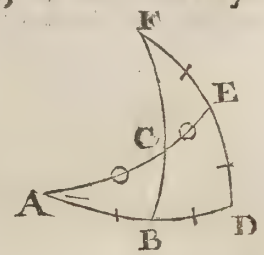
As the Radius,
To the Co-sine of one of the Sides B C;
So is the Co-sine of the other Side A B,
To the Co-sine of the Hypothenuse A C. *q. e. i.*

Case IX.

One Side and the adjacent Angle being given, to find the Hypothenuse.

Example.

Given $\left\{\begin{matrix} A\,B \\ <A \end{matrix}\right\}$ to find A C.



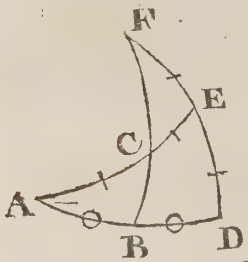
Produce the Sides and Hypothenuse to Quadrants, and join the Points F, E, and D, by an Arch; because F B is a Quadrant, F D shall be one likewise; and it will be, by *Prop. the 2d*,

s, F D : s, F E : : t, B D : t, C E. That is,

As the Radius,
To the Co-sine of the Given Angle B A C;
So is the Co-tangent of the Given Side A B,
To the Co-tangent of the Hypothenuse. *q. e. i.*

Case X.

The Hypothenuse and an Angle being given, to find the adjacent Side.



Example.

Given $\left\{\begin{matrix} A\,C \\ \sphericalangle A \end{matrix}\right\}$ to find A B.

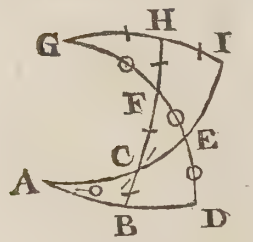
Having produced the Sides as usual, it will be, by *Prop. the 2d*,

s, F E : s, F D : : t, E C : t, D B. That is,

As the Co-sine of the Given Angle A,
To the Radius;
So is the Co-tangent of the Hypothenuse A C,
To the Co-tangent of the Side. *q. e. i.*

Case XI.

One of the Sides and the adjacent Angle being given, to find the other Angle.



Example.

Given $\left\{\begin{matrix} A\,C\,B \\ B\,C \end{matrix}\right\}$ to find the Angle A.

Having, as in the former *Case*, produced the Hypothenuse and Sides to Quadrants, and connected the Points F, E, and D by an Arch; produce the Sides and Hypothenuse of the Triangle C E F, till they are Quadrants; and join the Points G, H, I, by the Arch G H I, which will be a Quadrant; and it will be, by *Prop. the 1st*,

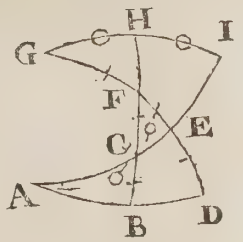
s C H : s C F : : s, H I : s F E. That is,

As the Radius,
To the Sine of the Angle F C E, equal to the Given Angle A C B;
So is the Co-sine of the Side Given B C,
To the Co-sine of the opposite Angle. A, *q. e. i.*

Case

Case XII.

One Side and the Angle oppoſite being given, to find the other Angle. *Example.*



Given $\left\{ \begin{matrix} < A \\ B C \end{matrix} \right\}$ to find the Angle at C.

Having made the ſame Preparations as in the former *Caſe*, it will be,

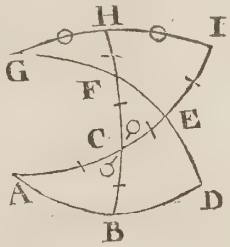
s, C F : s, C H : : s F E : s, H I,

by Prop. the 1ſt, That is,

As the Co-ſine of the Given Side B C,
To the Radius;
So is the Co-ſine of the Given Angle B A C,
To the Sine of the Angle A C B. *q. e. i.*

Case XIII.

One Side and the Hypothenuſe being given, to find the contained Angle. *Example.*



Given $\left\{ \begin{matrix} A C \\ B C \end{matrix} \right\}$ to find the Angle at C.

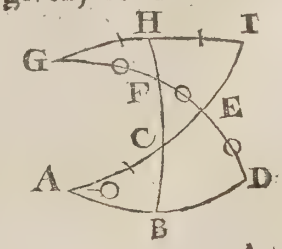
Continue the Sides as in the *11th Caſe*, and it will be, t, E I : t, F H : : s, I G : s, H G.

That is, *by Prop. the 2d*,

As the Tangent of the Hypothenuſe A C,
To the Tangent of the Side Given B C;
So is the Radius,
To the Co-ſine of the Angle at C. *q. e. i.*

Case XIV.

The Hypothenuſe and one of the Angles given, to find the other Angle. *Example.*



Given $\left\{ \begin{matrix} A C \\ > C \end{matrix} \right\}$ to find the Angle at A.

Produce the Sides as in *Caſe the 11th*, and it will be, *by Prop. the 2d*,

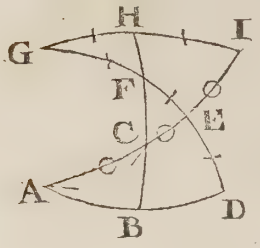
s, C I ; s, C E : : t, I H : t, E F. That is,

A

As the Radius,
To the Co-ſine of the Hypothenuſe A C;
So is the Tangent of the Angle at C,
To the Co-tangent of the Angle at A. *q. e. i.*

Caſe XV.

The Angles given, to find the Hypothenuſe.



Example.

Given $\left\{\begin{matrix}\angle A\\ \angle C\end{matrix}\right\}$ to find A C.

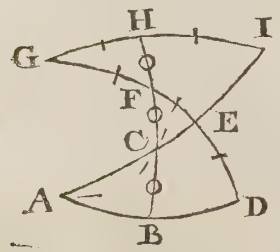
Having produced the Sides as in the 11*th Caſe*, it will be, *by Prop. the* 2*d*,

t, H I : t, F E : : s, I C : s, E C. That is,

As the Tangent of one of the Given Angles C,
To the Co-tangent of the other Angle A;
So is the Radius,
To the Co-ſine of the Hypothenuſe.

Caſe XVI.

The Angles given, to find one of the Sides.



Example.

Given $\left\{\begin{matrix}\angle A\\ \angle C\end{matrix}\right\}$ B C required.

Make the ſame Productions as in *Caſe the* 11*th*, and it will be, *by Prop. the* 1*ſt*,

s I H : s, E F : : s, H C : s, F C. That is,

As the Sine of the Angle adjacent to the Side required,
To the Co-ſine of the Angle oppoſite;
So is the Radius,
To the Co-ſine of the Side. *q. e. i.*

SECT.

SECT. IV.

Containing the Solution *of the* Twelve Cases *of* Oblique-angled Spherical Triangles.

EVery Oblique-angled *Spherical Triangle* may be reduced into Right-angled *Spherical Triangles*, by letting fall a Perpendicular from some one of its Angles upon the opposite Side produced, if need be.

The Perpendicular when let fall, either divides the Triangle into two Right-angled Triangles, or makes it two, by adding one to it.

If the Perpendicular falls within (as in the 1*st* and 2*d Figures*) it divides the Triangles B D C, B D C first given into two Right-angled ones, B A D and C A D. But,

If the Perpendicular falls without, (as in *Fig.* the 3*d*) it produces two Right-angled Triangles B A D and C A D, by adding one Right-angled Triangle C A D, to the Triangle B D C first given.

Fig. 1

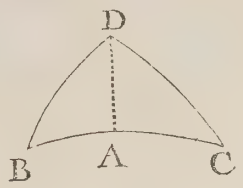


Fig. 2

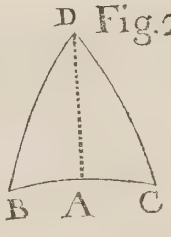
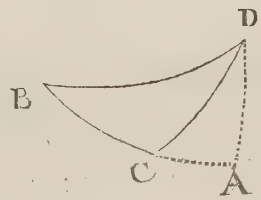


Fig. 3



If the Angles at the Base of the Oblique Triangle are Similar; *that is*, both Acute, (as in *Fig.* the 1*st.*) or both Obtuse, (as in *Fig.* the 2*d*) then the Perpendicular let fall from the Angle opposite to the Base, falls within the Triangle. But,

If the Angles at the Base are Dissimilar, *that is*, one be Obtuse, and the other be Acute, (as in *Fig.* the 3*d*) then the Perpendicular let fall from the Angle opposite to the Base will fall without the Triangle; and consequently the Base must be produced to meet it. These two last Inferences are abundantly manifest, from the *Fig.* in *Page* 253.

Hence,

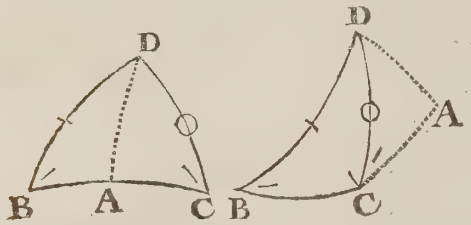
Hence, and by the Help of the former *Section*, in any Oblique-angled *Spherical Triangle* (having ſufficient *Data*) the Things ſought may be eaſily found; provided, that when the Perpendicular is let fall, there are two Things given beſides the Right Angle, in one of the Right-angled Triangles produced.

Wherefore it is neceſſary in all *Caſes*, that the Perpendicular fall from the End of a Given Side, and oppoſite to ſome Given Angle.

Hence thoſe Caſes wherein the Sides are given, and not the Angles, or the Angles and not the Sides, cannot be reſolved after this Manner, and therefore does not fall under the General Law; but require ſome other peculiar Contrivance, which ſhall be ſhewn in its due Place; and in the mean time we ſhall proceed to the *Solution* of ſuch *Caſes* as fall under the *General Head*.

Caſe I.

Two Angles and a Side oppoſite to one of them being given, to find the other oppoſite Side.



Example.

Given $\left\{\begin{matrix} \angle B \\ BD \\ \angle C \end{matrix}\right\}$ to find DC.

The Perpendicular DA being let fall from the end of the Given Side DB, and oppoſite to the Given Angle B, it will be, by *Caſe* the 1*ſt*, to find the Perpendicular,

R : s, BD : : s, $\angle$ B : s, DA. That is,

As the Radius,
To the Sine of the Given Side BD;
So is the Sine of the Adjacent Angle DBC,
To the Sine of the Perpendicular.

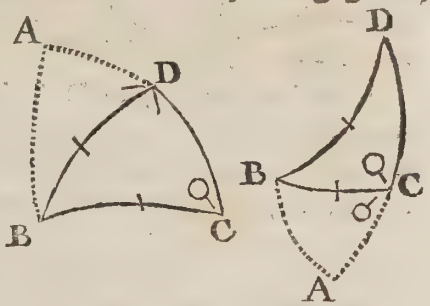
Which being obtain'd in the Triangle DAC, Right-angled at A, are given DA, the Perpendicular before found; and the Angle at the Baſe ACD, to find the Hypothenuſe DC, and it will be, by *Caſe* the 4*th* of *Right-angled*, &c.

s, $\angle$ C : s DA : : R : s, DC. That is,

As the Sine of the Angle oppoſite to the Given Side,
To the Sine of the Perpendicular;
So is the Radius,
To the Sine of the Side DC. *q. e. i.*

Caſe II.

Two Sides and an Angle oppoſite to one of them, being given, to find the other oppoſite Angle.



Example.

Given { BD, ∠D, BC } required ∠ at C.

The Perpendicular B A, being let fall from B the end of the Given Side B D, and oppoſite to the Given Angle B D C, falls without the Triangle, becauſe the Angles at D and C are diſſimilar; and therefore to find the Perpendicular, it will be, *by Caſe the 1ſt* of *Right-angled*, &c.

R: s, B D :: s, ∠ B D C *or* B D A: s, B A. That is,

As the Radius,
To the Sine of the Given Side B D, adjacent to the Given Angle D.
So is the Sine of the Given Angle B D C,
To the Sine of B A the Perpendicular.

Which being had, the Angle at C is eaſily obtained; by the Help of the *5th Caſe* of *Right-angled Spherical Triangles.*

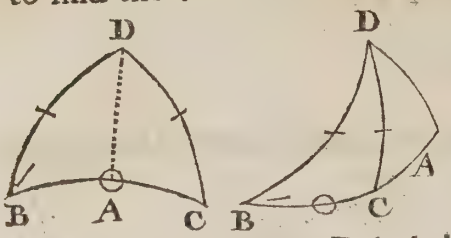
For ſince the Hypothenuſe BC of the Triangle B A C, and B A the Perpendicular are given, it will be, *by the ſame Caſe*,

s, B C: R :: s, B A : s ∠ C. That is,

As the Sine of the Side oppoſite to the Given Angle,
To the Radius;
So is the Sine of the Perpendicular,
To the Sine of the Angle at C. *q. e. i.*

Case III.

Two Sides and an Angle oppoſite to one of them, being given, to find the other Side.



Example.

Given $\left\{\begin{matrix} BD \\ \angle B \\ DC \end{matrix}\right\}$ to find BC.

The Perpendicular DA being let fall from the end of BD, and oppoſite to the Angle at B; BD and DC become Hypothenuſes in both Triangles, and therefore to find the Perpendicular DA, it will be, by *Caſe the 1ſt*, of Right-angled, &c.

R : s, BD : : s, $\angle$ B s, DA. That is,

As the Radius,
To the Sine of the Given Side BD;
So is the Sine of the adjacent Angle B,
To the Sine of the Perpendicular.

2. To find BA it will be,

R : c s, $\angle$ B : : t, BD : t, BA. That is,

As the Radius,
To the Co-ſine of the adjacent Angle B;
So is the Tangent of the Given Side BD,
To the Tangent of BA.
Which being obtain'd, AC is eaſily had.

For, As c s, DA : R : : c s, DC : c s, CA. That is,

As the Co-ſine of the Perpendicular DA,
To the Co-ſine of the Side oppoſite to the Given Angle;
So is the Radius,
To the Co-ſine of AC.

If the Perpendicular fall $\left\{\begin{matrix} \text{within} \\ \text{without} \end{matrix}\right\}$ then AC $\left\{\begin{matrix} \text{added to} \\ \text{ſub. from} \end{matrix}\right\}$ AB, gives BC. *q. e. i.*

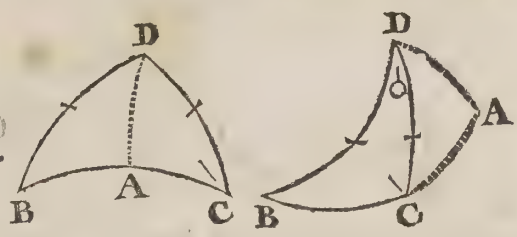
Caſe

Case IV.

Given two Sides and an Angle oppoſite to one of them, to find the contained Angle.

Example.

Given $\left\{\begin{matrix} B\,D \\ <C \\ D\,C \end{matrix}\right\}$ to find Angle D.



The Perpendicular being let fall, it will be, by *Caſe the 1ſt*,

1. R : s, D C : : s, $<$C : s, D A. That is,

As the Radius,
To the Sine of the adjacent Side C D;
So is the Sine of the Given Angle B C D,
To the Sine of the Perpendicular D A.

2. To find the Angle A D C, it will be, by *Caſe the 14th*,

R : c s D C : : t, $<$C : c t, $<$D. That is,

As the Radius,
To the Co-ſine of the Side D C, adjacent to the Given Angle C;
So is the Tangent of the Given Angle at C,
To the Co-tangent of the Angle A D C.

Again; In the Triangle B A D, having B D and D A, it will be, to find Angle B D A, by *Caſe the 13th*,

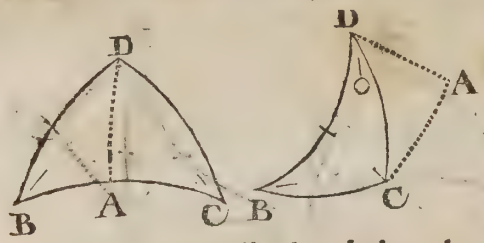
As t, B D : R : : t, D A : c s, $<$D. That is,

As the Tangent of the oppoſite Side B D,
To the Radius;
So is the Tangent of the Perpendicular D A,
To the Co-ſine of the Angle A D B.

If the Perpendicular fall $\left\{\begin{matrix} \text{within} \\ \text{without} \end{matrix}\right\}$ then A D C $\left\{\begin{matrix} \text{added to} \\ \text{taken from} \end{matrix}\right\}$ B D A, gives B D C. *q. e. i.*

Case V.

Two Angles and a Side oppoſite to one of them, being given, to find the other Angle.



Example.

Given $\left\{\begin{matrix}\angle B\\ B\,D\\ \angle C\end{matrix}\right\}$ to find the $\angle$ D.

The Perpendicular being let fall, it will be, by *Caſe the* 14*th*, to find B D A.

R : c s, B D : : t, $\angle$ B : c t, $\angle$ B D A.

As the Radius,
To the Co-ſine of the Given Side B D;
So is the Tangent of the Angle at B,
To the Co-tangent of the Angle B D A.

2. For the Perpendicular, it will be, by *Caſe the* 1*ſt*,

R : s, B D : : s, $\angle$ B : s, D A. That is,

As the Radius,
To the Sine of the Given Side B D;
So is the Sine of the Angle at B,
To the Sine of the Perpendicular.

Laſtly, To find A D C, it will be, by *Caſe the* 12*th*,

c s, D A : R : : c s, $\angle$ C : s A D C. That is,

As the Co-ſine of the Perpendicular,
To the Radius,
So is the Co-ſine of the Angle oppoſite to the Given Side,
To the Sine of the Angle A D C.

If the Perpendicular fall $\left\{\begin{matrix}\text{within}\\ \text{without}\end{matrix}\right\}$ then A D C $\left\{\begin{matrix}\text{added to}\\ \text{ſub. from}\end{matrix}\right\}$ A D B, gives C D B. *q. e. i.*

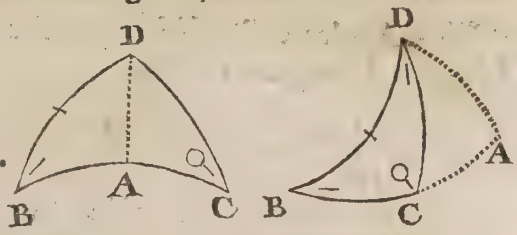
Caſe

Case VI.

Two Angles and the Interjacent Side given, to find the Opposite Angle

Example.

Given $\left\{\begin{matrix}\angle B \\ B\ D \\ \angle D\end{matrix}\right\}$ to find the $\angle$ at C.



The Perpendicular D A being let fall from D, and opposite to the Angle at B, it will be to find the Angle BDA, by *Case the* 14*th*,

R : c s, D B : : t, $\angle$ B : c t, $\angle$ B D A. That is,

As the Radius,
To the Co-sine of the Given Side D B;
So is the Tangent of the Angle at B,
To the Co-tangent of the Angle B D A.

If the Perpendicular fall within, then B D A taken from B D C, leaves A D C; but if the Perpendicular fall without, then B D C taken from B D A, leaves A D C.

2. For the Perpendicular D A, it will be, by *Case the* 1*st*,

R : s, B D : : s, $\angle$ B : s D A. That is,

As the Radius,
To the Sine of the Given Side B D;
So is the Sine of the Angle at B,
To the Sine of the Perpendicular.

3. For the Angle at C, it will be, by *Case the* 11*th*,

R : s, $\angle$ A D C : : c s, D A : c s, : $\angle$ C. That is,

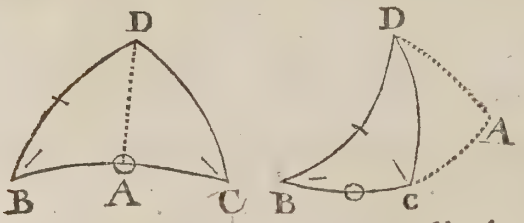
As the Radius,
To the Sine of the Angle A D C;
So is the Co-sine of the Perpendicular A D,
To the Co-sine of the Angle at C.

Which, if BDA be greater than DBC, and the $\angle$B $\left\{\begin{matrix}\text{Acute} \\ \text{Obtuse}\end{matrix}\right\}$ shall be $\left\{\begin{matrix}\text{Obtuse} \\ \text{Acute}\end{matrix}\right\}$ But if BDA be lesser than DBC and the $\angle$B, $\left\{\begin{matrix}\text{Acute} \\ \text{Obtuse}\end{matrix}\right\}$ shall be $\left\{\begin{matrix}\text{Acute} \\ \text{Obtuse}\end{matrix}\right\}$

Case

Case VII.

Two Angles and a Side oppofite to one of them, being given, to find the Interjacent Side.



Example.

Given $\left\{\begin{matrix}<B \\ BD \\ <C\end{matrix}\right\}$ to find BC.

Having let fall the Perpendicular D A,

In the Triangle B A D, are given the Hypothenufe B D, and the Angle at the Baſe B, to find B A and D A.

1. For D A the Perpendicular, it will be, by *Caſe the 1ſt*,

R : s, B D : : s, $<$ B : s, D A. That is,

As the Radius,
To the Sine of the Given Side B D;
So is the Sine of the Angle at B,
To the Sine of the Perpendicular D A.

2. For the Baſe B A, it will be, by *Caſe the 10th*,

R : c s, $<$ B : : t, B D : t, B A. That is,

As the Radius,
To the Co-ſine of the Angle at B;
So is the Tangent of the Side B D,
To the Tangent of B A.

Laſtly, In the Triangle A D C, are given A D, and the Angle at C; whence to find A C, it will be, by *Caſe the 3d*,

t, $<$ C : t, A D : : R : s, A C. That is,

As the Tangent of the Angle at C,
To the Tangent of the Perpendicular A D;
So is the Radius,
To the Sine of A C.

If the Perpendicular fall $\left\{\begin{matrix}\text{within} \\ \text{without}\end{matrix}\right\}$ then A C $\left\{\begin{matrix}\text{added to} \\ \text{ſub. from}\end{matrix}\right\}$ BA, gives BC. *q.e.i.*

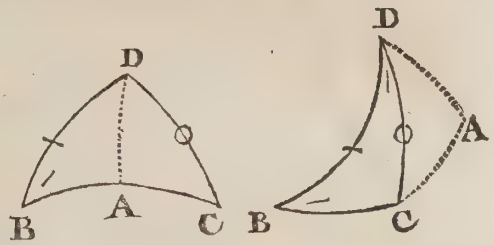
Caſe

Case VIII.

Two Angles and the Interjacent Side being given, to find either of the other opposite Sides.

Example.

Given $\left\{\begin{matrix}\angle B \\ B\,D \\ \angle D\end{matrix}\right\}$ to find D C.



1. To find B D A, it will be, by *Case the 14th*,

R : c s, B D : : t, $\angle$ B : c t, $\angle$ B D A. That is,

As the Radius,
To the Co-sine of the Given Side;
So is the Tangent of the Angle at B,
To the Co-tangent of the Angle B D A.

If the Perpendicular fall within,
Then B D A taken from B D C, leaves A D C.
If the Perpendicular fall without,
Then B D C taken from B D A, leaves A D C.

2. For the Perpendicular, it will be, by *Case the 1st*,

R : s, B D : : s, $\angle$ B : s A D. That is,

As the Radius,
To the Sine of the Given Side;
So is the Sine of the Angle at B,
To the Sine of the Perpendicular.

3. In the Triangle A D C, having the Perpendicular D A, and the Angle at D, it will be, to find D C, by *Case the 9th*,

R : c s, $\angle$ D : : c t, D A : c t, D C. That is,

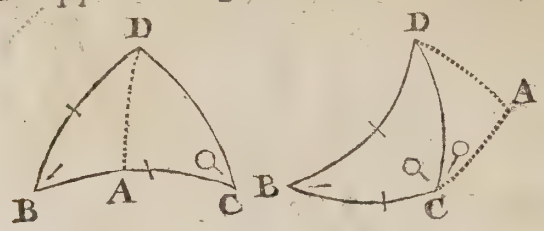
As the Radius,
To the Co-sine of the Angle C D A;
So is the Co-tangent of the Perpendicular D A,
To the Co-tangent of D C, the Side sought.

If C D A be $\left\{\begin{matrix}\text{Similar} \\ \text{Dissimilar}\end{matrix}\right\}$ $\left\}\begin{matrix}\text{to the} \angle \text{at B,} \\ \text{D C shall be}\end{matrix}\right.$ $\left\{\begin{matrix}\text{less} \\ \text{greater}\end{matrix}\right\}$ than a Quadrant.

Case

Case IX.

Two Sides and the contained Angle being given, to find one of the oppoſite Angles.



Example.

Given $\left\{\begin{matrix} BD \\ \angle B \\ BC \end{matrix}\right\}$ to find $\angle C$.

1. For B A, it will be, by *Caſe the* 10*th*,

R : c s, $\angle$B : : t, B D : t, B A. That is,

As the Radius,
To the Co-ſine of the Angle at B;
So is the Tangent of B D, the Side oppoſite to the Angle ſought,
To the Tangent of B A.

If the Perpendicular falls within the Triangle,
Then B A taken from B C, leaves A C.
But if the Perpendicular fall without,
Then B C taken from B A, leaves A C.

2. To find the Perpendicular D A, it will be, by *Caſe the* 1*ſt*,

R : s, B D : : s, $\angle$B : s, D A. That is,

As the Radius,
To the Sine of the Side oppoſite to the Angle ſought;
So is the Sine of the Given Angle,
To the Sine of the Perpendicular D A.

3. To find the Angle at C, by the Help of D A and A C, it will be, by *Caſe the* 6*th*,

s, C A : R : : t A D : t, $\angle$C. That is,

As the Sine of the adjacent Side C A,
To the Radius;
So is the Tangent of the Perpendicular D A,
To the Tangent of the Angle at C. *q. e. i.*

If B A be $\left\{\begin{matrix} \text{leſs} \\ \text{greater} \end{matrix}\right\}$ than BC, the Angle at C ſhall be $\left\{\begin{matrix} \text{Similar} \\ \text{Diſſimilar} \end{matrix}\right\}$ to the $\angle$ at B.

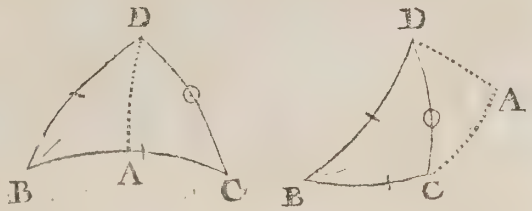
Caſe

Case X.

Two Sides and the contained Angle given, to find the other Side.

Example.

Given { B C, ∠B, B D } to find D C.



1. For B A, it will be, by *Case the* 10*th*,

R : c s, ∠ B : : t, B D : t, B A. That is,

As the Radius,
To the Co-sine of the Angle at B;
So is the Tangent of B D,
To the Tangent of B A.

If the Perpendicular fall within,
Then B A taken from B C, leaves A C.
If the Perpendicular fall without,
Then B C taken from B A, leaves A C.

2. For the Perpendicular D A, it will be, by *Case the* 1*st*,

R : s, B D : : s, ∠ B : s, D A. That is,

As the Radius,
To the Sine of the Given Side B D,
So is the Sine of the Angle at B;
To the Sine of the Perpendicular D A.

3. For the Side D C, it will be, by *Case the* 8*th*,

R : c s, A C : : c s, D A : c s, D C. That is,

As the Radius,
To the Co-sine of the Segment A C;
So is the Co-sine of the Perpendicular at D A,
To the Co-sine of the Side D C. *q. e. i.*

If A C be { Similar / Dissimilar } to D A, or the ∠ at B, then D C shall be { less / greater } than a Quadrant.

In the *Solution* of theſe *Ten Caſes* of *Oblique-angled Spherical Triangles*, the Perpendicular has been all along inveſtigated, as a Neceſſary Ingredient to diſcover the Thing ſought; which, though it be eſteemed Unartificial, yet has ſome peculiar Advantages attending it, as is well known to thoſe who are verſed in *Calculation*, and is not only very eaſy to be underſtood, but contributes very much to the better underſtanding of the Method generally made Uſe of, for finding the Thing required, without ſeeking the Length of the Perpendicular, which ſhall be ſhewn next; and to to that end, the two following *Lemmas* are premiſed.

Lemma the 1ſt.

If in two Sets of Proportionals, the two firſt Terms, or the firſt and third Terms in each be the ſame; the others ſhall be directly proportional. That is,

In Numbers.	*In Quantities.*
If it be as 2 to 6, ſo is 3 to 9.	If it be $a : b :: c : d$.
And as 2 to 6, ſo is 4 to 12.	And as $a : b :: m : n$.
Or,	Or,
If it be as 2 to 3, ſo is 6 to 9.	If it be $a : c :: b : d$.
And as 2 to 4, ſo is 6 to 12.	And as $a : m :: b : n$.
It ſhall be 3 : 9 :: 4 : 12.	It ſhall be $c : d :: m : n$.

Lemma the 2d.

If in two Sets of Proportionals, the two middle Terms in each be the ſame, the others will be reciprocally proportional.

In Numbers.	*In Quantities.*
If it be 12 : 6 :: 8 : 4.	If it be $c : a :: b : d$.
And as 24 : 6 :: 8 : 2.	And as $m : a :: b : n$.
It ſhall be 24 : 12 :: 4 : 2.	It ſhall be $m : c :: d : n$.

The

The Solution *of the first* Ten Cases, *without seeking the* Perpendicular.

Case I.

Two Angles and a Side opposite to one of them being given, to find the other opposite Side.

Given $\left\{\begin{matrix}\angle B\\ B\,D\\ \angle C\end{matrix}\right.$ to find D C. *See the* Fig. *of* Case the 1st.

The Perpendicular D A, being common to the Triangles D A B and D A C, it will be,

$$s, \angle B : R :: s, D\,A : s, B\,D.$$
$$s, \angle C : R :: s, D\,A : s, D\,C.$$

Wherefore, by *Lemma the 2d*, because the two middle Terms in each are the same, it will be,

$$s, \angle C : s, \angle B :: s, B\,D : s, D\,C.$$ That is,

As the Sine of the Angle at C,
To the Sine of the Angle at B;
So is the Sine of the Side B D,
To the Sine of the Side D C. *q. e. i.*

Again, in

Case II.

Where two Sides and an Angle opposite to one of them are given, to find the opposite Angle.

Given $\left\{\begin{matrix}B\,D\\ \angle D\\ B\,C\end{matrix}\right\}$ to find $\angle$ at C. *See the* Fig. *of* Case *the 2d.*

Because B A is common to the Triangles B A D and B A C, it will be,

$$s, D\,B : R :: s, B\,A : s, \angle D.$$
$$s, B\,C : R :: s, B\,A : s, \angle C.$$

Where, by *Lemma the 2d*, because the middle Terms in each are the same, it will be,

$$s, B\,C : s, B\,D :: s, \angle D : s, \angle C.$$ That is,

As the Sine of the Side B C,
To the Sine of the Side B D;
So is the Sine of the Angle at D,
To the Sine of the Angle at C. *q. e. i.*

Hence, and from the *former Case* it follows,

That in all *Spherical Triangles*, the Sines of the Sides are to each other, as the Sines of their oppoſite Angles, and the contrary.

Again, in

Caſe III.

Where two Sides and an Angle oppoſite to one of them, are given, to find the other Side that remains.

Having made the ſame Preparations, as in *Caſe the 3d*, and found the Baſe B A; becauſe the Perpendicular D A is common to both, it will be,

c s, D A : R : : c s, B D : c s, B A.
c s, D A : R : : c s, D C : c s, C A.

Now, becauſe the two firſt Terms in each are the ſame, it will be, by *Lemma the 1ſt*,

c s, B D : c s, B A : : c s, D C : c s, C A.

As the Co-ſine of the Side B D,
To the Co-ſine of the Segment B A;
So is the Co-ſine of the other Side D C,
To the Co-ſine of the Segment C A; which being obtain'd, the Side B C may be found, as in *Caſe the 3d of Sect. the 4th.*

Hence it is, That the Co-ſines of the Baſes are directly proportional to the Co-ſines of the Hypothenuſes, and the contrary.

Again, in

Caſe IV.

Where two Sides and an Angle oppoſite to one of them are given, to find the Contained Angle.

Given $\left\{\begin{matrix} BD \\ < C \\ DC \end{matrix}\right\}$ to find $< D$. *See the* Fig. *in* Page 267.

Having

Having let fall the Perpendicular D A, and found the Angle A D C, it will be,

t, D C : R : : t, D A : c s, A D C.
t, B D : R : : t, D A : c s, A D B.

Now, becauſe the two middle Terms in each are the ſame, it will be, by *Lemma the 2d*,

t, B D : t, D C : : c s, A D C : c s, A D B. That is,

As the Tangent of the Side B D,
To the Tangent of the Side D C;
So is the Co-ſine of the Angle A D C,
To the Co-ſine of the Angle A D B; by the Help of which, the Angle B D C may be found, as in *Caſe the 4th, Page 267.*

Hence it is, that the Tangents of the Sides or Hypothenuſes, are reciprocally proportional to the Co-ſines of the Angles at the *Vertex*, and the contrary.

And becauſe the Tangents and Co-tangents of Arches are reciprocally proportional, it follows,

That the Co-tangents of the Sides or Hypothenuſes, are directly proportional to the Co-ſines of the Angles at the *Vertex*, and the contrary; *that is*, in the ſame Triangle.

As the Co-tangent D C,
To the Co-tangent of D B;
So is the Co-ſine of the Angle A D C,
To the Co-ſine of A D B; and the contrary.

Likewiſe, in

Caſe V.

Where two Angles and a Side oppoſite to one of them, are given, to find the other Angle; that is,

Given $\left\{\begin{matrix} \angle B, \\ B D \\ \angle C, \end{matrix}\right.$ to find the $\angle$ D. *See the* Figures *in* Pag. 268.

The

The Perpendicular being let fall, and the Angle B D A found as formerly, it will be, to find A D C,

c s, D A : R : : c s, ∠ B : s, B D A.
c s, D A : R : : c s, ∠ C : s, A D C.

Wherefore, becauſe the two firſt Terms in each are the ſame, it will be, by *Lemma the 1ſt*,

c s, ∠ B : c s, ∠ C, : : s, B D A : s, A D C. That is,

As the Co-ſine of the Angle at B,
To the Co-ſine of the Angle at C;
So is the Sine of the Angle B D A,
To the Sine of the Angle A D C; by the Help of which, the Angle B D C may be found, according to the Methods already given in the preceding *Caſe*.

Hence it is, that the Co-ſines of the Angles at the Baſe, are directly proportional to the Sines of the Angles at the *Vertex*, and the contrary. That is,

c s, B : c s, C : : s, B D A : s, A D C.

Alſo, in

Caſe VI.

Where two Angles and the Interjacent Side are given, to find the other oppoſite Angle. That is,

Given { ∠B, B D, ∠D } to find the ∠ at C. *See the* Fig. *in* Page 269.

The Perpendicular being drawn as uſual, and the Angles B D A and A D C found, it will be,

R : c s, D A : : s, B D A : c s, D B C.
R : c s, D A : : s, C D A : c s, B C D.

Becauſe the two firſt Terms in each are the ſame, it will be, by *Lemma the 1ſt*,

s, B D A : s, C D A : : c s, D B C : c s, B C D. That is,

As the Sine of the Angle B D A,
To the Sine of the Angle C D A;
So is the Co-ſine of the Angle at B,
To the Co-ſine of the Angle at C. *q. e. i.*

To

To determine the Nature of which, the ſame Laws muſt be made uſe of, as is given in the *6th Caſe*, in *Page* 269.

And ſince the Angles at the *Vertex* are given, and one of the Angles at the Baſe to find the other; the Proportion might have been deduced immediately from the *Corollary* of the former *Caſe*, without any previous Inveſtigation.

Caſe VII.

Two Angles and a Side oppoſite to one of them, being given, to find the Interjacent Side.

Given $\left\{\begin{matrix}\angle B \\ B\,D \\ \angle C\end{matrix}\right\}$ to find B C. *See the* Figures *belonging to the* 7th Caſe, *in* Page 270.

The Perpendicular being let fall, and the Baſe B A of the Triangle B A D found, as in *Caſe the 7th*, it will be,

$$t, \angle B : R :: t, D\,A : s, B\,A.$$
$$t, \angle C : R :: t, D\,A : s, C\,A.$$

Wherefore, by *Lemma the 2d*, becauſe the middle Terms in each are the ſame, it will be,

$$t, \angle C : t, \angle B :: s, B\,A : s, C\,A.$$ That is,

As the Tangent of the Angle at C,
To the Tangent of the Angle at B;
So is the Sine of the Baſe B A,
To the Sine of the Baſe C A: Hence the Side ſought may be eaſily had, by the Directions given at the End of the *7th Caſe*, in *Page* 270.

Hence it is, that the Sines of the Baſes are reciprocally proportional to the Tangents of the Angles at the Baſe, and directly proportional to the Co-tangents of the ſame Angles; and the contrary.

Again, in

Caſe VIII.

Where two Angles and the Interjacent Side was given, to find either of the oppoſite Sides. That is,

Given $\left\{\begin{matrix}\angle B \\ B\,D \\ \angle D\end{matrix}\right\}$ to find D C. *See the* Figures *belonging to* Caſe the 8th, *in* Page 271.

Having

Having let fall the Perpendicular D A, and found the Angles B D A and C D A ſeparately, it will be for D C,

R : c t, D A : : c s, B D A : c t, B D.
R : c t, D A : : c s, A D C : c t, D C.

Becauſe the two firſt Terms in each Proportion are the ſame, it will be, by *Lemma the 1ſt*,

c s, B D A : c s, A D C : : c t, B D : c t, D C. That is,

As the Co-ſine of B D A,
To the Co-ſine of A D C;
So is the Co-tangent of B D,
To the Co-tangent of D C. *q. e. i.* The Nature of which may be eaſily known, by the Rules given at the End of the *8th Caſe*.

This Proportion for finding B D, might have been deduced from the *2d Corollary* of the *4th Caſe*.

Caſe IX.

Given two Sides and the Contained Angle, to find one of the Oppoſite Angles.

Given { B D, < B, B C } to find the < C. *See the* Figures *of Caſe the 9th, in* Page 272.

The Perpendicular being drawn, alſo B A and A C determined as in *Caſe the 9th*, it will be,

s, B A : R : : t, D A : t, < B.
s, C A : R : : t, D A : t, < C.

Hence, by *Lemma the 2d*,

s, C A : s, B A : : t, < B : t, < A. That is,

As the Sine of the Baſe C A,
To the Sine of the Baſe B A;
So is the Tangent of the Angle at B,
To the Tangent of the Angle at C. *q. e. i.* The Nature of which may be diſcovered by the Laws given at the End of the *9th Caſe*.

This Proportion might have been deduced from the *Corollary* of the *7th Caſe*.

Caſe X.

Given two Sides and the Contained Angle, to find the other Side that remains.

Given $\left\{\begin{matrix} B\,C \\ < B \\ B\,D \end{matrix}\right\}$ to find D C. *See the* Figures *of the* 10*th Caſe, in* Page 273.

The Preparation being made as uſual, and the Segments of the Baſe inveſtigated, it will be,

R : cs, D A : : cs, B A : cs, B D.
R : cs, D A : : cs, A C : cs, D C.

Wherefore, by *Lemma the* 1*ſt*,

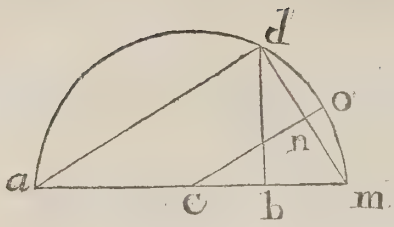
cs, B A : cs, A C : : cs, B D : cs, D C. That is,

As the Co-ſine of the Baſe B A,
To the Co-ſine of the Baſe C A;
So is the Co-ſine of the Side B D,
To the Co-ſine of the Side D C. *q. e. i.* The Property of which may be diſcovered by *Caſe the* 10*th*.

I might, indeed, have omitted the firſt Kind of *Solutions*, which were done by the Help of the Perpendicular firſt found: Or at leaſt have placed theſe latter Methods, at the End of each of the former; but I choſe rather to explain them both, and keep them ſeparate; to the end the Reader might not only uſe which Method would be moſt convenieut for his Purpoſe, but be throughly acquainted with this moſt uſeful Part of the *Mathematicks*.

In order to give a *Solution* of the 11*th Caſe* (where the three Sides are given to find the Angles) it is neceſſary to premiſe the following *Lemma's*; which, altho' they are Natural Conſequences of the Sinical Proportion, demonſtrated in the *Firſt Part*, and therefore require no other Proof; yet, ſince this *Caſe* is of ſo great Uſe in *Aſtronomy* and *Navigation*, I ſhall give a Demonſtration of them in this Place, that ſo the Reader may have the whole Proof before him in one View.

Lemma the 1ſt.



The Verſed Sine of the Supplement of an Arch, multiplied into half the Radius, is equal to the Square of the Co-ſine of half that Arch.

If the Given Arch be $d\,m$, $a\,b$ will be the Verſed Sine of its Supplement $d\,a$, and $c\,n$ the Co-ſine of $m\,o$ or $d\,o$ its half.

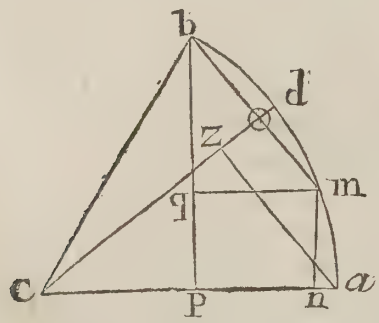
And becauſe the Triangles $a\,d\,b$ and $c\,n\,m$ are ſimilar, it will be,

$$a\,b : a\,d :: c\,n : c\,m.$$

But becauſe $a\,m$ is equal to twice $c\,m$, $a\,d$ is equal to twice $c\,n$.
Wherefore, $a\,b : 2\,c\,n :: c\,n : c\,m$. Or, as $a\,b : c\,n :: c\,n : \frac{1}{2}\,c\,m$.
Therefore, $a\,b \times \frac{1}{2}\,c\,m = c\,n \times c\,n = c\,n_{q}$. *q. e. d.*

Lemma the 2d.

The Difference of the Verſed Sines of two Arches multiplied into half the Radius, is equal to the Sine of half the Sum of thoſe Arches, multiplied into the Sine of half their Difference.



Let the given Arches be $a\,b$ and $a\,m$; then is $m\,o$ the Sine of $m\,d$, the Sine of half their Difference, and $a\,z$ the Sine of $a\,d$, the Sine of half their Sum: Alſo $q\,m$ equal to $p\,n$, equal to the Difference of the Verſed Sines $a\,p$ and $a\,n$, of the Arches $a\,b$ and $a\,m$.

Again; Becauſe $b\,m$ is parallel to $a\,z$, and $q\,m$ parallel to $a\,c$, the Triangles $b\,q\,m$ and $a\,c\,z$ are Similar, and it will be, $a\,c : a\,z :: b\,m : q\,m$.

But, as $\frac{1}{2}\,a\,c : a\,z :: \frac{1}{2}\,b\,m = b\,o = m\,o : q\,m$.

Wherefore, $\frac{1}{2}\,a\,c \times q\,m = m\,o \times a\,z$.

Theorem.

A Theorem.

In *Oblique Spherical Triangles*, the Rectangle, or Product of the Sines of any two Sides, is to the Square of the Radius; as the Difference of the Verſed Sines of the Sum of the two Sides and the Oppoſite Side, to the Verſed Sine of the Supplement of the Contained Angle.

Turn the upper Quadrant B C *q*, of the Semicircle *w* B *q*, about B *a m p* as an Axis (the

Laſtly, Draw C z a Perpendicular to P D, and join the Points z and *r*; then is D z the Verſed Sine of the Arch C D, or oppoſite Side.

And becauſe the Triangles *q* S *p* and C *r m* are Similar, it will be,

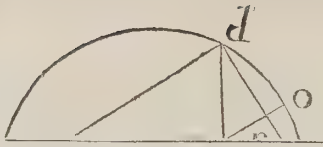
$$m\,C : p\,q :: m\,r : p\,S.$$

Wherefore, $m\,C : p\,q :: m\,C + m\,r : p\,q + p\,S.$

But $m\,C + m\,r = C\,r$. Alſo, $p\,q + p\,S = q\,S$, and $p\,q$ the Radius.

Wherefore, $m\,C : R :: C\,r : q\,S.$

Lemma the 1st.



The Verſed Sine of the Supplement of an Arch, multiplied into half the Radius, is equal to the Square of the Co-ſine of half that Arch.

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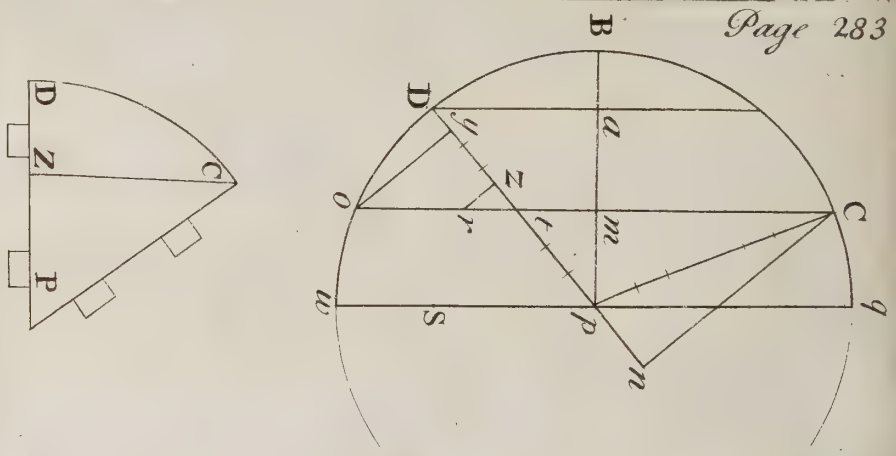
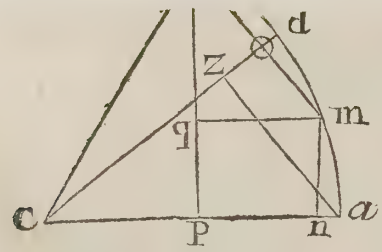




Sine of *a u*, the [illegible]

Alſo *q m* equal to *p n*, equal to the Difference of the Verſed Sines *a p* and *a n*, of the Arches *a b* and *a m*.

Again; Becauſe *b m* is parallel to *a z*, and *q m* parallel to *a c*, the Triangles *b q m* and *a c z* are Similar, and it will be, *a c* : *a z* : : *b m* : *q m*.

But, as $\frac{1}{2}$ *a c* : *a z* : : $\frac{1}{2}$ *b m* = *b o* = *m o* : *q m*.

Wherefore, $\frac{1}{2}$ *a c* × *q m* = *m o* × *a z*.

Theorem.

A Theorem.

In *Oblique Spherical Triangles*, the Rectangle, or Product of the Sines of any two Sides, is to the Square of the Radius; as the Difference of the Verſed Sines of the Sum of the two Sides and the Oppoſite Side, to the Verſed Sine of the Supplement of the Contained Angle.

Turn the upper Quadrant B C *q*, of the Semicircle *w* B *q*, about B *a m p* as an Axis (the Sector C D P being firſt erected obliquely above the Plane B *p w*) till the Point C of the Semicircle coincides with the Point C of the Sector; then will the Arches B C, C D, and D B, form an *Oblique-angled Spherical Triangle.*

Let C B D be the Angle pitched upon, then will C B and B D be the two containing Sides, and C D the Side oppoſite; and becauſe B O is equal to B C, D *o* will be equal to the Difference of the two containing Sides, B C and B D.

From *q* conceive a Perpendicular to fall upon *p w*, interſecting it in S, then will S *w* be the Verſed Sine of the Angle D B C, and S *q* the Verſed Sine of the Supplement; alſo from C let fall the Perpendicular C *r*.

Laſtly, Draw C z a Perpendicular to P D, and join the Points z and *r*; then is D z the Verſed Sine of the Arch C D, or oppoſite Side.

And becauſe the Triangles *q* S *p* and C *r m* are Similar, it will be,

$$m\,C : p\,q :: m\,r : p\,S.$$

Wherefore, $m\,C : p\,q :: m\,C + m\,r : p\,q + p\,S.$

But $m\,C + m\,r = C\,r$. Alſo, $p\,q + p\,S = q\,S$, and $p\,q$ the Radius.

Wherefore, $m\,C : R :: C\,r : q\,S.$

Reduce B C to the ſame Plane with B D, ſo that both may make one Plane; and make B *o* equal to B C, and let fall the Perpendiculars *oy* and C *n*; then will D *y* be the Verſed Sine of D *o*, the Difference of two containing Sides B C and B D, and D *n* the Verſed Sine of their Sum, and conſequently *z n* the Difference of the Verſed Sines of the Sum of the two Sides, and Side Oppoſite; and becauſe the Triangles D *ap*, *t zr* and *t n* C are ſimilar, it will be,

D *a* : D *p* : : *t z* : *t r* : : *t n* : *t* C : : *t z* + *t n* : *t r* + *t* C.

Wherefore, D *a* : D *p* : : *t z* + *t n* : *t r* + *t* C.

But *t z* + *t n* = *z n*. Alſo, *t r* + *t* C = *r* C, and D *p* equal to the Radius.

Wherefore, D *a* : R : : *z n* : *r* C.

But it was proved before, *m* C : R : : C *r* : *q* S.

Wherefore, D *a* × *m* C : R *q* : : C *r* × *z n* : C *r* × *q* S.

And conſequently, D *a* × *m* C : R *q* : : *z n* : *q s*.

But D *a* is the Sine of D B, one of the containing Sides, and *m* C the Sine of B C the other: Alſo *z n* the Difference of the Verſed Sines of the Sum of the Containing Sides, and Side Oppoſite; and *q* S the Verſed Sine of the Supplement of the Contained Angle.

Wherefore, as the Rectangle of the Sines, *&c.* as was aſſerted.

Again,

Becauſe, by the *Theorem*, s, B C × s, B D : R *q* : : *z n* : *q s*. multiplying each of the two laſt Terms by half R, or half the Radius, it will be,

s, B C × s, B D : R *q* : : *z n* × ½ R : *q s* × ½ R. That is,

The Product of the Sines of the two Containing Sides, is to the Square of the Radius, as the Rectangle or Product of the Difference of the Verſed Sines of the Sum of the Containing Sides, and the Oppoſite Side, into half the Radius, to the Product of half the Radius, into the Verſed Sine of the Complement of half the Contained Angle.

But by *Lemma the 2d*, the Difference of the Verſed Sines of two Arches, multiplyed into half the Radius, is equal to the Sine of half

half the Sum of thoſe Arches, into the Sine of half the Difference. That is,

$2n \times \frac{1}{2}R = s, \frac{1}{2}\overline{BC+BD+DC} \times (s, \frac{1}{2}\overline{BC+BD-DC} =)$

$s, \frac{BC+BD+DC}{2} - DC.$

And by *Lemma the 1ſt*, the Verſed Sine of the Supplement of an Arch, multiplyed into half the Radius, is equal to the Square of the Co-ſine of half that Arch. That is, $q\,s \times \frac{1}{2}R = \overline{c\,s\,q\,\frac{1}{2}} < B.$

Wherefore,

$s, BC \times s, BD : Rq :: s, \frac{1}{2}\overline{BC+BD+DC} \times s, \frac{1}{2}\overline{BC+BD+DC} - DC.$

$c\,s\,q\,\frac{1}{2} < B.$

That is, the Product of the Sines of the two containing Sides, is to the Square of the Radius, as the Product of the Sine of half the Sum of the three Sides, multiplyed into the Sine of half the Sum of the three Sides, leſſen'd by the oppoſite Side, to the Square of the Co-ſine of half the contained Angle.

Wherefore,

The Product of the Sines of the Sum of the three Sides, and the three Sides leſſened by the oppoſite Side, divided by the Quotient of the Rect-angle of the Sines of the two containing Sides, divided by the Radius, will produce a Sine; which multiplyed by the Radius, will give the Square of the Co-ſine of half the contained Angle.

Hence we are taught how, as in *Caſe the 11th*, from the three Sides of an *Oblique-angled Spherical Triangle*, to find any one of the Angles.

For from half the Sum of the three Sides, ſubſtract the Side oppoſite to the Angle required, and it will be,

As the Radius,
To the Sine of one of the Sides, containing the Angle required;
So is the Sine of the other containing Side,
To a 4th Sine.

As the 4th Sine,

Is to the Sine of half the Sum of the three Sides;

So is the Sine of the Difference between half the Sum of the three Sides, and the oppoſite Side,

To a 7th Sine.

To which 7th Sine add the Radius, (if you work by Artificial Sines) and half that Sum will be the Sine of the Complement of half the Angle required.

Now, If z be put equal to $\frac{1}{2}\overline{BC+BD+DC}$, and x equal to $\frac{1}{2}\overline{BC+BD+DC}-DC$; then s, BC × s, BD : R q :: s, z × s, x : c s q $\frac{1}{2}<B$. Wherefore $\frac{Rq}{s,BC\times s,BD}=\frac{cs\,q\frac{1}{2}<B}{s,z\times s,x.}$ That is, $\frac{Rq}{s,BC\times s,BD}\times s,z\times s,x=cs\,q\frac{1}{2}<B$, or, $\frac{R}{s,BC}\times\frac{R}{s,BD}\times s,z\times s,x=cs\,q\frac{1}{2}<B$. That is, Becauſe the Difference between the Logarithmic Radius and the Logarithmic Sine of any Arch is called the *Arithmetical Complement* of the Sine of that Arch,

To the *Arithmetical Complement* of the Sine of one of the containing Sides,

Add the *Arithmetical Complement* of the Sine of the other containing Side;

Alſo the Sine of half the Sum of the three Sides,

And the Sine of half the Sum leſſen'd by the Side oppoſite,

Half the Sum of theſe four Sines will be the Co-ſine of half the contained Angle.

And ſince (as has been already proved, in *Sect.* 2.) that the Angles of any *Oblique-angled Spherical Triangle*, conſtitute another Triangle, whoſe three $\left\{\begin{matrix}\text{Sides}\\\text{Angles}\end{matrix}\right\}$ are equal to the three $\left\{\begin{matrix}\text{Angles}\\\text{Sides};\end{matrix}\right\}$ except that the greateſt Side is the Supplement of the greateſt Angle, and the Supplement of the greateſt Angle the Supplement of the greateſt Side, We are taught how to reſolve the

XIIth Caſe.

Where the three Angles are given, to find the Sides, *viz.*

For from half their Sum, ſubtract the Angle oppoſite to the Side ſought, and it will be,

As

As the Radius,
To the Sine of one of the Angles adjacent to the Side required;
So is the Sine of the other adjacent Angle,
To a 4th Sine.

As this 4th Sine,
To the Sine of half the Sum of the three Angles;
So is the Sine of the Difference before found,
To a 7th Sine.
To which add the Radius (if you work by Artificial Sines) and half that Sum will be the Sine Complement of half the Side ſought.

Or, To the *Arithmetical Complement* of the Sine of one of the adjacent Angles,
Add the *Arithmetical Complement* of the Sine of the other adjacent Angle;
Alſo the Sine of half the Sum of the three Angles,
And the Sine of half the Sum leſſen'd by the oppoſite Angle;
Half the Sum of theſe four Sines will be the Co-ſine of half the interjacent Side.

N. B. If one or more of the Angles be Obtuſe, you muſt make uſe of their Supplements; and if the Side oppoſite to the Obtuſe Angle be required, the Quantity produced by the aforeſaid Analogy will be its Supplement.

SECT. V.

Containing ſome eaſy Stereographical Problems, *whoſe* Solutions *naturally flow from the Principles laid down in the* 1ſt Section; *of great Uſe in the Application of the ſame Section to* Aſtronomy.

PROBLEM I.

To find the Pole of any *Projected Great Circle.*

Of theſe *Projected Circles* there are three Sorts;

1. *The Primitive Circle*, or Periphery of the Plane of Projection.
2. *Right Circles*, or such as when Projected, are represented by strait Lines.
3. *Oblique Circles*, or the Representation of such great Circles as are not perpendicular to the *Plane of Projection*; which Circles, altho' they don't become Right Lines in the Projection, yet they divide the *Primitive Circle* into two equal Parts; and the Line connecting their Intersections passes through the Center of the same.

Case I.

To find the Pole of the *Primitive Circle*, or Periphery of the *Plane of Projection*.

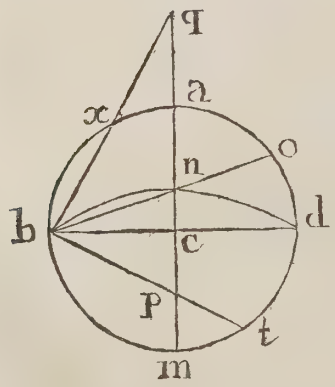
Solution.

Find the Center of the same Circle, and you have the Pole, *q. e. i.*

Case II.

To find the Pole of a Right Circle, or such a Circle as in the Projection is represented by a Right Line.

Let *b d* be the Circle given.



Solution.

Cross it with a Line at Right Angles, and the Intersections of the Diameter with the Primitive Circle, will be the two Poles.

Or,

Set off 90 Degrees from either Extremity in the Periphery of the *Plane of Projection*, and the Points so found, will be the Poles. Thus a and *m* are the Poles of the Circle *b d*, and *b* and *d* the Poles of the Circle a *m*.

Case

Case III.

To find the Pole of an *Oblique Circle*, as *b n d.*

It has been already proved, that the Poles will be found in that *Projected Diameter*, which cuts the Line connecting the Interſection of the *Oblique Circle* with the *Primitive* Circle at Right Angles, and are 90 Degrees diſtant from the Interſection of the *Oblique Circle* with the ſame Line, which Line is called the *Projected Axis.*

Solution.

1. Through *b* and *n*, the Interſection of the *Oblique Circle* with the *Primitive Circle* and *Projected Axis*, draw the Line *b n o*, cutting the *Primitive Circle* in *o.*

2. Set off 90 Degrees both Ways in the *Primitive Circle* from *o* to *t* and *x.*

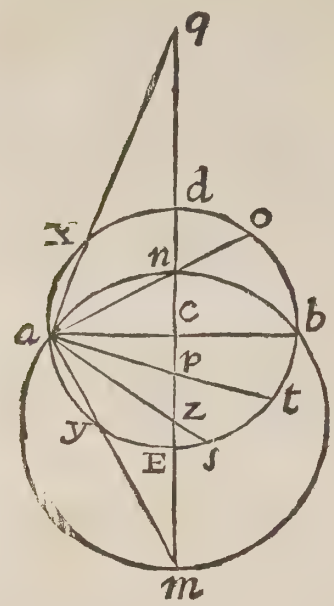
Then through *b*, and the Points laſt found, *viz. t* and *x*, draw the Lines *b t* and *b x*, continuing them, if need be, till they cut the *Projected Axis a m* in *p* and *q*, which Interſections will be the Poles required.

Or ſet off the Semi-tangents of the Sum and Difference of 90 Degrees, and *n c*, the neareſt Diſtance of the *Oblique Circle*, from the remote *Pole of Projection* each Way: That is, the Semi-tangent of the Difference from the Center, the contrary way with the *Oblique Circle*, and the Semi-tangent of the Sum, the ſame way with the *Oblique Circle* in the *Projected Axis*, and you have the Poles, *q. e. i.*

Thus *p* and *q* are the Poles of the *Oblique Circle b n d.*

And ſince, as has been proved, the Poles of all ſmall Circles are the ſame with thoſe great Circles to which they are parallel, it will be eaſy from hence, to find the Poles of any ſmall Circle whatſoever.

If it were required to describe a great Circle about any Point of the Pole.



1. If the Point given be in the *Center*, then the *Primitive Circle* is the great Circle.

2. If the Point given be in the *Primitive Circle*, as *d*,
Through *d* and the Center *c*, draw the Diameter *d* E, and at Right Angles to it the Diameter *a c b*, which will be the great Circle.

3. If the Point be taken any where within the Circle, as *p*,
Through the given Point *p*, draw the Diameter *d p* E, and cross it at Right Angles with the Diameter *a c b*. Then thro' the Point *a*, and the given Point *p*, draw the Line *a p t*, and set off the Arch *b t* from *t* to *s*, and draw the Line *a z s*; then will *z* be the Center, and *a z* the Radius of the Circle to be drawn.

PROBLEM II.

From any Point given, to lay down any Angle in the *Projection*.

This Problem admits of three Varieties;

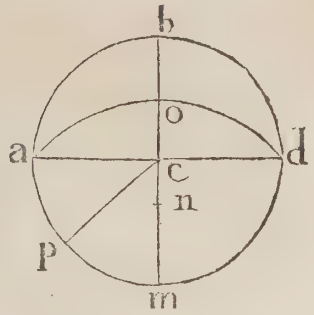
1. When the Angular Point is in the *Center of the Projection*.
2. When the Angular Point is somewhere in the Periphery of the *Plane of Projection*.
3. When the Angular Point is neither in the *Center*, nor in the *Periphery*, but somewhere in the *Plane*.

Case I.

To lay down an Angle, where the Angular Point is in the *Center* of the *Primitive Circle*.

Solution.

Solution.



From any Point of the *Periphery* as *a*, ſet off the Degrees of the Angle to *p*, and draw the Lines *c a* and *c p*, and the Thing is done.

Caſe II.

If the Angular Point be in the Periphery of the *Plane of Projection*, as ſuppoſe in *a*.

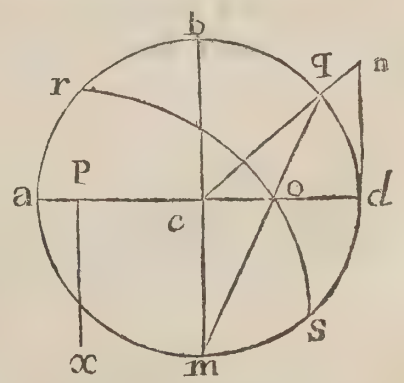
Solution.

1. Through *c*, the Center of the *Primitive Circle*, and *a* the Point given, draw the Line *a c d*, meeting the Periphery in *d*.
2. Take the Secant of the Given Angle to the Radius *c a* or *c d*, between the Points of the Compaſſes, and ſetting one Foot in *d*, with the other deſcribe the little Arch *n*; and with the ſame opening of the Compaſſes, ſetting one Foot in *a*, with the other croſs the former Arch in *n*.
3. With the ſame Diſtance, ſetting one Foot of the Compaſſes in *n*, ſweep the Arch *a o d*, and the Thing is done.

Caſe III.

If the Angular Point be not in the Center, neither in the *Periphery*, but in ſome other Point within the Circle, as *o*.

Solution.

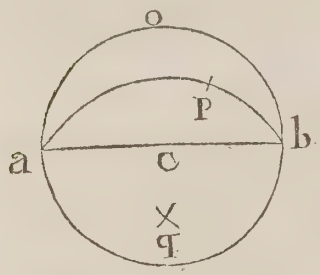


1. Through the Point given *o*, and the Center of the Projection *c*, draw the Diameter *a c o d*, and croſs it at Right Angles with another Diameter, as *b c m*.
2. Thro' *m* and the given Point *o*, draw the Line *m o q*, and thro' its Interſection *q* with the Periphery, draw *c q*, till it meet the Tangent in *n*.
3. Set off the Diſtance *d n*, from *c* to *p*, and draw *p x* parallel to *b m*.

4. Making *o p* the Radius of a Line of Tangents, ſet off the Tangent of the Complement of the Angle given from *p* to *x*; on which, as a Center with the Diſtance *x o*, deſcribe the Circle *r o s*, and the thing is done. All which is abundantly manifeſt from *Prop.* 1. *Sect.* 1.

PROBLEM III.

To draw a great Circle through any given Point, making with the *Primitive Circle* any given Angle.



Let the Point given be *p*, and the Circle to be deſcribed *a p b*, making a given Angle *p b o* with the *Primitive Circle*.

Solution.

1. With the Tangent of the given Angle, between the Points of the Compaſſes, ſetting one Foot in *c* the Center of the *Primitive Circle*, ſtrike the Arch *q*.

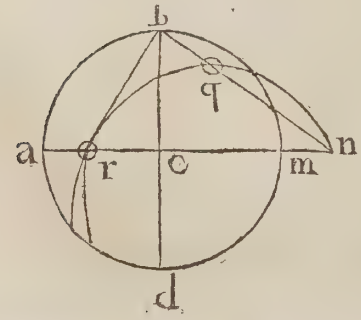
2. With the Secant of the ſame Angle between the Points of the Compaſſes, ſetting one Foot in *p* the Point given, with the other croſs the former Arch, in *q*.

3. With the ſame Diſtance between the Points of the Compaſſes, ſetting one Foot in *q*, with the other deſcribe the Circle *a p b*, and the Thing is done.

PROBLEM IV.

To draw a great Circle paſſing through any two Points given in the *Plane of Projection*.

Caſe I.



If one of the Points be in the Center, and the other in the Periphery as *a*, or within the Periphery as *r*; then through *c*, and the Point given, draw the Line *a r c m*, and the Thing is done.

Caſe

Case II.

If neither of the Points *r* or *q* be in the Center.

Solution.

1. Through one of the Points, *r*, draw a Diameter *a r c m*, and croſs it at Right Angles with the Diameter *b d*.
2. Join the Points *r* and *b*, and draw *b n* a Perpendicular to *b r*, and continue it till it meet the Diameter *a m*, produced in *n*.
3. Through the Points *r*, *q* and *n*, deſcribe a Circle, and the Thing is done.

PROBLEM V.

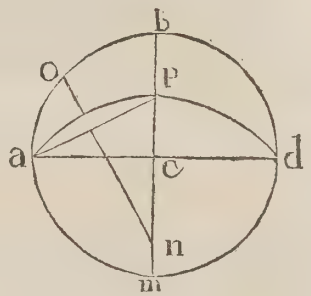
Through a Point given in a great Circle, to draw another great Circle, which ſhall cut it at Right Angles.

General Solution.

Through the Poles of the great Circle given, and the given Point, draw the Arch of a Circle, and the Thing is done.

Case I.

If it be required to draw a Circle that ſhall paſs through any given Point, as *p* or *b*, and be perpendicular to the *Primitive Circle.*



Solution.

Through the given Point *p* or *b*, and the Center *c*, draw the Diameter *b p c m*, and the Thing is done.

Case II.

If it be required to draw a Circle perpendicular to a Right Circle, which ſhall paſs through its Middle *c*, or the Center of the *Primitive Circle.*

Solution.

Through *c* the Center given, draw the Line *a d* at Right Angles, to the Right Circle given *b m*, and the Thing is done.

Case III.

If it be required to draw a Circle perpendicular to the Right Circle *b m*, and to pass through the Point *p* placed out of the Center.

Solution.

1. Cross the Diameter or great Circle given *b m*, at Right Angles with another Diameter, *a d*.
2. Connect the Points *a* and *p*, by a Right Line.
3. Bisect the Line *a p* by the Line *o n*, and continue it till it meet the Diameter *b m*, in *n*.
4. On *n* as a Center, with the Distance *n a* or *n p*, describe the Circle *a p d*, and the Thing is done.

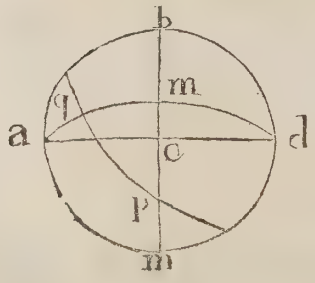
Case IV.

To draw a Circle perpendicular to an *Oblique Circle*, and to pass through a given Point.

If the Point given be in the Middle of the *Oblique Circle*, as at *m*, then through it, and the Center of the *Primitive Circle*, draw a Right Circle or Diameter, and the Thing is done.

But if the Point given be not in the Middle, as *m*, but in some other Part, as *q*;

Solution.



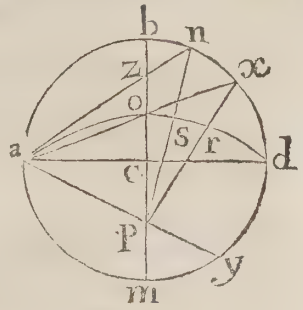
1. Find the Pole of the given *Oblique Circle*, by *Problem the 1st.*

2. Through the Pole last found, and the given Point *q*, draw a great Circle by the *4th Problem*, and the Thing is done.

PROBLEM VI.

From any Point given, to lay any Quantity of Degrees on a *Projected Great Circle.*

Case I.



If it be on the *Primitive Circle*, set off the Degrees from a Line of Chords in the Periphery from the Point given, and the Thing is done.

Case II.

If it be required to lay off a certain Number of Degrees in a Right Circle, from a given Point.

If the Point be in the Center.

Solution.

Set off the Semi-tangent of the given Arch in the Right Circle, from *c* the Center to *o*, and the Thing is done.

Or,

Set off the Number of Degrees in the *Primitive Circle*, from the other Pole of the Right Circle, as from *d* to *x*, and draw the Line *a x*; the Part *c o* cut off in the given great Circle, between the Center *c*, and the Intersection *o* will be the Part sought.

If the Point given in the Right Circle be not in the Center, but in some other Point, as *o*.

Solution.

To the Degrees answering to the Semi-tangent of the Arch *c o*, Add or Subtract the Degrees given, according as the *Case* requires. *That is*, If it be required to be set off from *o* towards *b*, then the Semi-tangent of the Sum must be set off from *c* towards *b*, to *z*. But if it be required to be set off towards the Center; *that is*, from *o* towards *c*, then the Semi-tangent of the Difference set off from *c* towards *o*, will give the Point sought.

But if in this Case it so happen, that the Degrees given, exceed in Number those answering to the Arch *c o*.

Solution.

Solution.

Then from the Number of Degrees given, Subtract the Degrees anſwering to the Arch *c o*, and ſet off the Semi-tangent of the Remainder from *c* towards *m*, to *p*; and the Arch *o p*, will be that which was required.

From *a*, one of the Poles of the Right Circle, through *c* or *o* the Points given, lay a Ruler, and obſerve where its cuts the *Primitive Circle*, as in *d* or ✕. From the Points thus found, *d* or ✕, as the *Caſe* requires, ſet off the Number of the Degrees, either towards *b* or *m*, *that is*, to *n* or *y*; and from the Points laſt found, to *a* lay a Ruler, and where it cuts the Right Circle, as in *z* and *p*, it gives the Points ſought.

Caſe III.

If it be required to lay off a certain Number of Degrees, in an *Oblique Circle*, as *a o d*.

Solution.

1. Find the Pole of the *Oblique Circle*.
2. Lay a Ruler from the Pole laſt found, over the Point *s*, and obſerve where it cuts the *Primitive Circle*, as in *n*.
3. Set off the Number of Degrees from the Interſection *n*, in the *Primitive Circle*, either Way, as the *Caſe* directs, either to *b* or ✕; and draw the Line *p b* or *p* ✕, and the Arch S *o*, or *s r*, will be that which was required.

PROBLEM VII.

To meaſure any Portion of a *Projected Great Circle*.

This *Problem* being no other than the Reverſe of the former, we ſhall make Uſe of the former *Figure*.

Caſe I.

If it be required to meaſure any Portion of the *Primitive Circle*, as *n* ✕; apply it to the Line of Chords, and the Number of Degrees is given.

Caſe

Case II.

If it be required to measure any Portion of a Right Circle, as *z p*.

Solution.

1. Having found one of its Poles as *a*, by *Problem the 1st*, lay a Ruler over the same Pole, and the Extremities *z* and *p* of the Arch you would measure, and the Arch *y n* cut off in the *Primitive Circle*, is equal to the Number of Degrees of the given Arch; which Number of Degrees of the Arch *n y*, may be measured by the *1st Case*.

Case III.

If it be required to measure any Portion, as *s r*, of an *Oblique Circle*, *a o d*.

Solution.

1. Find the Pole of the *Oblique Circle*, by *Problem the 1st*.
2. A Ruler laid from the Polar Point over the two Extremities of the Arch *s r*, will give the Points *n* and *x*; the Space intercepted between them being measured as in the *1st Case* of this *Problem*, will give the Quantity of the Arch, *q. e. i.*

PROBLEM VIII.

To measure a *Spherical Angle*.

To render this *Problem* more intelligible, it may not be amiss to repeat the *Definition* of an Angle given in the *Second Section*, viz.

That the Measure of a *Spherical Angle* is equal to the Arch of a great Circle (of which the *Angular Point* is the Pole) passing through the two Poles of those great Circles which constitute the Angle, and intercepted between them.

Hence, and from the *Seventh Problem*, follows,

The General Rule.

If a Ruler be laid over the *Angular Point*, and the two nearest Poles of the great Circles which constitute the Angle, the Arch cut off in the *Primitive Circle*, will give its Measure.

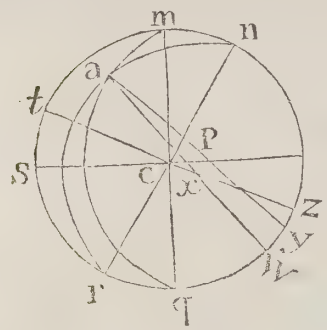
Case I.

If it be required to measure an Angle, when the *Angular Point* is in the Center.

Solution.

Continue the Lines that form it (if need be) till they cut the Circumference, and the intercepted Arch applied to a Line of Chords, will give the Measure.

Case II.



If it be required to measure an Angle, as *t m a*, when the *Angular Point* is in the *Periphery*.

Solution.

1. Find the Pole *p*, of the *Oblique Circle m a q*, and the Distance *p c* applied to a Line of Semi-tangents, will give the Measure of the Angle required.

Or,

1. From *m* the Angular Point, through *p* and *c* the Poles of the Circles *m a q* and *m t s q n*, lay a Ruler, and observe where it cuts the Periphery of the *Primitive Circle*, as in *q* and w.

2. Apply the Distance *q* w, to a Line of Chords, and the Number of Degrees that it measures will be the Quantity of the Angle.

Case III.

If the *Angular Point* be within the Periphery, and not in the Center as *a*, and it be required to determine the just Quantity of the Angle.

1. Find the Poles *p* and *x*, of the two Circles *m a q* and *n a r*, constituting the Angle.

2. Apply a Couple of Rulers, or draw a Couple of Lines from the Angular Point *a*, through the two Poles *p* and *x*, and the Arch *y* w applied to the Line of Chords, shews the Number of Degrees of the Angle. *q. e. i.*

PRO-

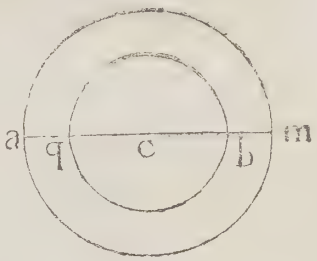
PROBLEM IX.

To draw a Parallel, or ſmall Circle.

Caſe I.

If the Circle to be projected be parallel to the *Primitive Circle*.

Solution.

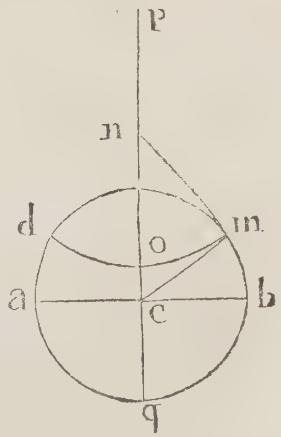


With the Semi-tangent of its Diſtance from the *Pole of Projection* or Center; or the Semi-tangent of the Complement of its Diſtance from the great Circle to which it is parallel, between the Points of the Compaſſes, ſetting one Foot in the Center or Pole *c*, with the other deſcribe the ſmall Circle *b q*, and the Thing is done.

Caſe II.

If the ſmall Circle *d o m*, to be projected, be parallel to the Right Circle *a b*.

Solution.



Having ſet off the Diſtance of the Parallel in the *Primitive Circle*, from *b* to *m*, or from *a* to *d*, draw *m n* a Tangent to the Point *m*; or make the Angle *c m n* Right, and the Interſection *n*, of the Line *m n*, with the Diameter, (which is at Right Angles to *a b*) is the Center, and *n m* the Radius; by the Help of which the Circle may be deſcribed.

Or,

Having found the Points of Interſection *m* and *d* as before, with the Tangent of the Complement of the ſmall Circles Diſtance from the Right Circle, to which it muſt be parallel, between the Points of the Compaſſes, ſetting one Foot in *m*, ſtrike a ſmall

Arch at *n*, then placing one Foot of the Compaſſes in the other Point *d*, croſs the former Arch in *n*, which will be the Center ſought; the Radius being the Diſtance between the Points of the Compaſſes. *Or*,

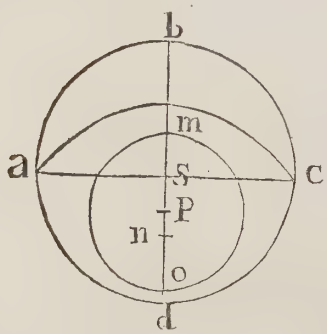
Having ſet off the Semi-tangent of the ſmall Circle's Diſtance from the Right Circle to which it is parallel, in the Line *c o*, from *c* to *o*; ſet off the Secant of the Complement of the ſame Diſtance from *c* to *n*, which Point *n* will be the Center; and the Diſtance between the Points found, *o* and *n*, will be the Semi-diameter; by the Help of which the Circle may be deſcribed.

Or,

Having ſet off the Semi-tangent of the neareſt Diſtance of the ſmall Circle from the *Pole of Projection*, from *c* to *o*, it will give one Interſection of the Diameter; and the Semi-tangent of the Complement of the ſame to 180 Degrees from the Center, the ſame Way to *p*, will give the other Extremity of the Diameter; the Middle of which, *n* will be the Center.

Caſe III.

To draw a Circle parallel to any *Oblique Circle* given.



1. Find the Pole of the *Oblique Circle* given.

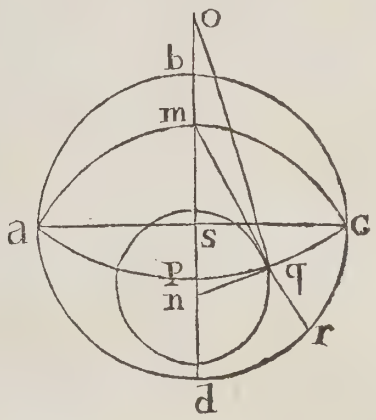
2. Set off the Semi-tangent of the greateſt and leaſt Diſtances of the ſmall Circle on its *Projected Axis*, from the Center or *Pole of Projection*; either on the ſame or contrary Sides, as the *Caſe* directs. *That is*, From *s* to *m*, and from *s* to *o*, and you have the two Extremities of its Diameter, the Middle of which, *viz. n*, will be the Center.

If the Diſtance of the ſmall Circle to be deſcribed from its Pole, be leſs than the Diſtance between its Pole and the *Pole of Projection*, then the Semi-tangent of the Sum and Difference of the Diſtance of the two Poles, and the ſmall Circles Diſtance from its Pole, ſet off on the ſame Side of the Center as the Pole is, will give the two Extremities of the ſmall Circles Diameter. But,

*

If

If the Diſtance of the ſmall Circle to be projected from its Pole be more than the Diſtance of the two Poles, *viz.* the *Pole of Projection*, and the Pole of the *Oblique Circle*, then the Semi-tangent of the Sum of the two Poles Diſtance, and the ſmall Circles Diſtance, ſet off on the ſame Side of the Center as the Pole is of, will give one Interſection; and the Semi-tangent of the Exceſs of the ſmall Circles Diſtance from its proper Pole, above the Diſtance of the two Poles, ſet off in the *Projected Axis*, on the contrary Side of the Center with the Pole, will give the other Interſection of the ſmall Circle, the Middle between which two Interſections will be the Center. *Or*,



1. Having found the Pole of the Circle given, *viz.* *p*, through it ſtrike a great Circle, as *a p c*, at Right Angles to the former.

2. Set off the Diſtance of the parallel to be projected, from *d* to *r*, and lay a Ruler over *m* and the Point *r*, and obſerve where it cuts the great Circle *a p c*, as in *q*.

3. From *o*, the Center of the Circle *a p c*, to *q* the Point laſt found, draw the Right Line *o q*, and make the Angle *o q n* a Right Angle.

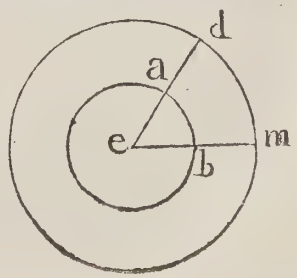
On *n*, as a Center with the Radius *q n*, deſcribe a ſmall Circle, and the Thing is done.

PROBLEM X.

To meaſure any Portion or Segment of a ſmall Circle, when projected.

Caſe I.

If the ſmall Circle given be parallel to the *Primitive Circle*.



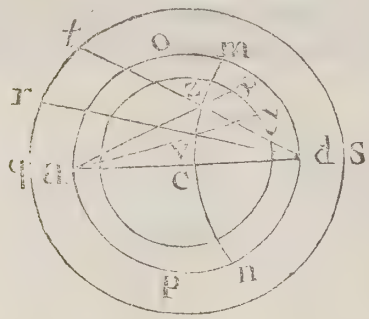
From *c* the Center of the *Primitive Circle*, through the two Extremities of the given Arch *a* and *b*, draw the Right Lines *c a* and *c b*, cutting the *Primitive Circle* in *d* and *m*; the Arch *d m* is the Meaſure of the given Arch *a b*.

By

By the Inverſe Method, may any Number of Degrees be laid down upon any ſmall Circle parallel to the *Primitive Circle*, from any Point given.

Caſe II.

To meaſure any Part or Portion of a ſmall Circle, parallel to any projected Right Circle.



1. Conſider the Diſtance of the ſmall Circle given from *d* its Pole, and deſcribe a ſmall Circle *q r t s*, as far diſtant from the under Pole of Projection *c*.

2. From *d* through the two Extremities *z* and *v*, of the Arch *z v* to be meaſured, draw the Lines *d z*, *d v*, and continue them till they interſect the Parallel *q r t s*, in *r* and *t*.

Or,

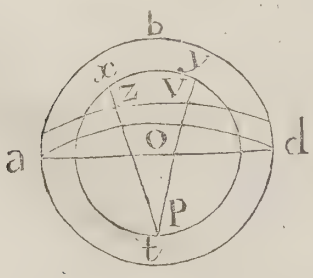
From *a* through *z* and *v*, the two Extremities of the Arch *z v*, draw the Right Lines *a z* and *a v*, to *x* and *y*.

3. Meaſure the Arches *y x* or *r t*, according to the Directions given in the 1*ſt Caſe*, and you have the Meaſure of the given Arch *z v*.

By the Inverſe Method, may any Number of Degrees be laid off, in any Circle parallel to a projected Right Circle from any Point given.

Caſe III.

To meaſure any Part or Portion of a ſmall Circle, parallel to any *Oblique Circle* given.



1. As in the former *Caſe*, ſo in this; conſider the Diſtance of the ſmall Circle, (a Part of which is to be meaſured) from either of its Poles; and deſcribe a ſmall Circle parallel to the *Primitive Circle*, as far diſtant from the under Pole.

2. Lay a Ruler over the Pole *p*, and the Extremities of the Arch of the ſmall Circle *z v*; and obſerve where it cuts the parallel Circle laſt drawn, in *x* and *y*.

3. Find

3. Find the Quantity of the Arch $x y$, by *Case the 1st* of this *Problem*, and the Thing is done.

By the Inverſe Method, may any Number of Degrees be laid off in any ſmall Circle parallel to an *Oblique Circle* from a given Point.

The *three Caſes* of this *Problem* may be reſolved by this one General Law, *viz.*

Through the Pole of the ſmall Circle given, and the two Extremities of the Arch to be meaſured, ſtrike two great Circles, and the Angle contained between them will be the Meaſure of the Arch given; the Quantity of which Angle may be diſcovered by the *8th Problem*.

And hence we are taught how to divide any Given Parallel into any Number of Parts, equal or unequal, as ſhall be deſired.

In the *Solutions* given to the ſeveral *Caſes* of theſe *Problems*, I have not referr'd to thoſe *Propoſitions* or *Corollaries* upon which they immediately depend; ſince he that is but moderately acquainted with the *1ſt Section* of this Part, will find it no difficult Task to do it himſelf.

SECT. VI.

Containing the Stereographic Projections *of the ſeveral* Caſes *of* Right-angled Spherical Triangles.

PROBLEM I.

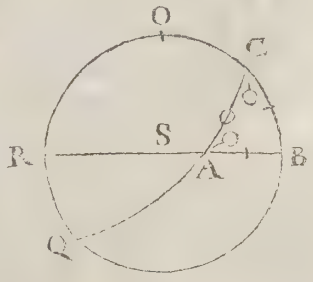
Caſes 6, 8.

Given the two Sides, to find the Hypothenuſe and Angles.

Example.

In the Triangle A B C, are

Given $\left\{\begin{matrix} A\,B \\ B\,C \end{matrix}\right\}$ required $\left\{\begin{matrix} <C \\ A\,C \\ <A \end{matrix}\right\}$



1. Deſcribe

1. Deſcribe the *Primitive Circle* R C B Q, and draw the Diameter R S A B.

2. Having found one of the Poles, as O, of the Circle R B, by *Prob. the 1ſt* of the *laſt Section*, by the Help of it, ſet off the Complement of A B, from S to A.

3. Set off the Number of Degrees of B C, in the Periphery of the *Primitive Circle*, from B to C.

4. Through the Points C and A, ſtrike the Arch of a great Circle, by *Prob. the 4th* of *Sect. the 5th*, and the Thing is done; and A B C is the Triangle ſought.

The Triangle being thus deſcribed, the Angles may be meaſur'd by *Prob. the 8th*, and the Hypothenuſe by *Prob. the 7th* of *Sect. the 5th*.

PROBLEM II.

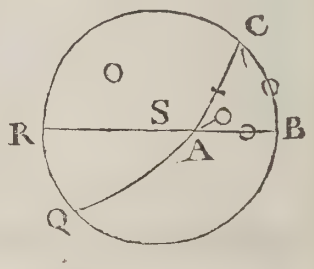
Caſes 1, 10, 14.

The Hypothenuſe and one of the Angles being given, to find the Sides and the other Angle.

Example.

In the Triangle A B C, are

Given $\left\{\begin{matrix} A\,C \\ <C \end{matrix}\right\}$ to find $\left\{\begin{matrix} A\,B \\ <A \\ B\,C \end{matrix}\right\}$



1. Through the Point C, (taken at pleaſure) ſtrike the Arch of a great Circle, as C A, making an Angle with the *Primitive Circle* equal to the Given Angle, by *Prob. the 2d*, and find its Pole O, by *Prob. the 1ſt* of *Sect. the 5th*.

2. By the Help of the Pole *o*, lay off the Number of Degrees of the Hypothenuſe in the great Circle laſt deſcribed from C to A, according to the Directions given in *Prob. the 6th of Sect. the 5th*.

3. Through the Center of the *Primitive Circle* S, and the Extremity A, of the Arch cut off, draw a Diameter or great Circle R A B, and the Thing is done; and the Sides A B and B C, may be meaſured by *Prob. the 7th*, and the Angle at A, by *Prob. the 8th* of *Sect. the 5th*.

PROB-

PROBLEM III.

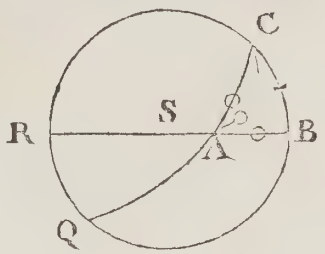
Cases 2, 9, 11.

One of the Angles and the Side adjacent being given, to find the remaining Parts of the Triangle.

Example.

In the Triangle A B C, are

Given $\left\{\begin{matrix} B C \\ <C \end{matrix}\right\}$ required $\left\{\begin{matrix} <A \\ A C \\ A B \end{matrix}\right\}$

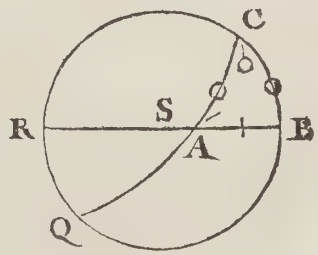


1. Through the Center of the *Primitive Circle* S, draw the Diameter R A B.
2. Set off the Side B C, in the *Primitive Circle*, from B to C.
3. Through C draw a great Circle, making with the *Primitive Circle* an Angle equal to the Given Angle C, by *Prob. the 2d* of *Sect. the 5th*, and A B C will be the Triangle.

The Quantity of the Sides A B and A C may be determined by *Prob. the 7th*, and the Angle B A C, by *Prob. the 8th* of the same *Section.*

If inſtead of the Angle C, the Angle at A had been given, as in *Caſes the 2d and 9th*, where are

Given $\left\{\begin{matrix} A B \\ <A \end{matrix}\right\}$ to find the reſt.



1. Having drawn the Diameter R B, ſet off the Complement of A B from S to A, by *Prob. the 6th* of the laſt *Section.*

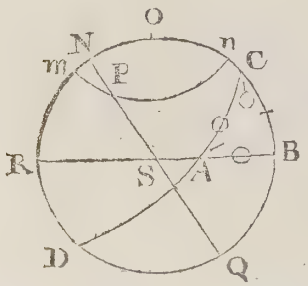
2. Through A, draw the Arch of a great Circle, making with the great Circle R B an Angle, equal to the Angle A, by *Prob. the 2d* of *Sect. the 5th*, and the Thing is done; and the required Parts of the Triangle A B C, may be meaſured by *Prob. the 7th and 8th* of *Sect. the 5th.*

PROBLEM IV.

Cases 3, 4, 12.

One Side and its oppoſite Angle being given, to find the other Parts of the Triangle.

Example.



In the Triangle A B C are,

Given $\left\{\begin{matrix} B\,C \\ <A \end{matrix}\right\}$ to find $\left\{\begin{matrix} A\,B \\ <C \\ A\,C \end{matrix}\right\}$

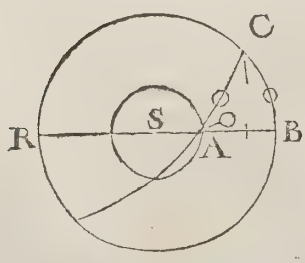
1. Having drawn the Right Circle R S A B, find its Pole by *Problem 1ſt* of *Sect. the 5th.*

2. Set off B C in the *Primitive Circle* from *B* to *C*, and from *O* to *N*, and draw the Right Circle N P S Q.

3. About the Point *O*, as a Pole, deſcribe the ſmall Circle at the Diſtance of the Angle A.

4. About the Point P, where this ſmall Circle interſects the Right Circle N P S Q, draw the great Circle *C A D*, and the Thing is done; and A B C is the Triangle, the Hypothenuſe A C, of which, and the Baſe A B, may be meaſured by *Prob. the 7th*, and the Angle at *C*, by *Problem the 8th of the laſt Section.*

If the Angle at C and A B had been given,



1. By *Problem the 9th* of the former *Section*, deſcribe the ſmall Circle S A, at the Diſtance of A B from the *Primitive Circle.*

2. At ſome Point as C, (taken at Pleaſure) make the Angle B C A, equal to the given Angle.

3. Through the Interſection A, draw the Diameter R B, and A B C will be the Triangle ſought: The Side B C, the Hypothenuſe A C, and Angle at C, may be meaſur'd by the *7th and 8th Problems* of *Section the 5th.*

PROB-

PROBLEM V.

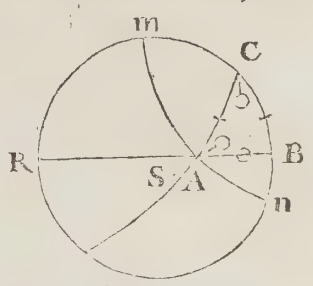
Cases 5, 7, 13.

The Hypothenuse and one of the Sides being given, to construct the Triangle, and determine the Length of the other Side, and the Quantity of the Angles.

Example.

In the Triangle A B C, are

$$\text{Given} \begin{Bmatrix} A\,C \\ C\,B \end{Bmatrix} \text{ to find} \begin{Bmatrix} <C \\ A\,B \\ <A \end{Bmatrix}$$

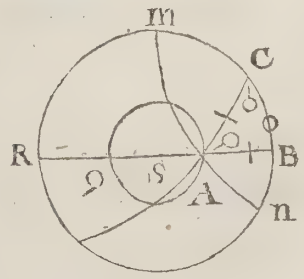


1. Having described the *Primitive Circle*, and drawn the Diameter R B, set off the Side C B from B to C.

2. About C, as a Pole, at the Distance of C A, describe the Parallel *m n*, by *Prob. the 9th* of the former *Section*.

3. Through the Intersection A, and the Point before found C, draw a great Circle as C A, and the Triangle A B C will be that which was required; the Side A B of which, may be measured by *Prob. the 7th*, and the Angles A and C, by *Prob. the 8th* of the last *Section*.

If in the Room of C B, the Side A B had been given.



1. About the Point C (taken at Pleasure) as a Pole, describe the small Circle *m n* at the Distance of A C the Hypothenuse, by *Prob. the 9th* of *Sect. the 5th*.

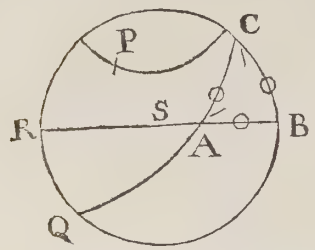
2. About the Point S as a Pole, describe the Parallel A Q at the Distance of Q S, the Complement of the given Side A B.

3. Through the Intersection A, and the Center *S*, draw the Diameter R B; also thro' the same Intersection A, and the Point C, draw the Arch of a great Circle as C A, by *Prob.* 4. of the former *Section*, and A B C will be the Triangle required; the Side C B of which, may be measur'd by *Prob. the 7th*, and the Angles at C and A, by *Prob. the 8th* of the former *Section*.

PROBLEM VI.

Cases 15, 16.

The Angles being given, to project the Triangle, and determine the Length of the Hypothenuse and Sides.



Example.

In the Triangle A B C, are

Given $\left\{\begin{matrix}<A\\<C\end{matrix}\right\}$ to find $\left\{\begin{matrix}CB\\CA\\AB\end{matrix}\right\}$

1. Draw a small Circle parallel to R B, at the Distance of the Complement of the Angle A.

2. About the Point S, the Center of the *Primitive Circle*, at the Distance of the Angle C, describe a small Arch cutting the Parallel in the Point P.

3. About the Point P as a Pole, describe the great Circle C A Q, and the Thing is done; and A B C will be the Triangle, the Sides of which may be measured by *Prob. the 7th* of *Sect the 5th*.

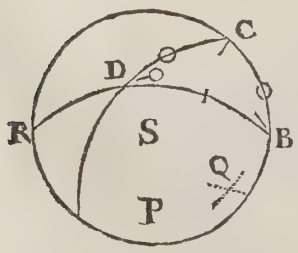
SECT. VII.

Containing the Stereographic Projection *of the* Twelve Cases *of* Oblique-angled Spherical Triangles.

PROBLEM I.

Cases 1, 5, 7.

Two Angles and a Side opposite to one of them being given, to find the other Angle and the two Sides.



Example.

In the Triangle B D C, are

Given $\left\{\begin{matrix}<B\\BD\\<C\end{matrix}\right\}$ required $\left\{\begin{matrix}DC\\<D\\BC\end{matrix}\right\}$

1. At any Point as B, (taken at Pleasure) in the Primitive Circle, form an Angle equal to the Given Angle B, by *Prob.* 2. of *Sect.* 5.

2. Find

2. Find the Pole P of the great Circle R D B, and by the Help of it, cut off the Arch B D equal to the Given Side B D, by *Prob. the 6th* of *Sect. the 5th.*

2. With the Tangent of the Angle C, between the Points of the Compaſſes, ſetting one Foot in S the Center, ſweep the little Arch Q; and with the Secant of the ſame Angle, ſetting one Foot in D, croſs the former Arch in Q.

4. On Q as a Center, with the ſame Extent of the Compaſſes, ſweep the Arch of the great Circle D C, and the Thing is done; and D C B is the Triangle required.

The Sides D C and B C may be meaſured by *Prob. the 7th* of *Sect. the 5th*, and the Angle at D, by *Prob.* 8, of the ſame *Section.*

PROBLEM II.

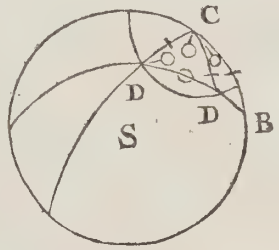
Caſes 2, 3, 4.

Two Sides and an Angle oppoſite to one of them being given, to find the other Parts of the Triangle.

Example.

In the Triangle D B C, are

Given { C D, ∠ B, B C } required { ∠ D, D B, ∠ C }



1. On the *Primitive Circle*, from any Point as B, ſet off the Number of Degrees of B C from B to C.

2. By the Help of the *2d Problem* of *Section the 5th*, form an Angle at the Point B, equal to the Given Angle B.

3. About the Point C as a Pole, draw a Parallel or ſmall Circle at the Diſtance of D C, by the Help of *Prob. the 9th* of *Sect the 5th.*

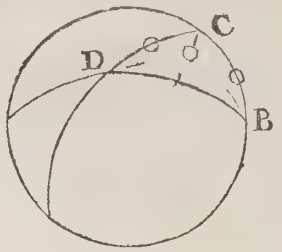
4. Through the Interſection D, of the Parallel laſt drawn, with the great Circle D B, and the Point C in the *Primitive Circle*; draw the Arch of a great Circle, and the Triangle D C B, is that that was required; and its unknown Side may be meaſured by *Prob. the 7th* of *Sect. the 5th*, and its Angles, by *Problem the 8th* of the ſame *Section.*

N. B. If the ſmall Circle cut the great Circle D B, in more Places than one, as it will do in all Caſes except one, it ſhews the Queſtion is capable of more than one Anſwer. And if thro' the two Interſections D, D, and the Point C, two great Circles are ſtruck, they will determine the ſeveral Values of the Things ſought.

PROBLEM III.

Cases 6, 8.

Two Angles and the interjacent Side being given, to find the other Parts of the Triangle.



Example.

In the Triangle C B D, are

Given { <B, B D, <D } required { D C, <C, B C }

1. At any Point as B (taken at Pleasure) in the *Primitive Circle*, form an Angle equal to the given Angle B, by *Problem the 2d* of *Section the 5th.*

2. By *Problem the 6th* of the same *Section*, lay off the Number of Degrees of B D, in the great Circle B D, from B to D.

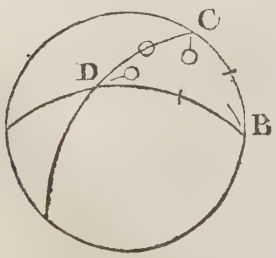
3. At the Point D, form an Angle equal in Quantity to the given Angle D; by drawing the great Circle D C, and the Thing is done; and the Sides D C and B C may be measured by *Problem the 7th*, and the Angle at C, by *Problem the 8th.*

This *Problem* might with as much Ease have been constructed, if B C had been given in the room of B D, and the Angle at C, in the room of D.

PROBLEM IV.

Cases 9, 10.

Two Sides and the contained Angle being given, to find the other Angle and the third Side.



Example.

In the Triangle B D C, are

Given { B C, <B, B D } required { <D, D C, <C }

1. By the Help of a *Line of Chords*, set off the Length of B C, (from any Point at pleasure as) B to C.

2. At the Point B, form an Angle equal to the given Angle B, by *Prob. the 2d* of *Sect. the 5th.*

3. Lay off the Number of Degrees of B D, in the great Circle B D, from B to D, by *Prob. the 6th of Sect. the 5th.*

4. Thro' the Points D and C, draw the Arch of a great Circle, as D C, by *Prob.* 4 of *Sect.* 5, and the Thing is done; and the Angles and Side required may be measured by the *7th* and *8th Prob.* of *Section the 5th.*

PROBLEM V.

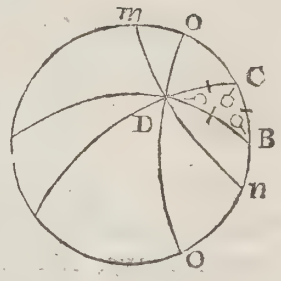
Case 11.

The three Sides given, to find the Angles.

Example.

In the Triangle B D C, are

Given { B C, B D, D C } required { ∠D, ∠C, ∠B }



1. Set off the Number of Degrees of B C in the *Primitive Circle*, from B to C, by *Problem* 9 of *Sect. the 5th.*

2. About the Point B as a Pole, describe the small Circle *o o*, at the Distance of B D.

3. About the Point C as a Pole, describe the small Circle *m n*, at the Distance of D C, by *Prob.* 9 of *Sect. the 5th.*

4. Through the Intersection D of the small Circles, and the Points C and B, strike two Arches of great Circles, as D C and D B, by by *Prob. the 4th* of *Sect. the 5th*, and the Thing is done; and the Angles at D, C, and B, may be measured by *Prob. the 8th* of the same *Section.*

PROBLEM VI.

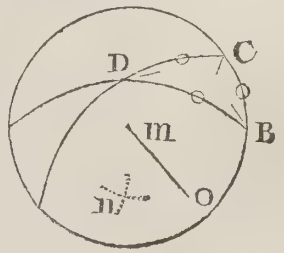
Case 12.

The three Angles being given, to find the three Sides.

Example.

In the Triangle B D C, are

Given { ∠B, ∠C, ∠D } required { D C, B D, C B }



1. Set off the Semi-tangent of the Supplement of the greateſt Angle C, from *m* the Center or Pole of the *Primitive Circle*, in the great Circle to *o*.

2. With the Semi-tangent of the Angle B, between the Points of the Compaſſes, ſetting one Foot in *m*, with the other ſweep a ſmall Arch at *n*.

3. About the Point *o* as a Pole, deſcribe the Arch of a Parallel as *n*, at a Diſtance equal to the Number of Degrees of the Angle D, by *Prob. the 9th of the 5th Section*; and obſerve where it cuts the Arch of the Parallel before drawn, as in *n*.

4. About the Points *n* and *o* as Poles, deſcribe the great Circles D B and D C, and the Thing is done; and the Sides of the Triangle D C B may be meaſured by the *7th Problem* of *the 5th Section.*

How all the Variety of *Caſes* in *Spherical Trigonometry* may be Projected from given Points, and their *Solutions* in Numbers different Ways, may be ſeen in the *5th* and *7th Sections* of the *Second Volume* of my *Syſtem of the Mathematicks.*

PART

PART IV.

CONTAINING

The Application of the Projection of the SPHERE, *and Doctrine of* Spherical Triangles, *to the* Solution *of such* Astronomical Problems *as are useful in* NAVIGATION.

SECT. I.

ASTRONOMICAL DEFINITIONS.

1. THE *Axis* is that imaginary Line, about which the Heavens seem to revolve; its two Extremities, are called the *Poles*; the Uppermoſt, or Viſible to us, the *Northern Pole*; the Lower, or Inviſible to Us, the *Southern Pole.*

2. The *Equinoctial*, or *Equator*, is that great Circle which is every Way equally diſtant from the Poles, and conſequently divides the Heavens into two equal Parts: It is ſo called, becauſe whenever the Sun is in this Circle, the Days and Nights are of an equal Length to all the Inhabitants of the Earth.

3. *Meridians*, or *Hour Circles*, are thoſe imaginary great Circles which interſect each other in the Poles of the World, and cut the *Equinoctial* at Right Angles.

4. The *Ecliptic* is a great Circle interſecting the *Equinoctial* in two oppoſite Points, and forming an Angle with it equal to the Sun's greateſt Declination, (which, by the moſt accurate Obſervations of Mr. *Flamſteed*, is found to be 23° 29′) and is that Circle which the

Sun deſcribes by his Annual Motion round the Earth, (according to the Syſtem of *Ptolomy*.)

This Circle is uſually divided into 12 equal Parts, called *Signs*, (and conſequently each Sign muſt contain 30 Degrees) beginning from the Interſection of the *Equinoctial*, and number'd as follows,

♈	♉	♊	♋	♌	♍
Aries	*Taurus*	*Gemini*	*Cancer*	*Leo*	*Virgo*
♎	♏	♐	♑	♒	♓
Libra	*Scorpio*	*Sagittary*	*Capricorn*	*Aquarius*	*Piſces*

The Firſt Six of theſe are called the *Northern Signs*, and poſſeſs that Half of the *Ecliptic* which is to the Northward of the *Equinoctial*, beginning at the firſt Point of *Aries*, and ending with the laſt Point of *Virgo*.

The latter Six are called the *Southern Signs*, becauſe they poſſeſs the Southern Half of the *Ecliptic*, beginning at the firſt Point of *Libra*, and ending with the laſt Point of *Piſces*.

5. *Circles of Longitude* in the Heavens, are thoſe great Circles which are imagined to paſs through, and conſequently interſect each other in, the Poles of the *Ecliptic*; and cut the *Ecliptic* at Right Angles, as the *Meridians* do the *Equinoctial*.

6. Of the *Meridians*, that which paſſes through the Interſection of the *Ecliptick* and *Equinoctial*, is called the *Equinoctial Colure*;

7. And that which cuts this at Right Angles, and paſſes thro' the Points of *Cancer* and *Capricorn*, is called the *Solſtitial Colure*.

8. The *Horizon* is a Circle encompaſſing the Earth, and dividing it into two equal Parts: It is ſo called, as being the Terminator or Boundary between the illuminated Part of the Hemiſphere and the luminated; and whenever the Sun, Moon, or Stars arrive at this Circle, they are then ſaid to Riſe or Set: It is Variable in different Latitudes, and has its Poles.

9. Its Uppermoſt or *Viſible Pole*, *viz.* the Point in the Heavens directy over our Head, is called the *Zenith*.

10. The Lowermoſt or *Inviſible*, *viz.* the Point directly under our Feet, is called the *Nadir*.

11. *Vertical* or *Azimuth Circles*, are thoſe Great Circles which interſect each other in the *Zenith* and *Nadir*, and cut the *Horizon* at Right Angles; as the *Meridians* do the *Equinoctial*, and the *Circles of Longitude* the *Ecliptic*.

* 12. Of

12. Of the *Vertical* or *Azimuth Circles*, that which paſſes thro' the Poles of the World, is called the *Meridian of the Place*; for this Reaſon, that when the Sun arrives at it, it is then Mid Day, or High Noon; and whenever the Moon, Stars, or other Planets are under it, they are then at the higheſt they can poſſible be in that Part of the Earth; and conſequently after that Time continually deſcend.

13. The *Prime Vertical*, or *Circle* of *Eaſt* and *Weſt*, is that *Vertical Circle* which cuts the *Meridian* at Right Angles.

14. Of the *Meridians* or *Hour Circles* (explain'd in *Def.* 3.) that which cuts the *Meridian* (deſcribed in *Def.* 12) at Right Angles, is called the *Six a-Clock Hour Circle*; becauſe when the Sun, Moon, or Stars, are under this Circle, they then have been ſix Hours abſent from, or want ſix Hours of arriving at the *Meridian*.

15. All Small Circles parallel to the *Equinoctial*, are called *Parallels of Declination*, when they are uſed with reſpect to the Sun, Moon or Stars; but *Parallels of Latitude*, when uſed with reſpect to the Places on the Earth. Of theſe,

16. That which is diſtant from the *Equinoctial*, by the Space of 23° 29′ towards the North, and which of conſequence muſt touch the *Ecliptic* in the firſt Point of *Cancer*, and which is the Parallel of the Sun's Diurnal Courſe when he is in that Point of the *Ecliptic*, is called the *Tropic of Cancer*.

17. That which is diſtant from the *Equinoctial* 23° 29′ towards the South Pole, and which therefore muſt touch the *Ecliptick* in the firſt Point of *Capricorn*, and become the Parallel of the Sun's Diurnal Courſe when he is in that Point, is called the *Tropic of Capricorn*.

18. That Small Circle which is diſtant from the North Pole by the Space of 23° 29′, is called the *Artic Circle*; and to any of thoſe Inhabitants whoſe *Zenith* falls within this Circle, the Sun is not viſible when he is in the *Tropic of Capricorn* before deſcribed.

19. That Small Circle which is as far diſtant from the Southern Pole, as the *Artic Circle* is from the Northern, is called the *Antartic Circle*: And to all thoſe Inhabitants (if any there be) whoſe *Zenith* falls between this Circle and the South Pole, the Sun never exhibits himſelf when he is in the firſt Point of *Cancer*.

20. All Small Circles parallel to the *Ecliptic* are called *Parallels of Latitude*, with regard to the Fixed Stars and Planets, as determining the Diſtance of any of them from the *Ecliptic.*

21. All Small Circles parallel to the *Horizon* are called *Almicanthers*, or *Parallels of Altitude*; as ſhewing how far the Sun, Moon, or Stars are above the *Horizon.*

22. *Longitude* of any Place upon the Earth, is meaſured by an Arch of the *Equator*, intercepted between the firſt *Meridian*, and the *Meridian* paſſing through the Place propoſed; *or*, it is equal to the Angle formed by the firſt *Meridian*, and the *Meridian* paſſing through the Place given.

23. The *Latitude* of any Place is meaſured by an Arch of the *Meridian* paſſing through the Place, and intercepted between the Place and the *Equator.*

24. The *Right Aſcenſion* of the Sun, Moon or any Star, is an Arch of the *Equinoctial*, intercepted between the firſt Point of *Aries* and the Interſection of the *Meridian* paſſing thro' the Sun, Moon or Star; and is the ſame with the Angle form'd by the *Equinoctial Colure*, and the *Meridian* paſſing through the Sun, Moon or Star: It is ſo called, becauſe it is the Point of the *Equinoctial*, which riſeth with the Sun or any Star, in a Right Sphere, or ſuch a Sphere where the Poles lye in the *Horizon*, and conſequently is the ſame Point that paſſes the *Meridian* with either of them.

25. The *Declination* of the Sun, Moon or any Star, is an Arch of the *Meridian* paſſing through the Sun, Moon or Star, and intercepted between either of them and the *Equinoctial*; and if the Sun, Moon or Star be to the Southward of the *Equinoctial*, it is called *South Declination*; but if they lye to the Northward, or betwixt the *Equinoctial* and the North Pole, it is called *North Declination.*

26. The *Place of the Sun*, is that Sign, Degree or Minute, *&c.* of the *Ecliptic*, that he poſſeſſes at any Time; and his *Longitude* is the Arch of the *Ecliptic*, intercepted between this Place and the firſt Point of *Aries.*

27. The *Longitude of the Moon*, or any Star, is an Arch of the *Ecliptic*, contain'd between the *Circle of Longitude*, paſſing thro' the Moon or Star and the firſt Point of *Aries*; and that Degree of any Sign over which the *Circle of Longitude* paſſing through the Moon, or any Star runs, is called the *Place of the Moon* or *Star.*

28. The

28. The *Latitude of the Moon*, or any Star, is equal to the Arch of the *Circle of Longitude*, paſſing over the *Moon*, or any Star, intercepted between the Moon or Star and the *Ecliptic*, and is called *South Latitude*, if the Moon or Star be to the Southward of the *Ecliptic*; but if to the Northward, it is termed *North Latitude*.

29. The *Azimuth*, is an Arch of the *Horizon* contain'd between the *Meridian* and *Vertical Circle* paſſing through the Sun, Moon or Star, at any Time propoſed, when they are above the *Horizon*, and is the ſame with the Angle at the *Zenith* made by the *Meridian* and the *Vertical Circle*.

30. The *Amplitude* is an Arch of the *Horizon*, contain'd between the Sun, Moon or Star, and the Eaſt Point of the *Horizon*, at their Riſing, or the Weſt Point at their Setting: Or it is equal to the Angle made by the *Prime Vertical* and the *Vertical Circle*, paſſing through the Sun, Moon or Star, at their Riſing or Setting.

31. The *Altitude of the Sun*, Moon or Star, is an Arch of the *Vertical Circle*, paſſing through the Sun, Moon or Star, and intercepted between the Sun, Moon or Star and the *Horizon*, and can never exceed 90 Degrees or a Quadrant.

32. *Zenith Diſtance* is that Arch of the *Vertical Circle* paſſing through the Sun, Moon or Star, which is contain'd between either of them and the *Zenith*; and therefore muſt alway, together with the *Altitude*, make 90 Degrees or a Quadrant.

33. *Meridian Altitude* is the Height of the Sun, Moon or Star, when they are upon the *Meridian*; and conſequently is the greateſt Height to which they can arrive at that Time.

34. *Oblique Aſcenſion*, or *Deſcenſion*, is that Degree of the *Equinoctial*, which Riſes or Sets with the Sun, Moon or any Star, in an *Oblique Sphere*, (or ſuch a Sphere where the *Equinoctial* cuts the *Horizon* at unequal Angles) which Poſition happens to all who live between the *Equator* and the Poles.

35. *Aſcenſional Difference* is the Difference between the *Right* and *Oblique Aſcenſion* or *Deſcenſion*; or is that Arch of the *Equinoctial*, which is contained between the Points of *Right* and *Oblique Aſcenſion* or *Deſcenſion*, and conſequently ſhews how long the Sun Riſes or Sets before or after the Hour of Six; or half the Time that the Moon or any Star is above or below the *Horizon*, exceeds or wants of Six Hours.

SECT.

SECT. II.

Containing the Chief *and* Primary Problems *ariſing from the* Diurnal Motion *of the* SUN, *or the Rotation of the* Earth *about her* Axis.

Which are of two Sorts:

1. GEneral, that is Such as require the Sun's Right Aſcenſion and Declination, his Place being firſt given.

Or,

2. Particular: *That is*, Variable in different Latitudes: Such are thoſe that from the Declination require,

1. His Amplitude and Aſcenſional Difference.
2. His Height and *Azimuth* at the Hour of Six.
3. His Height and the Time when he is due Eaſt or Weſt.
4. His Height and *Azimuth* at any Hour when he is in the *Equator*.
5. His Height and *Azimuth* at any Hour, when he is in any Point in the *Ecliptic*.

Or,

The Hour of the Day and *Azimuth*, when he is in any Point of the *Ecliptic*; from his Altitude, his Declination, and the Latitude of the Place.

Of each of theſe in their Order.

And becauſe the Sun's Place is ſuppoſed to be known or given in all *Problems of the Sphere*, let it be required in

PROBLEM I.

From the Sun's Longitude and greateſt Declination, (which is found by the moſt accurate Obſervations of Mr. *Flamſteed* to be 23° 29′) to find his preſent Declination and Right Aſcenſion.

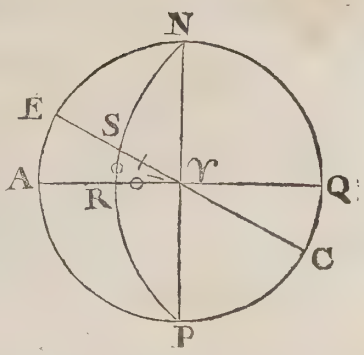
Example.

Suppoſe the Sun to be in 27° 30′ of *Taurus*, and his greateſt Declination as before fixed 23° 29′, what is his preſent Declination and Right Aſcenſion?

Geome-

Geometrically.

1. Having drawn the Solſtitial Colure N Q P A, the Equinoctial A Q, and at Right Angles to it the Equinoctial Colure N P, ſet off 23° 29′, the greateſt Declination from A to E, and draw the Ecliptic E C.



2. Set off the Semi-tangent of 57° 30′, the Sun's Longitude from the firſt Point of *Aries* from ♈ to S, then will S be the Place of the Sun.

3. Through S and the Poles N and P, draw the Circle of Right Aſcenſion N S R P, then will ♈ R be the Right Aſcenſion, and S R the preſent Declination; each of which may be meaſured by *Prob. the 7th, of Section the 5th, of Part the 3d.*

To determine their Quantities by Calculation.

In the Spherical Triangle S ♈ R, Right-angled at R, are given, S ♈, the Sun's Longitude from the next Equinoctial Point; and the Angle S ♈ R the greateſt Declination. Wherefore, by *Caſe the 1ſt, of Section the 3d, of Part the 3d,*

To find the Declination,

It will be, R : S ∠ S ♈ R :: s, ♈ S : s, S R. That is,

As the Radius ——	10.0000000
To the Sine of the greateſt Declination 23° 29′ ——	9.6004090
So is the Sine of the Sun's Long. from *Aries* 57° 30′	9.9260292
To the Sine of his preſent Declination 19° 38′ ——	9.5264382

Which is North, becauſe the Sun is in the *Northern Signs*, or between the firſt Point of *Aries* and the firſt Point of *Libra*.

Hence, and from the *Projection*, it is manifeſt, that when the Sun is at the ſame Diſtance from either of the Equinoctial Points, *Aries* or *Libra*, the Quantity of the Declination is the ſame; but its Denomination different, according as the Sun is converſant either in the *Northern* or *Southern* Signs.

So

So that this Declination in *Northern* Signs is common to

♉ 27° 30′, or ♌ 2° 30′.

In *Southern* Signs, to ♍ 27° 30′, or ♒ 2° 30′.

For the *Right Aſcenſion*, it will be, by *Caſe the* 10*th*, *of Section the* 3*d*, *of Part the* 3*d*,

R : cs, ∠ S ♈ R :: t, ♈ S : t, ♈ R. That is,

As the Radius	10.0000000
To the Co-ſine of the greateſt Declination 23° 29′	9.9623978
So is the Tang. of the Sun's Long. from *Aries* 57° 30′	10.1958127
To the Tang. of the R. Aſcenſion from *Aries* 55° 13′	10.1582105

Hence, and from the *Projection*, it is manifeſt, that the ſame Quantities of Right Aſcenſion, number'd from the neareſt Equinoctial Point, correſponds with the ſame Longitudes number'd from the ſame Equinoctial Point: Wherefore, to find the Right Aſcenſion from the firſt Point of *Aries*, Obſerve,

1. If the Sun be in the firſt Quadrant of the *Ecliptic*, that is, between the firſt Point of *Aries* and the firſt Point of *Cancer*, then the Right Aſcenſion found, is the Right Aſcenſion from the firſt Point of *Aries*.

2. If the Sun be in the ſecond Quadrant of the *Ecliptic*, that is, between the firſt Point of *Cancer* and the firſt Point of *Libra*, then the Right Aſcenſion found, ſubtracted from 180 Degrees, leaves the Right Aſcenſion from the firſt Point of *Aries*.

3. If the Sun be in the third Quadrant of the *Ecliptic*, that is, between the firſt Point of *Libra* and the firſt Point of *Capricorn*, then the Right Aſcenſion found, added to 180 Degrees, gives the Right Aſcenſion from the firſt Point of *Aries*.

4. If the Sun be in the fourth Quadrant of the *Ecliptic*, that is, between the firſt Point of *Capricorn* and the firſt Point of *Aries*, then the Right Aſcenſion found, ſubtracted from 360 Degrees, leaves the Right Aſcenſion from the firſt Point of *Aries*.

After this Manner, were the *Tables* of the Sun's Right Aſcenſion and Declination, at the End of this *Section*, computed from the Sun's Place in the *Ecliptic*, deliver'd in the firſt Table.

By

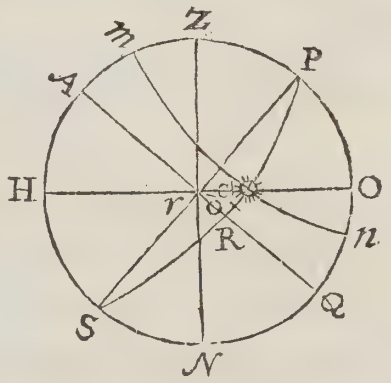
By the Reverſe of this *Problem*, may the Sun's Place and greateſt Declination be obtained from his preſent Declination and Right Aſcenſion; or his Longitude and Right Aſcenſion from the preſent and greateſt Declination; which *Problems* are ſometimes of Uſe in *Aſtronomy*.

PROBLEM II.

The Latitude of the Place and Declination of the Sun being given, to find his Amplitude and Aſcenſional Difference, Hour of Riſing and Setting, with the Length of the Day and Night.

Example.

In the Latitude of 51° 32′ North, the Sun having 19° 38′ North Declination, I demand his Amplitude and Aſcenſional Difference, with the Time of his Riſing and Setting, and the Length of the Day and Night.



1. Having drawn the Circle ZONH, repreſenting the Meridian, the Equinoctial A Q, and the Axis P S.

2. Set off the Latitude of the Place from P to O, and draw the Horizon H O.

3. Becauſe the Declination of the Sun is North, draw the Parallel *m n* of the Sun's Diurnal Courſe on the Northern Side of the Equator, at the Diſtance of 19° 38′, (by *Prob. the 9th of Sect. the 5th, of Part the 3d*) and obſerve where it cuts the Horizon, as at ☉.

4. Through the Interſection ☉, and the Poles P and S, draw the Meridian P ☉ R S, (*by Prob. the 4th, of Sect. the 5th, of Part the 3d*) then will ♈ ☉ be the Amplitude, and ♈ R the Aſcenſional Difference; each of which may be meaſured by *Prob. the 7th, of Section the 5th, of Part the 3d.*

But to determine their juſt Quantities by *Calculation*,

In the Spherical Triangle ☉ ♈ R, Right-angled at R, are given ☉ R the preſent Declination, and the Angle ☉ ♈ R the Complement of the Latitude of the Place. Whence,

To find the Amplitude, it will be, (by *Caſe the 4th, of Sect. the 5th, of Part the 3d.*)

s, $\angle \odot \Upsilon$ R : R : : s, $\odot$ R : s, $\Upsilon \odot$. That is,

As the Co-ſine of the Latitude 51° 32′ —— ——	9.7938317
To the Radius —— —— —— —— —— —— ——	10.0000000
So is the Sine of the Declination 19° 38′ —— ——	9.5263387
To the Sine of the Amplitude 32° 42′ —— —— ——	9.7325070

Which is always the ſame Way with the Declination, that is to ſay, if the Declination be South the Amplitude is Southerly, but if the Declination be North the Amplitude is Northerly; and conſequently in this Caſe, the Sun riſes and ſets 32° 42′ to the Northward of the Eaſt and Weſt Points, as it would have riſe and ſet 32° 42′ to the Southward of the Eaſt and Weſt Points if the Declination had been South.

For the Aſcenſional Difference it will be, (by *Caſe the 3d, of Sect. the 3d, of Part the 3d.*)

t, $\angle \odot \Upsilon$ R : R :: t, $\odot$ R : s, R Υ. That is,

As the Co-tangent of the Latitude 51° 32′ —— ——	9.9000865
To the Radius —— —— —— —— —— —— ——	10.0000000
So is the Tangent of the Declination 19° 38′ —— ——	9.5523514
To the Sine of the Aſcenſional Difference 26° 41′	9.6522649

Which (if the Latitude and Declination be both the ſame Way, *that is*, both North or both South) converted into Hours and Minutes (by allowing 15 Degrees to an Hour, and 15 Minutes of Right Aſcenſion to one Minute of Time) ſhews how long the Sun Riſes before Six in the Morning, and ſets after Six at Night.

But (if the Declination of the Sun, and the Latitude of the Place are contrary; that is, if one be North and the other South, then the Aſcenſional Difference converted into Time, ſhews how many Hours and Minutes the Sun riſes after Six in the Morning, and ſets before Six at Night; which is abundantly manifeſt from the Figure.

Hence in this Caſe, becauſe the Latitude is North, and the Declination North.

	h.	m.	ſec.
From Six Hours	6	00	00
Take the Aſcenſional Difference equal to	1	46	44
There Remains the Time of the Sun's Riſing	4	13	16

For the Setting,

	h.	m.	ſec.
To Six Hours	6	00	00
Add the Aſcenſional Difference equal to	1	46	44
The Sum is the Time of the Sun's Setting	7	46	44

Now becauſe the Hour of the Sun's ſetting ſhews the Time elapſed between Noon and the End of the Day, which is therefore equal to Half the Length of the Day: If the Hour of the Sun's ſetting be doubled, it gives the Length of the Day, or Number of Hours the Sun is viſible above the Horizon.

Wherefore in the preſent Caſe,

If the Hour of Sun-ſetting 7 h. 46 m. 44 ſec. be multiplied by 2, or added to its ſelf, it gives 15 h. 33 m. 28 ſec. the Length of the Day at that Time of the Year; in which the Declination is the ſame in the Queſtion.

Again, Becauſe the Hour of Sun-riſing ſhews the Quantity of Time elapſed, between the Time of the Sun-riſing and Midnight, which is therefore equal to Half the Length of the Night; if the Hour of Sun-riſing be doubled, it gives the Length of the Night.

Wherefore in the preſent Caſe,

If the Hour of Sun's riſing 4 h. 13 m. 16 ſec. be doubled, or added to its ſelf, it gives the Length of the Night 8 h. 26 m. 32 ſec. which being added to the Length of the Day before found, makes 24 Hours.

Hence it is obſervable, that when the Sun is at equal Diſtances from the firſt Point of *Cancer* or *Capricorn*, the Amplitudes Aſcenſional Differences, Time of the Sun's riſing and ſetting, Length of the Day and Night, are the ſame; ſince the Sun's Declination from which they are computed, is the ſame.

Hence it is farther obſervable, that the Hour of Sun-riſing when the Sun hath any determinate Degree and Minute of North Declination, is the ſame with the Hour of Sun-ſetting at that time of the Year when the Sun has the ſame Number of Degrees South Declination; alſo the Time of Setting when the Declination is North, the ſame with the Time of Riſing when the Declination is South; and laſtly, the Length of the Day when the Declination is North, is the ſame with the Length of the Night when the Declination is South; and the Length of the Day when the Declination is South, is the ſame with the Length of Night when the Declination is North.

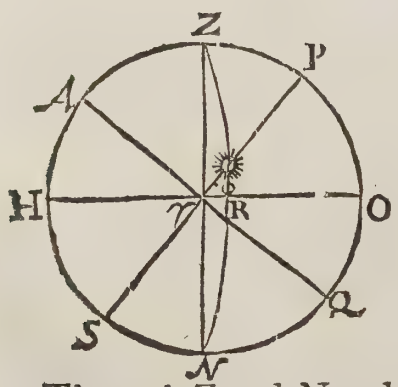
By the inverſe Method, If the true Amplitude, or the Time of the Sun's Riſing and Setting, can be obtained by Obſervation, it will be eaſy to find the Latitude of the Place.

PROBLEM III.

The Latitude of the Place and the Declination of the Sun being given, to find the Sun's Height and Azimuth at the Hour of Six.

Example.

Given the Latitude 51° 32′ North, the Sun's Declination 19° 38′ North, to find the Height and Azimuth of the Sun at the Hour of Six.



1. Having drawn the Meridian ZONH, the Horizon HO, and at Right-angles to it the Prime Vertical ZN, ſet off the Latitude from O to P, and draw the Axis or Six a Clock Hour Circle P ♈ S.

2. Set off the Semi-tangent of the Declination, becauſe it is North, in the Axis from ♈ towards P to ☉, and it gives the Place of the Sun.

Through Z and N and the Point ☉, draw a Vertical Circle *(by Prob. the 4th, of Sect. the 5th, of Part the 3d)* and the Arch of it contained between the Place of the Sun at ☉, and the Horizon, that is to ſay, ☉R is the Sun's Height at that Time; and the Arch of

 the

the Horizon H R or R O, is the Azimuth from the Meridian; each of which may be measured *(by Prob. the 7th, of Sect. the 5th, of Part the 3d.)*

But to obtain their just Quantities by *Calculation*,

In the Sperical Triangle ☉ ♈ R, Right-angled at R, are given ☉ ♈ the present Declination, and ∠ ☉ ♈ R the Latitude of the Place, whence it will be *(by Case the 1st, of Sect. the 3d, of Part the 3d)*

For the Sun's Height at Six.

R : s, ☉ ♈ R :: s, ♈ ☉ : s, ☉ R. That is,

As the Radius	10.0000000
To the Sine of the Latitude 51° 32′	9.8937452
So is the Sine of the Sun's Declination 19° 38′	9.5263387
To the Sine of the Sun's Height at the Hour of Six 15° 15′ either in the Morning or in the Afternoon	9.4200839

If the Declination be contrary to the Latitude, that is, if one be North and the other South, then it gives the Depression, and shews how many Degrees the Sun is below the Horizon at that time.

For the Azimuth at the Hour of Six, it will be *(by Case the 10th, of Sect. the 3d, of Part the 3d.)*

R : cs, ☉ ♈ R :: t, ☉ ♈ : t, ♈ R. That is,

As the Radius	10.0000000
To the Co-sine of the Latitude 51° 32′	9.7938317
So is the Tangent of the Sun's Declination 19° 38′	9.5523514
To the Co-tangent of the Azimuth from the Meridian 77° 29′	9.3461831

Which must be counted from the Visible Pole; wherefore if it be in the Morning the Azimuth is North 77° 29′ East, but if it be in the Afternoon the Azimuth is North 77° 29′ West.

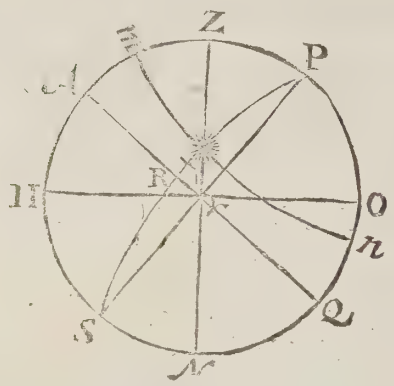
PROB-

PROBLEM IV.

The Latitude of the Place, and the Declination of the Sun being given, to find his Height, and the Time when he will appear due East or West.

Example.

Let the Latitude be 51° 32′ North, the Declination of the Sun as before 19° 38′ North; to find the Sun's Height, and Time when he will appear due East or West.



1. Having, as in the former Figure, drawn the Meridian Z O N H, the Prime Vertical Z N, the Horizon H O, the Axis P S, and the Equinoctial A Q, draw the Parallel of Declination *m n*, and where it cuts the Prime Vertical, Z N as in ☉, it gives the Place of the Sun.

2. Through ☉ and the Poles P and S, draw a Meridian (*by Prob. the 4th, of Sect. the 5th, of Part the 3d*) and ☉ ♈ will be the Sun's Height, and R ♈ the Time after Six in the Morning, or before Six in the Afternoon: Both which may be measured (*by Prob. the 7th, of Sect. the 5th, of Part the 3d.*)

But for obtaining their Lengths by *Calculation*,

In the Right-angled Spherical Triangle ♈ ☉ R, Right-angled at R, are given, R ☉ the Sun's Declination, the Angle ☉ ♈ R the Latitude of the Place; and therefore (*by Case the 4th, of Sect. the 5th, of Part the 3d*).

To find the Height when he will appear due East or West, it will be,

S ∠ ☉ ♈ R : R : : s, R ☉ : s, ☉ ♈. That is,

As the Sine of the Latitude 51° 32′ ——	9.8937452
To the Radius ——	10.0000000
So is the Sine of the Sun's Declination 19° 38′ ——	9.5263387
To the Sine of the Height when due E. or W. 15° 15′	9.6325935

But

But for the Time when he will appear due East or West, it will be (*by Case the 3d, of Sect. the 3d, of Part the 3d.*)

t, $< \odot \Upsilon$ R : R :: t, $\odot$ R : S, R Υ. That is,

As the Tangent of the Latitude 51° 52′ ——	10.0999135
To the Radius —— —— —— —— —— ——	10.0000000
So is the Tangent of the Declination 19° 38′ ——	9.5523514
To the Sine of the Time 16° 28′ —— —— ——	9.4524379

Which converted into Hours and Minutes of Time, by allowing 15 Degrees to an Hour, shews how long after Six in the Morning it is ere the Sun will be due East; and how long before Six in Afternoon the Sun will appear due West.

Wherefore,

	h.	m.	sec.
To six Hours —— —— —— —— —— ——	6	00	00
Add the Time last found —— —— —— ——	1	05	52
It gives the Time the Sun appears due E. in the Morn.	7	04	52

Or,

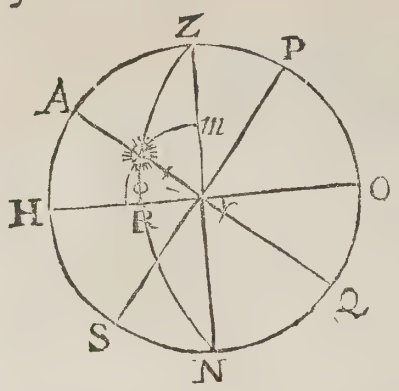
	h.	m.	sec.
From six Hours —— —— —— —— —— ——	6	00	00
Take the Time last found —— —— —— ——	1	05	52
It gives the Time when the Sun is due W. in the After.	4	54	08

PROBLEM V.

The Sun being in the Equator, the Latitude of the Place is given, and the Hour of the Day; to find the Sun's Height and Azimuth at that Time.

Example.

I demand the Sun's Height and Azimuth, at half an Hour past Three in the Afternoon, or at half an Hour past Eight in the Morning; each of which Times are three Hours and an half distant from Noon, in the Latitude of 51° 32′ North. 1. Having



1. Having drawn the Meridian Z O N H, the Horizon H O, the Prime Vertical Z N, the Axis P S, and the Equinoctial A Q, as in the former *Case*; set off the Semi-tangent of the Complement of the Hour from Noon, in the Equinoctial from ♈ to ☉, then will ☉ be the Place of the Sun.

2. Through ☉ and the Points Z and N, draw the Vertical Circle Z ☉ R N, then will ☉ R be the Height, and H R or R O the Azimuth from the Meridian; each of which may be measured by *Prob. the 7th, of Sect. the 5th, of Part the 3d.*

But to determine their Lengths by *Calculation*,

In the Triangle ♈ ☉ R, Right-angled at R, are given ♈ ☉ the Complement of the Hour from Noon, and the Angle ☉ ♈ R the Complement of the Latitude of the Place: Wherefore (*by Case the 1st, of Section the 3d, of Part the 3d,*) it will be to find the Height at that Time,

As R : S, ☉ ♈ :: S, ☉ ♈ R : S, ☉ R. That is,

As the Radius	10.0000000
To the Co-sine of the Hour from Noon, 52° 32′ —	9.7844471
So is the Co-sine of the Latitude 51° 32′	9.7938317
To the Sine of the Height at that Time 22° 15′ —	9.5782788

And for the Azimuth it will be (*by Case the 10th, of Sect. the 5th, of Part the 3d.*)

R : cs, ☉ ♈ R :: t, ☉ ♈ : t, ♈ R. That is,

As the Radius	10.0000000
To the Sine of the Latitude 51° 32′	9.8937452
So is the Co-tangent of the Hour from Noon 52° 30′	9.8849805
To the Co-tang. of the Azim. from the Mer. 59° 00′	9.7787257

Which is South 59° 00′ Eastwardly, if it be in the Forenoon; but South 59° 00′ Westwardly, if the Time given be in the Afternoon.

PROB-

PROBLEM V.

CASE 2.

If instead of the Hour of the Day, the Altitude of the Sun was given, that is,

If the Latitude of the Place 51° 32′ North, and the Altitude of the Sun 22° 15′ be given, to find the Hour of the Day and Azimuth from the Meridian, then in the Right-angled Spherical Spherical Triangle ⊙ ♈ R Right-angled at R, (see the Figure in the preceding Page) are given, ⊙ ♈ R the Complement of the Latitude, and ⊙ R the Altitude; whence to find the Azimuth it will be (by *Case the 3d, of Section the 3d, of Part the 3d.*)

t, < ⊙ ♈ R : R :: t, ⊙ R : S, R ♈. That is,

As the Co-tangent of the Latitude 51° 32′ ——	9.9000865
To the Radius ——	10.0000000
So is the Tangent of the Altitude 22° 15′ ——	9.6118409
To the Co-sine of the Azimuth from the Mer. 59° 00′	9.7117544

Which is South 59° 00′ Eastwardly, if the Altitude was taken in the Morning; but South 59° 00′ Westwardly, if the Altitude was observed in the Afternoon.

For the Hour of the Day it will be (*by Case the 4th, of Sect. the 3d, of Part the 3d.*)

s, < ⊙ ♈ R : R :: s, ⊙ R :: s, ⊙ ♈. That is,

As the Co-sine of the Latitude 51° 32′ ——	9.7938317
To the Radius ——	10.0000000
So is the Sine of the Height 22° 15′ ——	9.5782364
To the Co-sine of the Hour from Noon 52° 30′ —	9.7844047

Which being converted into Time, will give 3 h. 30 m. for the Hour of the Day, if the Altitude was taken in the Afternoon; but being subtracted from Twelve Hours will give 8 h. 30 m. for the Hour in the Forenoon, if the Altitude was observed in the Morning

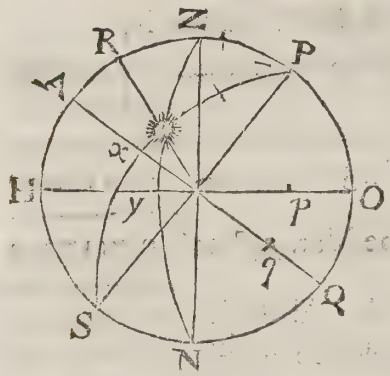
PROBLEM VI.

The Sun being in any Point of the Ecliptick, his Declination is given, together with the Hour of the Day and Latitude of the Place; to find his Altitude and Azimuth at that Time.

Example.

In the Latitude of 51° 32′ North, the Sun having 19° 38′ North Declination; his Altitude and Azimuth is required at half an Hour after Three in the Afternoon, or at half an Hour after Eight in the Morning.

Geometrically.



1. Having drawn the Meridian Z P O N H, the Horizon H O, and the Prime Vertical Z N.

2. Set off the Latitude from O to P, and draw the Axis P A, and at Right-angles to it the Equator A Q.

3. Through P and S draw the Arch of an Hour Circle P ⊙ S, forming an Angle with the Meridian, equal to 52° 30′ the Hour from Noon converted into Time.

4. Set off the Sun's Declination 19° 38′ in the Hour Circle P S from ϒ to ⊙ (*by Case the 3d, of Prob. the 6th, of Sect. the 5th,*) then will ⊙ be the Place of the Sun.

5. Through Z and ⊙ draw the Vertical Circle Z ⊙ N, then will ⊙ *y* be the Altitude, and the Angle ⊙ Z P the Azimuth from the Meridian; which may be measured by *Problems the 7th and 8th, of Section the 5th, of Part the 3d.*

But to determine their Quantities by *Calculation*,

In the Spherical Triangle Z P ⊙ are given, Z P the Complement of the Latitude; P ⊙ in this *Case*, the Complement of the Declination, and Z P ⊙ the Hour from Noon. Whence, to find the Azimuth from the Meridian, it will be (*by Case the 9th, of Sect. the 4th, of Part the 3d.*)

R :

R : cs, ZP ⊙ : : t, ⊙P : t, PR. That is,

As the Radius ——	10.0000000
To the Co-sine of the Hour from Noon 52° 30′ —	9.7844471
So is the Co-tangent of the Declination 19° 38′ —	10.4476486
To the Tangent of PR, or a fourth Arch 59° 37′ 45″	10.2320957

Now PR—ZP=ZR, that is, the fourth Arch 59° 37′ 40″ leſſened by the Complement of the Latitude 38° 28′, leaves ZR 21° 9′ 45″, which we call a fifth Arch. Wherefore,

s, ZR : s, PR : : t, < P : t, < Z. That is,

As the Sine of the fifth Arch 21° 09′ 45″ ——	9.5575244
To the Sine of the fourth Arch 59° 37′ 45″ ——	9.9358916
So is the Tangent of the Hour from Noon 52° 30′	10.1150195
To the Tang. of the Azim. from the Merid. 72° 11′ 57″	10.4933867

Wherefore, if the Time given be in the Morning it is South 72° 11′ 57″ Eaſt, but if the Time given be in the Afternoon the Azimuth is South 72° 11′ 57″ Weſt.

For the Altitude it will be (*by Caſe the 1ſt, of Sect. the 4th, of Part the 3d.*)

s, < Z : s, ⊙P : : s, < P : s, Z ⊙. That is,

As the Sine of the Azim. from the Merid. 72° 11′ 57″	9.9786940
To the Co-ſine of the Declination 19° 38′ ——	9.9739873
So is the Sine of the Hour from Noon 52° 30′ ——	9.8994667
To the Co-ſine of the Height at that Time 38° 17′ 52″	9.8947600

PROBLEM VI.

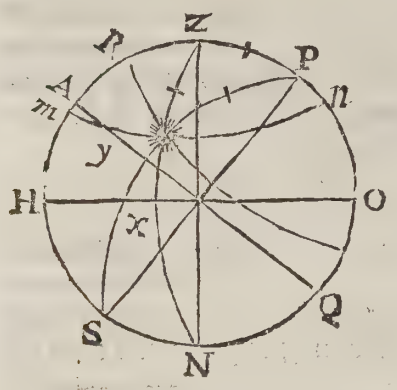
CASE 2.

Given the Latitude of the Place, the Declination of the Sun, and his Altitude at any Time; to find the Hour of the Day, and his Azimuth from the Meridian.

Example.

In the Latitude of 51° 32′ North, the Sun having 19° 38′ North Declination, his Altitude was found by Observation to be 38° 17′ 52″: I demand the Hour of the Day, and his Azimuth from the Meridian.

Geometrically.



Having drawn the Meridian Z O-N H, the Horizon H O, the Prime Vertical Z N, the Axis P S, and the Equinoctial A Q, after the manner taught in the former *Case*, draw the Parallel of Northern Declination R ⊙, as also the Parallel of Altitude *m n*, according to the Directions given in *Problem the 9th, of Section the 5th, of Part the 3d*, and the Point ⊙, where these two Parallels intersect will give the Place of the Sun; through which and the Poles P and S, as also the *Zenith* and *Nadir* Z and N, if an Hour Circle as P ⊙ S, and an Azimuth Circle as Z ⊙ N be drawn (*by Prob. the 4th, of Sect. the 5th, of Part the 3d*) the Thing is done; and the Angle Z P ⊙ will be the Hour from Noon, as will the Angle Z be the Azimuth from the Meridian; each of which may be measured (*by Prob. the 8th, of Sect the 5th, of Part the 3d.*)

But to find their just Quantities by *Calculation*,

In the Spherical Triangle Z P ⊙ are given, Z P the Complement of the Latitude, Z ⊙ the Complement of the Altitude, and P ⊙ the Complement of the Declination; whence (*by Case the 11th, of Right-angled Spherical Triangles.*

To find the Azimuth it will be,

As R : s, Z ⊙ :: s, Z P : a fourth Sine. That is,

As the Radius	10.0000000
To the Co-sine of the Altitude 38° 17′ 52″	9.8947593
So is the Co-sine of the Latitude 51° 32′	9.7938317
To a fourth Sine	9.6885910

Again,

Again,

As the 4th Sine : S, $\frac{1}{2}$ ZP+ZO+PO :: S, $\frac{1}{2}$ ZP+ZO+PO − P O : 7th Sine. That is,

As the 4th Sine	9.6885910
Is to the Sine of half the Sum of the Complement of the Latitude, Complement of the Altitude, and Complement of the Declination 80° 16′ 4″	9.9937044
So is the Sine of half the ſame Sum leſſened by the Complement of the Declination 9° 54′ 4″	9.2353976
To a ſeventh Sine	9.5405110
To which add the Radius	19.5405110
The Sine Complement of half that Sum 53° 54′ 02″ being doubled, gives 107° 48′ 04″ for the Azimuth from the North	9.7702555

Or,

To the Arith. Comp. of the Co-ſine of the Lat. 51° 32′	0.2061683
Add the Arith. Comp. of the Co-ſine of the Alt. 38° 17′ 52″	0.1052407
Alſo the Sine of the Half Sum 80° 16′ 04″	9.9937044
And the Sine of the Remainder 9° 54′ 04″	9.2353976
Half the Sum of theſe four Sines	19.5405110
Will give the Co-ſine of 53° 54′ 02″	9.7702555

Which being doubled, will give 107° 48′ 04″, the Azimuth from the North, the ſame as before found.

Wherefore,

To find the Azimuth, the General Rule is,

To the Complement of the Latitude of the Place, add the Complement of the Sun's Altitude, and the Complement of the Sun's Declination in to one Sum: From half this Sum ſubtract the Complement of the Sun's Declination, and keep the Difference or Remainder.

 Then

Then ſay, As the Radius to the Co-ſine of the Altitude, ſo is the Co-ſine of the Latitude, *&c.* Or,

To the Arithmetical Complement of the Co-ſine of the Latitude,
Add the Arithmetical Complement of the Co-ſine of the Altitude;
Alſo the Sine of the Half Sum,
And the Sine of the Difference or Remainder.

Half the Sum of theſe four Sines will give the Co-ſine of Half the Azimuth; which being doubled, will give the Azimuth from the Viſible Pole. Or,

By Gunter's *Scale.*

Extend the Compaſſes from the Sine of 90 Degrees, to the Sine of the Complement of the Altitude on the ſame Line; the ſame Extent will reach from the Sine of the Complement of the Latitude, to a fourth Sine; then Extend the Compaſſes from the fourth Sine, to the Sine of the Half Sum; the ſame Extent of the Compaſſes will reach from the Sine of the Difference or Remainder, to a ſeventh Sine; againſt which on the Line of Verſed Sines ſtands the Azimuth from the Meridian.

From the Azimuth thus determined, it will be eaſy by the help of *Caſe the 1ſt, of Sect. the 4th, of Part the 3d.*

To find the Hour,

For, s, ⊙P : s, ⊙ZP :: s, Z⊙ : s, ZP⊙. That is,

As the Co-ſine of the Declination 19° 38′ ——	9.9739873
To the Sine of the Azim. from the Merid. 72° 11′ 56″	9.9786937
So is the Co-ſine of the Altitude 38° 17′ 52″ ——	9.8947592
To the Sine of the Time from Noon 52° 30′ ——	9.8994656

Which converted into Hours and Minutes, gives the Time of the Day, *viz.* 3 h. 30 m. if the Obſervation was made in the Afternoon; but if the Obſervation be made in the Morning, then,

	h.	m.	ſec.
From ——	12	00	00
Take the ſame Arch converted into Hours and Min.	3	30	00
The Remainder will give the Hour in the Forenoon	8	30	00

By

By *the* 11*th Case of Sect. the* 4*th, of Part the* 3*d*, the Hour of the Day may be investigated directly without the help of the Azimuth, after the manner following.

To the Complement of the Latitude add the Complement of the Declination, and the Complement of the Altitude into one Sum; From Half that Sum take the Complement of the Altitude, and keep the Difference. Then say,

As the Radius	10.0000000
To the Co-sine of the Latitude 51° 32′	9.7938317
So is the Co-sine of the Declination 19° 38′	9.9739373
To a fourth Sine	9.7678190

Again,

As the fourth Sine	9.7678190
To the Sine of the Half Sum before determined 80° 16′ 04″	9.9937044
So is the Sine of the Difference 28° 33′ 56″	9.6795768
To a seventh Sine	9.9054622
To which add the Radius	19.9054622
The Sine Comp. of Half that Sum 26° 15′ doubled	9.9527311

gives the Time from Noon equal to 3 h. 30 m. as before: Whence the Time of the Day may be easily found by the Rules laid down in the preceding Page.

Or,

To the Arith. Comp. of the Co-sine of the Lat. 51° 32′	0.2061683
Add the Arith. Comp. of the Co-sine of the Al. 19° 38′	0.0260127
Also the Sine of the Half Sum 80° 16′ 04″	9.9937044
And the Sine of the Remainder 28° 33′ 56″	9.6795768
Half the Sum of these four Sines	19.9054622
Will give the Co-sine of 26° 15′	9.9527311

Which being doubled gives 52° 30′, equal in Time to 3 h. 30 m.

* Or,

Or, *By* Gunter's Scale.

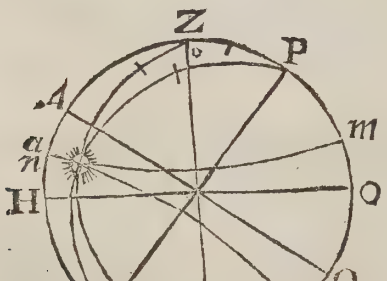
Extend the Compaſſes from the Sine of 90 Degrees on the Line of Sines, to the Sine of the Complement of the Latitude on the ſame Line; the ſame Extent will reach from the Sine of the Complement of the Declination, to a fourth Sine; then Extend the Compaſſes from the fourth Sine, to the Sine of the Half Sum; the ſame Extent of the Compaſſes will reach from the Sine of the Difference or Remainder, to a ſeventh Sine; againſt which on the Line of Verſed ſtands the Hour from Noon.

Hence the *Azimuth* may be eaſily determined,

For, s, Z ☉ : s, Z P ☉ :: s, P ☉ : s, P Z ☉. That is,

As the Co-ſine of the Altitude 38° 17′ 52″ ——	9.8947592
To the Sine of the Time from Noon 52° 30′ ——	9.8994667
So is the Co-ſine of the Declination 19° 38′ ——	9.9739873
To the Sine of the Azimuth from the Merid. 72° 12′	9.9786948

In the preceding *Example* the Latitude of the Place and Declination of the Sun have been ſuppoſed both the ſame Way, that is, both North; but if they be contrary, that is, if one be North and the other South, then inſtead of the Complement of the Declination you muſt make uſe of the Declination added to 90 Degrees, for in the adjacent Figure where the Latitude is ſuppoſed to be 51° 32′ North, and the Declination 19° 39′ South, it is evident that the Side P ☉ oppoſite to the Angle Z, is equal to P A + A *a*, that is, to the Declination increaſed by a Quadrant or 90 Degrees; now as in either Caſe P ☉ is equal to the Sun's Diſtance from the Viſible Pole, if inſtead of the Expreſſion Complement of the Declination, or the Declination more 90 Degrees, we uſe the Term the Sun's Diſtance from the Pole; then the *General Rule* for finding the Sun's Azimuth

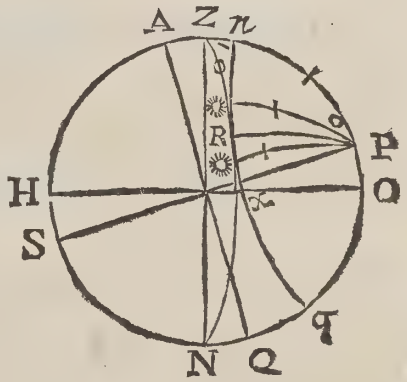


Azimuth, will in each Caſe run thus; to the Complement of the Latitude add the Complement of the Altitude, and the Sun's Diſtance from the Pole into one Sum; from Half the Sum ſubtract the Sun's Diſtance from the Pole, and keep the Remainder; then ſay, As the Radius, *&c.*

When the Latitude of the Place is leſs than the Declination of the Sun, and both the ſame Way, which will happen at certain Times in all Places ſituated between the Tropics, the Sun will appear upon the ſame Azimuth Circle twice in the Morning and twice in the Afternoon.

For *Example.*

Let it be required to find the Hour of the Day, and Altitude of the Sun, when he will appear upon the Weſt North Weſt, or Eaſt North Eaſt Azimuth Circle at *Barbadoes*, in the Latitude of 13° 30′ North, at the Time of the Summer Soliſtice, or when the Sun has 23° 29′ of Northern Declination, which happens upon the 10*th* of *June*.



1. Having drawn the Meridian Z O N H, the Horizon H O, and at Right Angles to it the Prime Vertical Z N, ſet off the Latitude of the Place 13° 30′ from O to P, and draw the Six a-Clock Hour Circle P S, and at Right Angles to it the Equator A Q.

2. Thro' the *Zenith* and *Nadir*, draw the Vertical Circle Z ☉ ☉ N, forming an Angle of 67° 30′ 00″, equal to the Azimuth, by *Prob. the 2d, of Sect. the 5th, of Part the 3d.*

3. About the Pole P, at the Diſtance of the Complement of the Sun's Declination 66° 31′ 00 ſec. draw the ſmall Circle *n* ☉ *q*, by *Prob. the 9th, of Sect. the 5th, of Part the 3d.*

Where this interſects the Vertical Circle, before drawn in the Points ☉, ☉, it gives the two different Places in the Heavens where the Sun is when he appears under the ſame Azimuth; and ✕ ☉, ✕ ☉, will be the different Altitudes of the Sun at thoſe Times, each of which may be meaſured by *Prob. the 7th, of Sect. the 5th, of Part the 3d.*

4. Through the Points ⊙, ⊙, and P, draw the Hour Circles P⊙, P⊙, and the Angles ZP⊙, ZP⊙, will give the Times from Noon, each of which may be measured by *Prob. the 8th, of Sect. the 5th, of Part the 3d.*

But to find their Quantities by *Calculation*,

In the *Oblique-angled Spherical Triangle* ZP⊙ are given, ZP equal to the Complement of the Latitude, the Angle Z the Azimuth, P⊙ the Sun's Distance from the North Pole; whence to find Z⊙, the Complement of the Altitude, it will be, by *the 3d Case* of *Oblique-angled Spherical Triangles*, having first let fall the Perpendicular PR,

As R : cs, ∠Z :: t, ZP : t, ZR. That is,

As the Radius	10.0000000
To the Co-sine of the Azimuth 67° 30′	9.5828397
So is the Co-tangent of the Latitude 13° 30′	10.6196463
To the Tangent of a fourth Arch 57° 53′ 51″ ½	10.2024860

And again,

As cs, ZP : cs, P⊙ :: cs, ZR : cs, ⊙R. That is,

As the Sine of the Latitude 13° 30′	9.3681853
To the Sine of the Declination 23° 29′	9.6004090
So is the Co-sine of the fourth Arch ZR 57° 53′ 51″ ½	9.7254489
To the Co-sine of the fifth Arch ⊙R 24° 53′ 14″ ¾	9.9576726

But ZR + ⊙R = Z⊙. That is, If to the fourth Arch 57 deg. 53 min. 51 sec. ½ be added, the fifth Arch 24 deg. 53 min. 14 sec. ¾, the Sum 82 deg. 47 min. 06 sec. ¼, taken from 90 Degrees, will leave 7 deg. 12 min. 53 sec. ¾ for the least Altitude ⊙. But,

If from the fourth Arch 57 deg. 53 min. 51 sec. ½ be taken, the fifth Arch 24 deg. 53 min. 14 sec. ¾, the Remainder 33 deg. 00 min. 36 sec. ¾, taken from 90 Deg. will leave 56 deg. 59 min. 23 sec. ¼ for the greater Altitude.

Whence,

Whence, to find the Time from Noon, it will be,

As the Co-ſine of the Declination 23° 29′ —— 9.9624527

To the Sine of the Azimuth 67° 30′ —— 9.9656153
So is the Co-ſine of the leaſt Altitude 7° 12′ 53″ ¾ 9.9965476

To the Sine of the correſpondent Time 87° 54′ 26″ 9.9997102
anſwering to 5 hours 51 min. 37 ſec. ¾.

Again,

As the Co-ſine of the Sun's Declination 23° 29′ —— 9.9624527

To the Sine of the Azimuth 67° 30′ —— 9.9656153
So is the Co-ſine of the greateſt Altitude 56° 59′ 23″ ¼ 9.7362193

To the Sine of the correſpondent Time 33° 16′ 55″ 9.7393819
anſwering to 2 hours 13 min. 07 ſec. ¾.

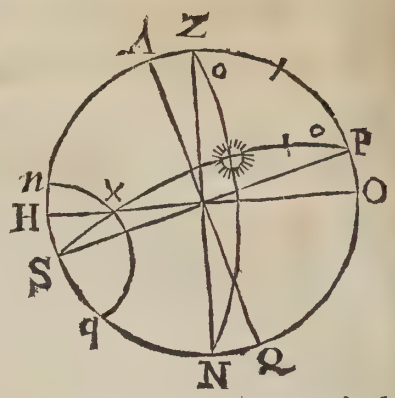
Whence it is manifeſt, that at *Barbadoes* on the 10*th* of *June*, when the Sun has his greateſt Declination, he appears upon the ſame Point of the Compaſs twice in the Forenoon and twice in the Afternoon; that is, he appears on the E N E. Azimuth at 6 hours 8 min. 22 ſec. ¼ in the Morning, and again at 9 hours 46 min. 52 ſec. ¼ the ſame Morning; at the firſt of which Times he has 7° 12′ 53″ ¾ of Altitude, and at the ſecond Time 56° 59′ 23″ ¾.

And again,

He appears on the WNW. Azimuth at 2 hours 13 min. 07 ſec. ¾ in the Afternoon, and again at 5 hours 51 min. 37 ſec. ¾ the ſame Afternoon; at the firſt of which Times he has 56 deg. 59 min. 23 ſec. ¼ of Altitude, and at the ſecond only 7 degrees 12 min. 53 ſec. ¾.

Again,

Suppoſe it were required to find, at the ſame Place and on the ſame Day, what ſhall be the Sun's greateſt Azimuth from the Meridian, at what Time it ſhall happen, and at what Altitude.



1. Having drawn the Meridian Z O N H, the Horizon H O, the Prime Vertical Z N, the Six a-Clock Hour Circle P S, and the Equinoctial A Q, about the South Pole S, draw a ſmall Circle *n x q*, at the Diſtance of the Sun's Declination (in this *Caſe* 23 deg. 29 min.) to interſect the Horizon in *x*, by *Prob. the 9th, of Sect. the 5th, of Part the 3d.*

2. About the Point *x* as a Pole, deſcribe the Vertical Circle Z ⊙ N, by the Directions given in *Page* 290, and through the Points P, *x*, and S, draw the Hour Circle P ⊙ S, by *Prob. the 4th, of Sect. the 5th, of Part the 3d*, where theſe Circles interſect each other, as in the Point ⊙, it will give the Place of the Sun; then will Z ⊙ be the Complement of the Altitude (which may be meaſured by *Prob. the 7th, of Sect. the 5th, of Part the 3d*) the Angle P Z ⊙ the Azimuth, and the Angle Z P ⊙ the Hour from Noon; each of which may be meaſured by *Problem the 8th, of the ſame Section.*

But to determine them by *Calculation*,

In the Spherical Triangle Z P ⊙, Right-angled at ⊙, are given Z P equal to the Complement of Latitude, P ⊙ equal to the Complement of the Declination; whence to find, firſt, the Hour from Noon, it will be, by *Caſe the 13th* of *Right-angled Spherical Triangles*,

As c t, P ⊙ : c t, Z P : : R : c s, ∠P. That is,

As the Tangent of the Declination 23° 29′ ——	9.6379563
To the Tangent of the Latitude 13° 30′ ——	9.3803537
So is the Radius ——	10.0000000
To the Co ſine of the Hour from Noon 56° 27′ 20″ ¼	9.7423974

which converted into Time, will give 3 h. 45 min. 49 ſec. ½.

2. To

2. To find the Angle Z, or the Azimuth, it will be, by *Caſe the 5th* of *Right-angled Spherical Triangles*,

As S. ZP : S. P⊙ :: R : S, < Z. That is,

As the Co-ſine of the Latitude 13° 30′ 00″ ——	9.9878315
To the Co-ſine of the Declination 23° 31′ 29″ —	9.9624527
So is the Radius —— —— —— —— ——	10.0000000
To the Sine of the Azimuth 70° 36′ 09″ ½ ——	9.9746212

3. To find Z⊙, the Complement of the Altitude, it will be, by *Caſe the 7th* of *Right-angled Spherical Triangles*,

As cs, P⊙ : cs, ZP : : R : cs, Z⊙. That is,

As the Sine of the Declination 23° 29′ 00′ ——	9.6004090
To the Sine of the Latitude 13° 30′ 00″ ——	9.3681853
So is the Radius —— —— —— —— —— ——	10.0000000
To the Sine of the Altitude 35° 51′ 43″ ½ ——	9.7677763

Whence it follows, that the Sun's greateſt Azimuth happens at 8 hours 14 min. 10 ſec. ⅔ in the Morning, when he appears North 70 deg. 36 min. 09 ſec. Eaſt, or Eaſt North Eaſt a quarter Eaſt nearly, or at 3 hours 45 min. 49 ſec. ⅔ in the Afternoon, when he appears North 70 deg. 36 min. 09 ſec. Weſt, or Weſt North Weſt half Weſt nearly; at each of which Times he has 35 deg. 51 min. 43 ſec. ½ of Altitude, which is therefore the propereſt Time for obſerving the Azimuth of the Sun, becauſe the Variation is then the leaſt.

The Sun's Amplitude, by *Prob. 2. of this Sect.* (ſee *Page* 322) will be found to be 24 deg. 11 min. 33 ſec. and conſequently his Diſtance from the North Point of the Horizon will be 65 deg. 48 min. 27 ſec. whence, by the Help of the preceding Calculation, he will be found to have the ſame Azimuth at 10 h. 03 min. 36 ſec. ½ in the Morning, or at 1 h. 56 min. 23 ſec. ½ in the Afternoon, when his Altitude will be found to be 50 deg. 43 min. 36 ſec. So that between the Time of the Sun's Riſing at 5 h. 36 min. 00 ſec. and 8 h. 14 min. 10 ſec. when he has the greateſt Azimuth, which is equal to 2 h. 38 min. 10 ſec. he alters his Azimuth but 4 deg.

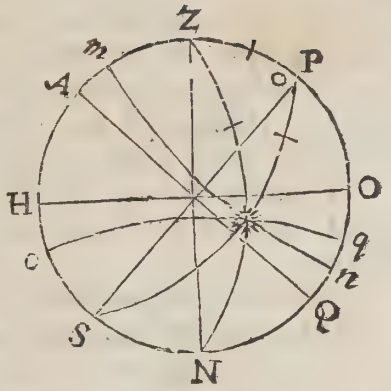
* 27 min.

47 min. 42 ſec. that is, not quite half a Point, during which Time he aſcends 35 deg. 51 min. 43 ſec. above the Horizon; and again, between 8 h. 14 min. 10 ſec. and 10 h. 03 min. 36 ſec. where his Azimuth becomes equal to the Complement of the Amplitude, and which is equal to 1 h. 49 min. 26 ſec. during which Time he gains 14 deg. 51 min. 53 ſec. of Altitude, he alters his Azimuth again but 4 h. 47 min. 42 ſec. and the ſame in the Afternoon between the Hours 1 h. 56 min. 27 ſec. and 3 h. 45 min. 49 ſec. and again, between 3 h. 45 m. 49 ſec. and the Time of the Sun's Setting; ſo that between the Time of the Sun's Riſing at 5 h. 36 min. 00 ſec. and 10 h. 03 min. 36 ſec. in the Forenoon, and between 1 h. 56 min 24 ſec. in the Afternoon, and the Time of the Setting of the Sun, which is at 6 h. 24 min. 00 ſec. which is equal to 4 hours 27 min. 36 ſec. during which Time the Sun aſcends and deſcends 50 deg. 43 min. 36 ſec. he alters his Azimuth but 4 $\frac{3}{4}$; whence it is, that the Variation in theſe Places may be obſerved with the greateſt Exactneſs, ſince a conſiderable Error in Time or Altitude will make little or no Error in the Azimuth; and 'tis for this Reaſon that our Mariners ought to omit no Opportunity at ſuch Times and in ſuch Places, to make all the Obſervations they can poſſible.

It is confirm'd by daily Experience, that the Atmoſphere continues illuminated for ſome Time after the Sun is ſet, and begins to be illuminated before he riſes; and by Aſtronomers it is found to laſt in the Evening 'till the Sun is deſcended 18 Degrees below the Horizon, when the ſmalleſt Fix'd Stars that can be ſeen with the Naked Eye, and ſuch are thoſe of the 6th Magnitude, become Viſible, and that it begins in the Morning when the Sun is within 18 Degrees of aſcending the Horizon, and when theſe Stars begin to vaniſh, this Commencement of the Illumination of the Atmoſphere when the Sun begins to ſcatter his Light through the Heavens, or firſt Dawning of the Day, is commonly called the Break of the Day, or Beginning of Morning Twilight; as in the Evening when this Illumination ceaſes, and the Atmoſphere is wholly deſtitute of the Solar Rays, it is called the End of Evening Twilight, or Beginning of Total Darkneſs.

Let it therefore be required to find the Beginning of Twilight in the Morning, and the End of Twilight in the Evening, in the Latitude of 51° 32′ North, when the Sun's Declination is 19° 38′ North.

1. Having



1. Having drawn the Meridian Z O N H, the Horizon H O, the Prime Vertical Z N, the Axis P S, the Equator A Q, and the Parallel of Northern Declination $m \odot n$, as in the *first Figure*, about the Zenith Z, draw the ſmall Circle $o \odot q$, at the Diſtance of 108 deg. or which is the ſame Thing, a Parallel of Depreſſion of 18 Degrees, where this interſects the Parallel of Declination before drawn, as in ⊙, gives the Place of the Sun at the Beginning of Twilight in the Morning, or at the End of Twilight in the Evening.

2. Through the Point ⊙, and the Zenith and Nadir, draw the Vertical Circle Z ⊙ N; alſo through the Poles P and S, and the ſame Point ⊙, draw the Hour Circle P ⊙ S, and the Thing is done; and the Angle ⊙ P Q, is equal to the Time of the Beginning of Twilight in the Morning, and the Angle Z P ⊙ will give the End of the Twilight in the Evening; each of which may be meaſured by *Prob. the 8th, of Sect. the 5th, of Part the 3d.*

But to find it by *Calculation*, by the *11th Caſe* of *Oblique-angled Spherical Triangles*,

To P ⊙ equal to the Complement of the Declination (in this *Caſe*) equal to 70 deg. 22 min. add Z P equal to the Complement of the Latitude of 38 deg. 28 min. and Z ⊙ equal to the Depreſſion of the Sun plus 90 Degrees, equal to 108 Degrees, from half the Sum equal to 108 deg. 25 min. ſubtract Z ⊙ the Diſtance of the Sun from the Zenith, equal to 108 deg. and keep the Remainder 00 deg. 25 min. Then

To the Arith. Comp. of the Co-ſine of the Latit. 51° 32′	0.2061683
Add the Arith. Comp. of the Co-ſine of the Decl. 19° 38′	0.0260127
Alſo the Sine of the half Sum 108° 25′ —— ——	9.9771674
And the Sine of the Remainder 00° 25′ —— ——	7.8616623
Half the Sum of theſe Four Sines —— —— —— ——	9.0355053

will be the Sine of 6 deg. 14 min. which being doubled gives 12 deg.

*

12 deg. 28 min. for the Angle ⊙ P Q, and being reduced into Time gives 0 h. 49 min. 52 sec. for the Beginning of Twilight in the Morning, and taken from 12 h. gives the Angle Z P ⊙, equal to 11 h. 10 min. 08 sec. for the End of Twilight in the Evening.

It has been already found, when the Sun has 19° 38′ of North Declination, he rises at 4 h. 13 min. 16 sec. when the Morning Twilight ends; wherefore, to find the Duration of Twilight in the Morning,

	H.	*M.*	*S.*
From the End of Morning Twilight ——	4	13	16
Take the Beginning of Morning Twilight ——	0	49	52
There remains the Duration of Twilight in the Morn.	3	23	24

And because when the Sun sets, the Evening Twilight begins,

	H.	*M.*	*S.*
From the End of the Evening Twilight ——	11	10	08
Take the Time of the Sun's Setting ——	7	46	44
There remains the Duration of Twilight in the Even.	3	23	24

And because the Sun has the same Northern Declination on the 14*th* of *July*, that he has on the 7*th* of *May*, it follows, that on each of those Days, the Day breaks or begins to appear at 0 h. 49 min. 52 sec. in the Morning; and the Sun rises at 4 h. 13 min. 16 sec. and sets at 7 h. 46 min. 44 sec. and the Twilight ends at 11 h. 10 min. 08 sec. at Night, when the Duration of Twilight is 3 h. 23 min. 24 sec.

When the Declination of the Sun becomes equal to the Difference between the Complement of the Latitude and the Depression, and both the same Way, the Solar Rays never quit the Hemisphere; therefore from the Complement of the Latitude of *London*, equal to 38 deg. 28 min. take the Depression 18 deg 00 min. the Remainder 20 deg. 28 min. is equal to the Declination when the Total Darkness ceases; and this answering to the 11*th* of *May* and the 11*th* of *July*, it follows, that in *London*, from the 11*th* of *May* to the 11*th* of *July* there is no Total Darkness, and that the Twilight continues from the Time of the Setting of the Sun till the Time of his Rising again.

I have here inserted such *Problems* only as are useful in the *Art of Navigation*, but if the Reader's Curiosity should excite him to know how all the *Various Cases* of the *Problems* arising from the Diurnal Motion of the Sun or Stars may be resolved, he may consult the *5th Part of my System of the Mathematicks, Volume the Second.*

TABLES

TABLES

OF THE

SUN's true Place

Every Day at Noon,

Under the *Meridian* of his Majesty's *Observatory*;

Calculated from

Mr. *Flamsteed*'s SOLAR TABLES,

For the Years 1737, 1738, 1739, 1740.

TOGETHER WITH

His *Right Ascension* in Time, and *Declination* from the *Equator*; with *Variations* to make them serve for Twenty Years to come: And a TABLE of the *Right Ascensions* and *Distances* from the Pole of some eminent *Fixed Stars*, deduced from the *Flamsteedian* OBSERVATIONS, and fitted to the Year 1738.

Days	Jan. ♑ ° ′	Febr. ♒ ° ′	March ♓ ° ′	April ♈ ° ′	May ♉ ° ′	June ♊ ° ′	Days
1	22 27	23 55	22 00	22 35	21 41	21 23	1
2	23 29	24 56	22 59	23 34	22 39	22 21	2
3	24 30	25 56	23 59	24 32	23 37	23 18	3
4	25 31	26 57	24 59	25 31	24 34	24 15	4
5	26 32	27 57	25 58	26 29	25 32	25 12	5
6	27 33	28 58	26 58	27 28	26 30	26 10	6
7	28 34	29 58	27 58	28 26	27 28	27 7	7
8	29 36	♓ 58	28 57	29 25	28 25	28 4	8
9	♒ 36	1 59	29 57	♉ 23	29 23	29 1	9
10	1 37	2 59	♈ 56	1 22	♊ 20	29 58	10
11	2 38	3 59	1 55	2 20	1 18	♋ 56	11
12	3 39	5 00	2 55	3 18	2 16	1 53	12
13	4 40	6 00	3 54	4 17	3 13	2 50	13
14	5 41	7 00	4 53	5 15	4 11	3 47	14
15	6 42	8 00	5 52	6 13	5 8	4 44	15
16	7 43	9 00	6 52	7 11	6 6	5 42	16
17	8 44	10 01	7 51	8 10	7 3	6 39	17
18	9 45	11 01	8 50	9 8	8 1	7 36	18
19	10 46	12 01	9 49	10 6	8 58	8 33	19
20	11 47	13 01	10 48	11 4	9 55	9 30	20
21	12 48	14 01	11 47	12 2	10 53	10 28	21
22	13 48	15 01	12 46	13 0	11 50	11 25	22
23	14 49	16 01	13 45	13 58	12 48	12 22	23
24	15 50	17 01	14 44	14 56	13 45	13 19	24
25	16 51	18 01	15 43	15 54	14 42	14 16	25
26	17 52	19 01	16 42	16 52	15 40	15 13	26
27	18 52	20 00	17 41	17 50	16 37	16 11	27
28	19 53	21 00	18 40	18 48	17 34	17 8	28
29	20 53		19 39	19 46	18 32	18 5	29
30	21 54		20 37	20 44	19 29	19 2	30
31	22 55		21 36		20 26		31

Days	July ♋ °	′	August ♌ °	′	Septemb ♍ °	′	October ♎ °	′	Novemb ♏ °	′	Decemb ♐ °	′	Days
1	19	59	19	39	19	39	19	10	20	12	20	37	1
2	20	57	20	37	20	38	20	10	21	12	21	38	2
3	21	54	21	34	21	36	21	9	22	13	22	39	3
4	22	51	22	32	22	35	22	9	23	13	23	40	4
5	23	48	23	30	23	34	23	9	24	14	24	41	5
6	24	45	24	28	24	32	24	8	25	15	25	42	6
7	25	43	25	26	25	31	25	8	26	15	26	44	7
8	26	40	26	23	26	30	26	8	27	16	27	45	8
9	27	38	27	21	27	29	27	8	28	17	28	46	9
10	28	35	28	19	28	27	28	8	29	17	29	47	10
11	29	32	29	17	29	26	29	7	♐	18	♑	48	11
12	♌	29	♍	15	♎	25	♏	7	1	19	1	49	12
13	1	27	1	13	1	24	1	7	2	20	2	50	13
14	2	24	2	11	2	23	2	7	3	20	3	52	14
15	3	21	3	9	3	22	3	7	4	21	4	53	15
16	4	19	4	7	4	21	4	7	5	22	5	54	16
17	5	16	5	5	5	20	5	7	6	23	6	55	17
18	6	14	6	3	6	19	6	7	7	24	7	56	18
19	7	11	7	1	7	18	7	7	8	25	8	58	19
20	8	8	7	59	8	17	8	8	9	26	9	59	20
21	9	6	8	57	9	17	9	8	10	27	11	0	21
22	10	3	9	56	10	16	10	8	11	28	12	1	22
23	11	1	10	54	11	15	11	8	12	29	13	2	23
24	11	58	11	52	12	14	12	8	13	30	14	3	24
25	12	56	12	50	13	14	13	9	14	31	15	5	25
26	13	53	13	49	14	13	14	9	15	32	16	6	26
27	14	51	14	47	15	12	15	9	16	33	17	7	27
28	15	49	15	45	16	12	16	10	17	34	18	8	28
29	16	46	16	44	17	11	17	10	18	35	19	9	29
30	17	44	17	42	18	11	18	11	19	36	20	10	30
31	18	41	18	41			19	11			21	12	31

Days	Jan. ♑		Febr. ♒		March ♓		April ♈		May ♉		June ♊		Days
	°	′	°	′	°	′	°	′	°	′	°	′	
1	22	13	23	40	21	45	22	21	21	27	21	10	1
2	23	14	24	41	22	45	23	19	22	25	22	7	2
3	24	15	25	41	23	45	24	18	23	23	23	4	3
4	25	16	26	42	24	44	25	17	24	20	24	1	4
5	26	17	27	42	25	44	26	15	25	18	24	59	5
6	27	18	28	43	26	44	27	14	26	16	25	56	6
7	28	19	29	43	27	43	28	12	27	14	26	53	7
8	29	20	♓	44	28	43	29	11	28	11	27	50	8
9	♒	21	1	44	29	42	♉	9	29	9	28	47	9
10	1	22	2	44	♈	42	1	7	♊	7	29	45	10
11	2	23	3	45	1	41	2	6	1	4	♋	42	11
12	3	24	4	45	2	40	3	4	2	2	1	39	12
13	4	25	5	45	3	40	4	2	2	59	2	36	13
14	5	26	6	46	4	39	5	1	3	57	3	33	14
15	6	27	7	46	5	38	5	59	4	55	4	31	15
16	7	28	8	46	6	37	6	57	5	52	5	28	16
17	8	29	9	46	7	37	7	55	6	49	6	25	17
18	9	30	10	46	8	36	8	54	7	47	7	22	18
19	10	31	11	46	9	35	9	52	8	44	8	19	19
20	11	32	12	46	10	34	10	50	9	42	9	17	20
21	12	33	13	46	11	33	11	48	10	39	10	14	21
22	13	33	14	46	12	32	12	46	11	36	11	11	22
23	14	34	15	46	13	31	13	44	12	34	12	8	23
24	15	35	16	46	14	30	14	42	13	31	13	5	24
25	16	36	17	46	15	29	15	40	14	28	14	2	25
26	17	36	18	46	16	28	16	38	15	26	15	0	26
27	18	37	19	46	17	27	17	36	16	23	15	57	27
28	19	38	20	46	18	26	18	34	17	20	16	54	28
29	20	39			19	24	19	32	18	18	17	51	29
30	21	39			20	23	20	29	19	15	18	48	30
31	22	40			21	22			20	12			31

Days	July ♋ °	′	August ♌ °	′	Septemb ♍ °	′	October ♎ °	′	Novemb ♏ °	′	Decemb ♐ °	′	Days
1	19	46	19	25	19	25	18	56	19	57	20	22	1
2	20	43	20	23	20	24	19	55	20	57	21	23	2
3	21	40	21	20	21	22	20	55	21	58	22	24	3
4	22	37	22	18	22	21	21	55	22	59	23	25	4
5	23	35	23	16	23	19	22	54	23	59	24	26	5
6	24	32	24	14	24	18	23	54	25	0	25	27	6
7	25	29	25	11	25	17	24	54	26	0	26	28	7
8	26	26	26	9	26	15	25	53	27	1	27	30	8
9	27	24	27	7	27	14	26	53	28	2	28	31	9
10	28	21	28	5	28	13	27	53	29	3	29	32	10
11	29	18	29	3	29	12	28	53	♐	3	♑	33	11
12	♌	16	♍R	1	♎	11	29	53	1	4	1	34	12
13	1	13	0	59	1	10	♏	53	2	5	2	36	13
14	2	10	1	57	2	9	1	53	3	6	3	37	14
15	3	8	2	55	3	8	2	53	4	6	4	38	15
16	4	5	3	53	4	7	3	53	5	7	5	38	16
17	5	2	4	51	5	6	4	53	6	8	6	40	17
18	6	0	5	49	6	5	5	53	7	9	7	41	18
19	6	57	6	47	7	4	6	53	8	10	8	43	19
20	7	55	7	45	8	3	7	53	9	11	9	44	20
21	8	52	8	43	9	2	8	53	10	12	10	45	21
22	9	50	9	42	10	1	9	53	11	13	11	46	22
23	10	47	10	40	11	1	10	54	12	14	12	47	23
24	11	44	11	38	12	0	11	54	13	15	13	48	24
25	12	42	12	36	12	59	12	54	14	16	14	50	25
26	13	40	13	35	13	59	13	54	15	17	15	51	26
27	14	37	14	33	14	58	14	55	16	18	16	52	27
28	15	35	15	31	15	57	15	55	17	19	17	53	28
29	16	32	16	30	16	57	16	56	18	20	18	54	29
30	17	30	17	28	17	56	17	56	19	21	19	56	30
31	18	27	18	27			18	56			20	57	31

Days	*Jan.* ♑		*Febr.* ♒		*March* ♓		*April* ♈		*May* ♉		*June* ♊		Days
	°	′	°	′	°	′	°	′	°	′	°	′	
1	21	57	23	26	21	31	22	6	21	13	20	56	1
2	22	59	24	26	22	31	23	5	22	11	21	53	2
3	24	0	25	27	23	30	24	4	23	9	22	50	3
4	25	1	26	27	24	30	25	2	24	7	23	47	4
5	26	2	27	28	25	30	26	1	25	4	24	45	5
6	27	3	28	29	26	29	26	59	26	2	25	42	6
7	28	4	29	29	27	29	27	58	27	0	26	39	7
8	29	5	♓	29	28	28	28	56	27	57	27	36	8
9	♒	6	1	30	29	28	29	55	28	55	28	34	9
10	1	7	2	30	♈	27	♉	53	29	53	29	31	10
11	2	9	3	30	1	27	1	52	♊	50	♋	28	11
12	3	10	4	30	2	26	2	50	1	48	1	25	12
13	4	11	5	31	3	25	3	48	2	45	2	22	13
14	5	12	6	31	4	25	4	47	3	43	3	20	14
15	6	12	7	31	5	24	5	45	4	40	4	17	15
16	7	13	8	31	6	23	6	43	5	38	5	14	16
17	8	14	9	31	7	22	7	41	6	35	6	11	17
18	9	15	10	32	8	22	8	40	7	33	7	8	18
19	10	16	11	32	9	21	9	38	8	30	8	6	19
20	11	17	12	32	10	20	10	36	9	28	9	3	20
21	12	18	13	32	11	19	11	34	10	25	10	0	21
22	13	19	14	32	12	18	12	32	11	22	10	57	22
23	14	20	15	32	13	17	13	30	12	19	11	54	23
24	15	20	16	32	14	19	14	28	13	17	12	51	24
25	16	21	17	32	15	15	15	26	14	15	13	49	25
26	17	22	18	31	16	14	16	24	15	12	14	46	26
27	18	22	19	31	17	13	17	22	16	9	15	43	27
28	19	23	20	31	18	11	18	20	17	7	16	40	28
29	20	24			19	10	19	18	18	4	17	37	29
30	21	24			20	9	20	15	19	1	18	35	30
31	22	25			21	8			19	59			31

Days	July ♋ ° ′	August ♌ ° ′	Septemb ♍ ° ′	October ♎ ° ′	Novemb ♏ ° ′	Decemb ♐ ° ′	Days
1	19 32	19 11	19 11	18 41	19 42	20 7	1
2	20 29	20 9	20 10	19 41	20 43	21 8	2
3	21 26	21 7	21 8	20 41	21 43	22 9	3
4	22 24	22 4	22 7	21 40	22 44	23 10	4
5	23 21	23 2	23 5	22 40	23 44	24 12	5
6	24 18	24 0	24 4	23 39	24 45	25 13	6
7	25 15	24 58	25 3	24 39	25 46	26 14	7
8	26 13	25 55	26 1	25 39	26 46	27 15	8
9	27 10	26 53	27 0	26 39	27 47	28 16	9
10	28 7	27 51	27 59	27 39	28 48	29 17	10
11	29 4	28 49	28 38	28 38	29 48	♑ 18	11
12	♌ 2	29 47	29 56	29 38	♐ 49	1 20	12
13	0 59	♍ 45	♎ 55	♏ 38	1 50	2 21	13
14	1 56	1 43	1 54	1 38	2 51	3 22	14
15	2 54	2 41	2 53	2 38	3 52	4 23	15
16	3 51	3 39	3 52	3 38	4 52	5 24	16
17	4 48	4 37	4 51	4 38	5 53	6 25	17
18	5 46	5 35	5 50	5 38	6 54	7 27	18
19	6 43	6 33	6 50	6 38	7 55	8 28	19
20	7 41	7 31	7 49	7 38	8 56	9 29	20
21	8 38	8 29	8 48	8 39	9 57	10 30	21
22	9 36	9 27	9 47	9 39	10 58	11 31	22
23	10 33	10 26	10 46	10 39	11 59	12 33	23
24	11 31	11 24	11 46	11 39	13 0	13 34	24
25	12 28	12 22	12 45	12 40	14 1	14 35	25
26	13 26	13 21	13 44	13 40	15 2	15 36	26
27	14 23	14 19	14 44	14 40	16 3	16 37	27
28	15 21	15 17	15 43	15 41	17 4	17 38	28
29	16 18	16 16	16 42	16 41	18 5	18 40	29
30	17 16	17 14	17 42	17 41	19 6	19 41	30
31	18 13	18 13		18 42		20 42	31

Days	*Jan.* ♑		*Febr.* ♒		*March* ♓		*April* ♈		*May* ♉		*June* ♊		Days
	°	′	°	′	°	′	°	′	°	′	°	′	
1	21	43	23	11	22	16	22	51	21	57	21	39	1
2	22	44	24	12	23	16	23	50	22	55	22	36	2
3	23	45	25	12	24	15	24	48	23	53	23	34	3
4	24	46	26	13	25	15	25	47	24	50	24	31	4
5	25	47	27	13	26	15	26	45	25	48	25	28	5
6	26	48	28	14	27	14	27	44	26	46	26	25	6
7	27	49	29	14	28	14	28	42	27	43	27	22	7
8	28	51	♓	14	29	13	29	41	28	41	28	20	8
9	29	52	1	15	♈	13	♉	39	29	39	29	17	9
10	♒	53	2	15	1	12	1	38	♊	36	♋	14	10
11	1	54	3	16	2	12	2	36	1	34	1	11	11
12	2	55	4	16	3	11	3	34	2	31	2	9	12
13	3	56	5	16	4	10	4	33	3	29	3	5	13
14	4	57	6	16	5	10	5	31	4	27	4	3	14
15	5	58	7	17	6	9	6	29	5	24	5	0	15
16	6	59	8	17	7	8	7	27	6	21	5	57	16
17	8	0	9	17	8	7	8	26	7	19	6	55	17
18	9	0	10	17	9	6	9	24	8	16	7	52	18
19	10	1	11	17	10	5	10	22	9	14	8	49	19
20	11	2	12	17	11	5	11	20	10	11	9	46	20
21	12	3	13	17	12	4	12	18	11	9	10	43	21
22	13	4	14	17	13	3	13	16	12	6	11	41	22
23	14	5	15	17	14	2	14	14	13	3	12	38	23
24	15	5	16	17	15	1	15	12	14	1	13	35	24
25	16	6	17	17	15	59	16	10	14	58	14	32	25
26	17	7	18	17	16	58	17	8	15	55	15	29	26
27	18	8	19	17	17	57	18	6	16	53	16	26	27
28	19	8	20	17	18	56	19	4	17	50	17	24	28
29	20	9	21	16	19	55	20	1	18	47	18	21	29
30	21	10			20	54	20	59	19	45	19	18	30
31	22	11			21	52			20	42			31

Days	July ♋ °	′	August ♌ °	′	September ♍ °	′	October ♎ °	′	November ♏ °	′	December ♐ °	′	Days
1	20	15	19	55	19	55	19	25	20	28	20	53	1
2	21	12	20	53	20	54	20	26	21	29	21	54	2
3	22	10	21	50	21	53	21	26	22	29	22	56	3
4	23	7	22	48	22	51	22	25	23	30	23	57	4
5	24	4	23	46	23	50	23	25	24	30	24	58	5
6	25	1	24	44	24	48	24	25	25	31	25	59	6
7	25	59	25	41	25	47	25	24	26	32	27	0	7
8	26	56	26	39	26	46	26	24	27	32	28	1	8
9	27	53	27	37	27	45	27	24	28	33	29	2	9
10	28	51	28	35	28	44	28	24	29	34	♈	4	10
11	29	48	29	31	29	42	29	24	♐	35	1	5	11
12	♌	45	♍	31	♎	41	♏	24	1	35	2	6	12
13	1	42	1	29	1	40	1	24	2	36	3	7	13
14	2	40	2	27	2	39	2	24	3	37	4	8	14
15	3	37	3	25	3	38	3	24	4	38	5	9	15
16	4	35	4	23	4	37	4	24	5	39	6	11	16
17	5	32	5	21	5	36	5	24	6	40	7	12	17
18	6	29	6	19	6	35	6	24	7	40	8	14	18
19	7	27	7	17	7	34	7	24	8	41	9	14	19
20	8	24	8	15	8	34	8	24	9	42	10	15	20
21	9	22	9	13	9	33	9	24	10	43	11	16	21
22	10	19	10	11	10	32	10	24	11	44	12	18	22
23	11	17	11	10	11	31	11	25	12	45	13	19	23
24	12	14	12	8	12	31	12	25	13	46	14	20	24
25	13	12	13	6	13	30	13	25	14	47	15	21	25
26	14	9	14	5	14	29	14	26	15	48	16	22	26
27	15	7	15	3	15	29	15	26	16	49	17	24	27
28	16	4	16	1	26	18	16	26	17	50	18	25	28
29	17	2	17	0	17	28	17	27	18	51	19	26	29
30	18	0	17	58	18	27	18	27	19	52	20	27	30
31	18	57	18	57			19	28			21	28	31

Days	*Jan.* S		*Febr.* S		*March* S *		*April* N		*May* N		*June* N		Days
	°	′	°	′	°	′	°	′	°	′	°	′	
1	21	36	13	34	3	10	8	48	18	13	23	12	1
2	21	26	13	14	2	47	9	9	18	28	23	16	2
3	21	15	12	54	2	24	9	31	18	44	23	19	3
4	21	4	12	33	2	0	9	53	18	57	23	22	4
5	20	53	12	12	1	36	10	14	19	11	23	24	5
6	20	41	11	51	1	12	10	35	19	24	23	26	6
7	20	29	11	30	0	48	10	56	19	38	23	27	7
8	20	16	11	9	0	25	11	17	19	41	23	28	8
9	20	3	10	47	0	1	11	38	20	3	23	29	9
10	19	50	10	25	N.	22	11	58	20	16	23	29	10
11	19	36	10	3	0	46	12	18	20	28	23	29	11
12	19	22	9	42	1	9	12	38	20	39	23	28	12
13	19	8	9	20	1	33	12	58	20	50	23	27	13
14	18	53	8	57	1	57	13	17	21	1	23	26	14
15	18	38	8	35	2	20	13	37	21	11	23	25	15
16	18	22	8	14	2	43	13	36	21	22	23	24	16
17	18	6	7	50	3	7	14	15	21	32	23	20	17
18	17	49	7	22	3	30	14	34	21	41	23	19	18
19	17	34	7	4	3	54	14	52	21	50	23	12	19
20	17	17	6	41	4	17	15	10	21	59	23	12	20
21	17	0	6	18	4	40	15	28	22	7	23	4	21
22	16	43	5	55	5	3	15	46	22	15	23	0	22
23	16	25	5	31	5	26	16	4	22	22	22	54	23
24	16	7	5	8	5	49	16	21	22	29	22	49	24
25	15	49	4	45	6	12	16	38	22	36	22	43	25
26	15	31	4	22	6	35	16	54	22	43	22	37	26
27	15	12	3	58	6	57	17	11	22	48	22	30	27
28	15	53	3	35	7	20	17	27	22	54	22	23	28
29	14	34			7	42	17	43	22	29	22	16	29
30	14	14			8	4	17	58	23	4	22	8	30
31	13	54			8	26			23	8			31

Days	July		August		Septemb		October		Novemb		Decemb		Days
	N		N		N *		S		S		S		
	°	′	°	′	°	′	°	′	°	′	°	′	
1	21	59	14	57	4	6	7	31	17	49	23	9	1
2	21	51	14	39	3	43	7	54	18	5	23	13	2
3	21	42	14	20	3	20	8	16	18	21	23	17	3
4	21	33	14	1	2	57	8	38	18	37	23	20	4
5	21	23	13	43	2	34	9	1	18	51	23	23	5
6	21	13	13	23	2	10	9	23	19	6	23	25	6
7	21	2	13	4	1	47	9	45	19	21	23	27	7
8	20	52	12	45	1	24	10	6	19	35	23	28	8
9	20	40	12	25	1	0	10	28	19	49	23	29	9
10	20	20	12	5	0	37	10	50	20	2	23	29	10
11	20	17	11	45	0	13	11	11	20	15	23	29	11
12	20	5	11	25	S	10	11	32	20	28	23	28	12
13	19	52	11	4	0	34	11	53	20	40	23	27	13
14	19	39	10	43	0	57	12	14	20	52	23	25	14
15	19	26	10	22	1	20	12	34	21	3	23	23	15
16	19	13	10	1	1	44	12	55	21	14	23	21	16
17	18	59	9	40	2	7	13	15	21	24	23	18	17
18	18	47	9	19	2	31	13	35	21	35	23	15	18
19	18	30	8	57	2	54	13	55	21	45	23	11	19
20	18	16	8	25	3	18	14	15	21	54	23	7	20
21	18	1	8	16	3	41	14	34	22	3	23	2	21
22	17	45	7	51	4	4	14	53	22	12	22	56	22
23	17	30	7	30	4	27	15	12	22	20	22	51	23
24	17	14	7	8	4	51	15	31	22	28	22	45	24
25	16	57	6	45	5	34	15	49	22	35	22	38	25
26	16	41	6	23	5	17	16	7	22	42	22	31	26
27	16	24	6	0	6	0	16	25	22	48	22	23	27
28	16	8	5	38	6	23	16	43	22	54	22	15	28
29	15	50	5	15	6	46	17	0	23	0	22	7	29
30	15	33	4	52	7	9	17	17	23	5	21	58	30
31	15	15	4	29			17	34			21	49	31

Days	*Jan.* S °	′	*Febr.* S °	′	*March* S * °	′	*April* N °	′	*May* N °	′	*June* N °	′	Days
1	21	39	13	39	3	17	8	43	18	10	23	11	1
2	21	29	13	20	2	53	9	4	18	25	23	15	2
3	21	18	12	59	2	29	9	26	18	39	23	18	3
4	21	7	12	38	2	6	9	48	18	53	23	21	4
5	20	56	12	17	1	42	10	9	19	7	23	23	5
6	20	44	11	57	1	18	10	30	19	21	23	25	6
7	20	32	11	35	0	55	10	51	19	35	23	27	7
8	20	19	11	14	0	31	11	12	19	48	23	28	8
9	20	6	10	53	C	7	11	33	20	1	23	29	9
10	19	53	10	31	N	16	11	53	20	13	23	29	10
11	19	40	10	9	0	40	12	13	20	25	23	29	11
12	19	26	9	47	1	4	12	33	20	37	23	28	12
13	19	11	9	25	1	27	12	53	20	48	23	27	13
14	18	56	9	3	1	51	13	13	20	59	23	26	14
15	18	41	8	40	2	14	13	32	21	10	23	24	15
16	18	26	8	17	2	38	13	51	21	20	23	22	16
17	18	10	7	55	3	2	14	10	21	30	23	19	17
18	17	54	7	32	3	25	14	29	21	39	23	16	18
19	17	38	7	10	3	48	14	48	21	48	23	13	19
20	17	21	6	47	4	12	15	6	21	57	23	9	20
21	17	4	6	24	4	35	15	24	22	5	23	5	21
22	16	47	6	0	4	58	15	42	22	13	22	1	22
23	16	30	5	37	5	21	15	59	22	21	22	56	23
24	16	12	5	14	5	44	16	17	22	28	22	51	24
25	15	53	5	51	6	6	16	34	22	35	22	45	25
26	15	34	4	27	6	29	16	51	22	41	22	38	26
27	15	16	4	4	6	52	17	7	22	47	22	32	27
28	14	57	3	40	7	14	17	23	22	53	22	25	28
29	14	38			7	37	17	39	22	58	22	18	29
30	14	19			7	59	17	55	23	3			30
31	13	59			8	21			23	7			31

Days	*July* N		*August* N		*Septemb* N*		*October* S		*Novemb* S		*Decemb* S		Days
	°	′	°	′	°	′	°	′	°	′	°	′	
1	22	1	15	1	4	12	7	26	17	46	23	8	1
2	21	52	14	43	3	49	7	48	18	2	23	12	2
3	21	44	14	25	3	26	8	11	18	17	23	16	3
4	21	35	14	6	3	3	8	33	18	33	23	19	4
5	21	25	13	47	2	40	8	55	18	48	23	22	5
6	21	15	13	28	2	16	9	17	19	3	23	24	6
7	21	5	13	9	1	53	9	39	19	18	23	26	7
8	20	54	12	50	1	30	10	1	19	32	23	28	8
9	20	43	12	30	1	7	10	23	19	45	23	29	9
10	20	32	12	10	0	43	10	44	19	59	23	29	10
11	20	20	11	50	0	19	11	6	20	12	23	29	11
12	20	8	12	30	S	4	11	27	20	25	23	29	12
13	20	56	11	9	0	28	11	48	20	37	23	28	13
14	19	43	10	49	0	51	12	9	20	49	23	26	14
15	19	30	10	27	1	15	12	30	20	1	23	24	15
16	19	16	10	6	1	38	12	50	21	12	23	22	16
17	19	2	9	45	2	1	13	10	21	22	23	19	17
18	18	48	9	24	2	25	13	30	21	32	23	16	18
19	18	34	9	2	2	49	13	50	21	42	23	12	19
20	18	19	8	41	3	12	14	10	21	52	23	8	20
21	18	4	8	19	3	35	14	29	22	1	23	3	21
22	17	49	7	57	3	59	14	48	22	10	22	58	22
23	17	34	7	35	4	22	15	7	22	18	22	52	23
24	17	18	7	13	4	45	15	26	22	26	22	46	24
25	17	2	6	51	5	8	15	44	22	33	22	40	25
26	16	46	6	28	5	31	16	2	22	40	22	33	26
27	16	29	6	6	5	54	16	20	22	47	22	25	27
28	16	12	5	43	6	17	16	38	22	53	22	17	28
29	15	55	5	20	6	40	16	55	22	58	22	9	29
30	15	37	4	58	7	3	17	12	22	3	22	0	30
31	15	19	4	35			17	29			21	51	31

Days	*Jan.* S		*Febr.* S		*March* S *		*April* N		*May* N		*June* N		Days
	°	′	°	′	°	′	°	′	°	′	°	′	
1	21	42	13	44	3	22	8	37	18	6	23	10	1
2	21	31	13	24	2	59	8	59	18	21	23	14	2
3	21	21	13	4	2	35	9	21	18	36	23	17	3
4	21	10	12	43	2	11	9	42	18	15	23	20	4
5	20	59	12	22	1	48	10	13	19	4	23	23	5
6	20	47	12	1	1	24	10	25	19	18	23	25	6
7	20	35	11	41	1	0	10	46	19	31	23	27	7
8	20	23	11	19	0	37	11	7	19	44	23	28	8
9	20	10	10	58	0	13	11	27	19	57	23	29	9
10	19	57	10	36	N	11	11	48	20	9	23	29	10
11	19	43	10	15	0	34	12	8	20	21	23	29	11
12	19	29	9	53	0	58	12	28	20	33	23	29	12
13	19	15	9	8	1	22	12	48	20	45	23	28	13
14	19	0	8	46	1	45	13	8	20	56	23	27	14
15	18	45	8	24	2	9	13	27	21	7	23	25	15
16	18	30	8	1	2	32	13	47	21	17	23	23	16
17	18	14	7	38	2	54	14	6	21	27	23	21	17
18	18	57	7	15	3	19	14	25	21	36	23	18	18
19	17	42	6	52	3	42	14	43	21	46	23	14	19
20	17	25	6	29	4	6	15	2	21	55	23	11	20
21	17	8	6	6	4	29	15	20	22	3	23	7	21
22	16	51	5	43	4	52	15	38	22	11	23	2	22
23	16	34	5	20	5	15	15	55	22	19	22	57	23
24	16	16	4	56	5	38	16	12	22	26	22	52	24
25	15	58	4	33	6	1	16	30	22	33	22	46	25
26	15	40	4	9	6	24	16	46	22	40	22	40	26
27	15	21	3	46	6	46	17	3	22	46	22	33	27
28	15	2			7	9	17	19	22	51	22	27	28
29	14	43			7	31	17	35	22	57	22	19	29
30	14	24			7	53	17	51	23	2	22	12	30
31	14	4			8	15			23	6			31

Days	July		August		Septemb		October		Novemb		Decemb		Days
	N		N		N*		S		S		S		
	°	′	°	′	°	′	°	′	°	′	°	′	
1	22	3	15	6	4	17	7	20	17	41	23	7	1
2	21	55	14	48	3	54	7	43	17	57	23	11	2
3	21	46	14	29	3	31	8	5	18	13	23	15	3
4	21	37	14	10	3	8	8	27	18	29	23	19	4
5	21	28	13	51	2	45	8	50	18	44	23	22	5
6	21	18	13	31	2	22	9	12	18	59	23	24	6
7	21	7	13	13	1	58	9	34	19	14	23	26	7
8	20	57	12	54	1	35	9	56	19	28	23	28	8
9	20	46	12	34	1	12	9	18	19	42	23	29	9
10	20	35	12	14	0	48	10	39	19	55	23	29	10
11	20	23	11	54	0	25	11	1	20	8	23	29	11
12	20	11	11	34	0	1	11	22	20	21	23	29	12
13	19	59	11	14	S	22	11	43	20	34	23	28	13
14	19	46	10	53	0	46	12	4	20	46	23	26	14
15	19	33	10	32	1	9	12	25	20	57	23	24	15
16	19	20	10	11	1	33	12	45	21	9	23	22	16
17	19	6	9	50	1	56	13	5	21	20	23	19	17
18	18	52	9	29	2	19	13	25	21	30	23	16	18
19	18	38	9	7	2	43	13	45	21	40	23	13	19
20	18	23	8	46	3	6	14	5	21	49	23	9	20
21	18	8	8	24	3	30	14	24	21	59	23	4	21
22	17	53	8	2	3	53	14	44	22	7	22	59	22
23	17	37	7	40	4	16	15	3	22	16	22	53	23
24	17	22	7	18	4	50	15	21	22	24	22	47	24
25	17	6	6	56	5	3	15	40	22	31	22	40	25
26	16	49	6	34	5	26	15	58	22	38	22	34	26
27	16	33	6	11	5	49	16	16	22	45	22	27	27
28	16	16	5	48	6	12	16	34	22	51	22	19	28
29	15	59	5	26	6	35	16	51	22	57	22	11	29
30	15	41	5	3	6	58	17	8	23	2	22	2	30
31	15	24	4	41			17	25			21	53	31

Days	Jan. S °	Jan. S ′	Febr. S °	Febr. S ′	March S * °	March S * ′	April N °	April N ′	May N °	May N ′	June N °	June N ′	Days
1	21	43	13	49	3	5	8	54	18	17	23	13	1
2	21	33	13	29	2	41	9	16	18	32	23	17	2
3	21	23	13	9	2	17	9	37	18	46	23	20	3
4	21	12	12	48	1	54	9	59	19	0	23	22	4
5	21	1	12	28	1	30	10	20	19	14	23	24	5
6	20	50	12	7	1	6	10	41	19	28	23	26	6
7	20	38	11	46	0	42	11	2	19	41	23	28	7
8	20	25	11	25	0	19	11	23	19	54	23	29	8
9	20	13	11	3	N	5	11	43	20	6	23	29	9
10	20	0	10	42	0	29	12	4	20	18	23	29	10
11	19	46	10	20	0	52	12	24	20	30	23	29	11
12	19	32	9	58	1	16	12	44	20	42	23	28	12
13	19	18	9	36	1	40	13	4	20	53	23	27	13
14	19	3	9	14	2	1	13	23	21	4	23	25	14
15	18	48	8	51	2	27	13	42	21	15	23	23	15
16	18	33	8	29	2	50	14	1	21	25	23	21	16
17	18	18	8	6	3	14	14	20	21	34	23	18	17
18	18	2	7	44	3	37	14	39	21	43	23	15	18
19	17	46	7	21	4	0	14	57	21	52	23	11	19
20	17	29	6	58	4	23	15	15	22	1	23	7	20
21	17	13	6	35	4	47	15	33	22	9	23	3	21
22	16	56	6	12	5	10	15	51	22	17	22	58	22
23	16	38	5	49	5	33	16	8	22	24	22	53	23
24	16	20	5	26	5	55	16	25	22	31	22	47	24
25	16	2	5	2	6	18	16	42	22	38	22	41	25
26	15	44	4	39	6	41	16	59	22	44	22	35	26
27	15	26	4	15	7	3	17	15	22	50	22	28	27
28	15	7	3	52	7	26	17	31	22	56	22	21	28
29	14	48	3	28	7	48	17	47	23	1	22	14	29
30	14	28			8	10	18	2	23	5	22	6	30
31	14	9			8	32			23	9			31

Days	*July* N °	′	*August* N °	′	*Septemb* N * °	′	*October* S °	′	*Novemb* S °	′	*Decemb* S °	′	Days
1	21	57	14	52	4	0	7	37	17	54	23	10	1
2	21	48	14	34	3	37	7	59	18	10	23	14	2
3	21	39	14	15	3	14	8	22	18	25	23	18	3
4	21	30	13	57	2	51	8	44	18	41	23	21	4
5	21	20	13	38	2	28	9	6	18	56	23	23	5
6	21	10	13	18	2	4	9	27	19	11	23	25	6
7	20	59	12	59	1	41	9	49	19	25	23	27	7
8	20	48	12	39	1	17	10	11	19	39	23	28	8
9	20	37	12	19	0	54	10	33	19	52	23	29	9
10	20	26	11	59	0	31	10	55	20	5	23	29	10
11	20	14	11	39	0	8	11	16	20	18	23	29	11
12	20	2	11	19	S	16	11	38	20	31	23	28	12
13	19	49	10	58	0	40	11	59	20	43	23	27	13
14	19	36	10	37	1	3	12	20	20	55	23	25	14
15	19	23	10	16	1	27	12	40	21	6	23	23	15
16	19	9	9	55	1	50	13	0	21	17	23	20	16
17	18	55	9	34	2	14	13	20	21	28	23	17	17
18	18	41	9	13	2	37	13	40	21	38	23	14	18
19	18	26	8	51	3	0	14	0	21	48	23	10	19
20	18	11	8	30	3	24	14	20	21	57	23	5	20
21	17	56	8	8	3	47	14	39	22	6	23	0	21
22	17	41	7	46	4	11	14	58	22	14	22	55	22
23	17	25	7	24	4	34	15	17	22	22	22	49	23
24	17	9	7	2	4	57	15	35	22	30	22	43	24
25	16	53	6	39	5	20	15	54	22	37	22	36	25
26	16	37	6	17	5	43	16	12	22	43	22	29	26
27	16	20	5	54	6	6	16	29	22	50	22	21	27
28	16	3	5	31	6	29	16	47	22	56	22	13	28
29	15	46	5	9	6	52	17	4	23	1	22	5	29
30	15	28	4	46	7	15	17	21	23	6	21	56	30
31	15	10	4	23			17	38			21	46	31

Days	*Jan.* h.	m.	*Febr.* h.	m.	*March* h.	m.	*April* h.	m.	*May* h.	m.	*June* h.	m.	Days
1	19	37	21	45	23	31	1	24	3	17	5	22	1
2	19	41	21	49	23	34	1	27	3	21	5	27	2
3	19	46	21	53	23	38	1	31	3	25	5	31	3
4	19	50	21	57	23	42	1	35	3	29	5	35	4
5	19	54	22	1	23	45	1	38	3	33	5	39	5
6	19	58	22	5	23	49	1	42	3	37	5	43	6
7	20	3	22	8	23	53	1	46	3	41	5	47	7
8	20	7	22	12	23	56	1	49	3	45	5	51	8
9	20	11	22	16	24	0	1	53	3	49	5	56	9
10	20	15	22	20	0	3	1	57	3	53	6	0	10
11	20	20	22	24	0	7	2	1	3	57	6	4	11
12	20	24	22	27	0	11	2	4	4	1	6	8	12
13	20	28	22	31	0	14	2	8	4	5	6	12	13
14	20	32	22	35	0	18	2	12	4	9	6	16	14
15	20	36	22	39	0	22	2	16	4	13	6	21	15
16	20	41	22	43	0	25	2	19	4	17	6	25	16
17	20	45	22	46	0	29	2	23	4	21	6	29	17
18	20	49	22	50	0	32	2	27	4	25	6	33	18
19	20	53	22	54	0	36	2	30	4	29	6	37	19
20	20	57	22	58	0	40	2	34	4	33	6	41	20
21	21	1	23	1	0	43	2	38	4	37	6	45	21
22	21	5	23	5	0	47	2	42	4	41	6	50	22
23	21	9	23	9	0	50	2	46	4	45	6	54	23
24	21	13	23	12	0	54	2	50	4	49	6	58	24
25	21	17	23	16	0	58	2	54	4	54	7	2	25
26	21	21	23	20	1	1	2	58	4	58	7	6	26
27	21	25	23	23	1	5	3	1	5	2	7	10	27
28	21	29	23	27	1	9	3	5	5	6	7	14	28
29	21	33			1	13	3	9	5	10	7	18	29
30	21	37			1	16	3	13	5	14	7	22	30
31	21	41			1	20			5	18			31

Days	July h.	July m.	August h.	August m.	Septemb h.	Septemb m.	October h.	October m.	Novemb h.	Novemb m.	Decemb h.	Decemb m.	Days
1	7	27	9	28	11	22	13	11	15	11	17	19	1
2	7	31	9	32	11	26	13	14	15	15	17	24	2
3	7	35	9	36	11	29	13	18	15	19	17	28	3
4	7	39	9	40	11	33	13	22	15	23	17	32	4
5	7	43	9	43	11	36	13	26	15	27	17	37	5
6	7	47	9	47	11	40	13	29	15	32	17	41	6
7	7	51	9	51	11	44	13	33	15	36	17	46	7
8	7	55	9	55	11	47	13	37	15	40	17	50	8
9	7	59	9	58	11	51	13	41	15	44	17	54	9
10	8	3	10	2	11	54	13	44	15	48	17	59	10
11	8	7	10	6	11	58	13	48	15	52	18	3	11
12	8	11	10	9	12	1	13	52	15	57	18	8	12
13	8	15	10	13	12	5	13	56	16	1	18	12	13
14	8	19	10	17	12	9	14	0	16	5	18	17	14
15	8	23	10	20	12	12	14	4	16	9	18	21	15
16	8	27	10	24	12	16	14	7	16	14	18	26	16
17	8	31	10	28	12	20	14	11	16	18	18	30	17
18	8	35	10	31	12	23	14	15	16	22	18	35	18
19	8	38	10	35	12	27	14	19	16	27	18	39	19
20	8	42	10	39	12	30	14	23	16	31	18	43	20
21	8	46	10	42	12	34	14	27	16	35	18	46	21
22	8	50	10	46	12	37	14	31	16	40	18	52	22
23	8	54	10	50	12	41	14	35	16	44	18	57	23
24	8	58	10	53	12	45	14	39	16	48	19	1	24
25	9	2	10	57	12	49	14	43	16	52	19	5	25
26	9	5	11	0	12	52	14	47	16	57	19	10	26
27	9	9	11	4	12	56	14	51	17	1	19	14	27
28	9	13	11	8	13	0	14	55	17	6	19	19	28
29	9	17	11	11	13	3	14	59	17	10	19	23	29
30	9	21	11	15	13	7	15	3	17	15	19	27	30
31	9	25	11	18			15	7			19	32	31

Days	*Jan.* h.	m.	*Febr.* h.	m.	*March* h.	m.	*April* h.	m.	*May* h.	m.	*June* h.	m.	Days
1	19	36	21	43	23	30	1	23	3	16	5	21	1
2	19	40	21	48	23	33	1	26	3	20	5	26	2
3	19	45	21	52	23	37	1	30	3	24	5	30	3
4	19	49	21	56	23	41	1	34	3	28	5	34	4
5	19	53	22	0	23	44	1	37	3	32	5	38	5
6	19	57	22	4	23	48	1	41	3	36	5	42	6
7	20	2	22	7	23	52	1	45	3	40	5	46	7
8	20	6	22	11	23	55	1	48	3	44	5	51	8
9	20	10	22	15	23	59	1	52	3	48	5	55	9
10	20	14	22	19	0	3	1	56	3	52	5	59	10
11	20	19	22	23	0	6	2	0	3	56	6	3	11
12	20	23	22	26	0	10	2	3	4	0	6	7	12
13	20	27	22	30	0	13	2	7	4	4	6	11	13
14	20	31	22	34	0	17	2	11	4	8	6	15	14
15	20	35	22	38	0	21	2	15	4	12	6	20	15
16	20	40	22	42	0	24	2	18	4	16	6	24	16
17	20	44	22	45	0	28	2	22	4	20	6	28	17
18	20	48	22	49	0	32	2	26	4	24	6	32	18
19	20	52	22	53	0	35	2	30	4	28	6	36	19
20	20	56	22	57	0	39	2	34	4	32	6	40	20
21	21	0	23	0	0	42	2	38	4	36	6	44	21
22	21	4	23	4	0	46	2	41	4	40	6	49	22
23	21	8	23	8	0	50	2	45	4	44	6	53	23
24	21	12	23	11	0	53	2	49	4	48	6	57	24
25	21	16	23	15	0	57	2	53	4	52	7	1	25
26	21	20	23	19	1	0	2	57	4	57	7	5	26
27	21	24	23	22	1	4	3	0	5	1	7	9	27
28	21	28	23	26	1	8	3	4	5	5	7	13	28
29	21	32			1	12	3	8	5	9	7	17	29
30	21	36			1	15	3	12	5	13	7	21	30
31	21	40			1	19			5	17			31

Days	*July* h.	m.	*Auguſt* h.	m.	*Septemb* h.	m.	*Octoter* h.	m.	*Novemb* h.	m	*Decemb* h	m.	Days
1	7	26	9	27	11	21	13	10	15	10	17	18	1
2	7	30	9	31	11	25	13	14	15	14	17	22	2
3	7	34	9	35	11	28	13	17	15	18	17	27	3
4	7	38	9	39	11	32	13	21	15	22	17	31	4
5	7	42	9	42	11	36	13	25	15	26	17	36	5
6	7	46	9	46	11	39	13	28	15	31	17	40	6
7	7	50	9	50	11	43	13	32	15	35	17	44	7
8	7	54	9	54	11	46	13	36	15	39	17	49	8
9	7	58	9	57	11	50	13	40	15	43	17	53	9
10	8	2	10	1	11	53	13	44	15	47	17	58	10
11	8	6	10	5	11	57	13	47	15	51	18	2	11
12	8	10	10	8	12	1	13	51	15	56	18	7	12
13	8	14	10	12	12	4	13	55	16	0	18	11	13
14	8	18	10	16	12	8	13	59	16	4	18	16	14
15	8	22	10	19	12	11	14	3	16	8	18	20	15
16	8	26	10	23	12	15	14	6	16	13	18	25	16
17	8	30	10	27	12	19	14	10	16	17	18	29	17
18	8	34	10	30	12	22	14	14	16	21	18	33	18
19	8	37	10	34	12	26	14	18	16	26	18	38	19
20	8	41	10	38	12	30	14	22	16	30	18	42	20
21	8	45	10	41	12	33	14	26	16	34	18	47	21
22	8	49	10	45	12	37	14	30	16	39	18	51	22
23	8	53	10	49	12	40	14	34	16	43	18	56	23
24	8	57	10	52	12	44	14	38	16	47	19	0	24
25	9	1	10	56	12	48	14	42	16	52	19	4	25
26	9	5	10	59	12	51	14	46	16	56	19	9	26
27	9	8	11	3	12	55	14	50	17	0	19	13	27
28	9	12	11	7	12	59	14	54	17	5	19	17	28
29	9	16	11	10	13	2	14	58	17	9	19	22	29
30	9	20	11	14	13	6	15	2	17	13	19	26	30
31	9	24	11	18			15	6			19	31	31

Days	Jan.		Febr.		March		April		May		June		Days
	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	
1	19	35	21	43	23	29	1	22	3	15	5	20	1
2	19	39	21	47	23	33	1	25	3	19	5	25	2
3	19	44	21	51	23	36	1	29	3	23	5	29	3
4	19	48	21	55	23	40	1	33	3	27	5	33	4
5	19	52	21	59	23	43	1	36	3	31	5	37	5
6	19	56	22	3	23	47	1	40	3	35	5	41	6
7	20	1	22	6	23	51	1	44	3	39	5	45	7
8	20	5	22	10	23	54	1	48	3	43	5	50	8
9	20	9	22	14	23	58	1	51	3	47	5	54	9
10	20	13	22	18	0	2	1	55	3	51	5	58	10
11	20	18	22	22	0	5	1	59	3	55	6	2	11
12	20	22	22	26	0	9	2	2	3	59	6	6	12
13	20	26	22	29	0	13	2	6	4	3	6	10	13
14	20	30	22	33	0	16	2	10	4	7	6	14	14
15	20	34	22	37	0	20	2	14	4	11	6	19	15
16	20	39	22	41	0	23	2	17	4	15	6	23	16
17	20	43	22	44	0	27	2	21	4	19	6	27	17
18	20	47	22	48	0	31	2	25	4	23	6	31	18
19	20	51	22	52	0	34	2	29	4	27	6	35	19
20	20	55	22	56	0	38	2	33	4	31	6	39	20
21	20	59	22	59	0	42	2	36	4	35	6	43	21
22	21	3	23	3	0	45	2	40	4	39	6	48	22
23	21	7	23	7	0	49	2	44	4	43	6	52	23
24	21	11	23	10	0	52	2	48	4	47	6	56	24
25	21	15	23	14	0	56	2	52	4	52	7	0	25
26	21	19	23	18	1	0	2	56	4	56	7	4	26
27	21	23	23	21	1	3	3	0	5	0	7	8	27
28	21	27	23	25	1	7	3	3	5	4	7	12	28
29	21	31			1	11	3	7	5	8	7	16	29
30	21	35			1	14	3	11	5	12	7	20	30
31	21	39			1	18			5	16			31

Days	*July* h.	m.	*Auguſt* h.	m.	*Septemb* h.	m.	*Octoter* h.	m.	*Novemb* h	m.	*Decemb* h.	m.	Days
1	7	25	9	26	11	20	13	9	15	9	17	17	1
2	7	29	9	30	11	24	13	13	15	13	17	21	2
3	7	33	9	34	11	27	13	16	15	17	17	26	3
4	7	37	9	38	11	31	13	20	15	21	17	30	4
5	7	41	9	42	11	35	13	24	15	25	17	35	5
6	7	45	9	46	11	38	13	28	15	29	17	39	6
7	7	49	9	49	11	42	13	31	15	34	17	44	7
8	7	53	9	53	11	45	13	35	15	38	17	48	8
9	7	57	9	56	11	49	13	39	15	42	17	52	9
10	8	1	10	0	11	53	13	43	15	46	17	57	10
11	8	5	10	4	11	56	13	46	15	50	18	1	11
12	8	9	10	8	12	0	13	50	15	55	18	6	12
13	8	13	10	11	12	3	13	54	15	59	18	10	13
14	8	17	10	15	12	7	13	58	16	3	18	15	14
15	8	21	10	19	12	11	14	2	16	7	18	19	15
16	8	25	10	22	12	14	14	6	16	12	18	24	16
17	8	29	10	26	12	18	14	9	16	16	18	28	17
18	8	33	10	30	12	21	14	13	16	20	18	32	18
19	8	36	10	33	12	25	14	17	16	25	18	37	19
20	8	40	10	37	12	29	14	21	16	29	18	41	20
21	8	44	10	41	12	32	14	25	16	33	18	40	21
22	8	48	10	44	12	36	14	29	16	37	18	50	22
23	8	52	10	48	12	40	14	33	16	42	18	54	23
24	8	56	10	51	12	43	14	37	16	46	18	59	24
25	9	0	10	55	12	47	14	41	16	51	19	3	25
26	9	4	10	59	12	51	14	45	16	55	19	8	26
27	9	7	11	2	12	54	14	49	16	59	19	12	27
28	9	11	11	6	12	58	14	53	17	4	19	16	28
29	9	15	11	9	13	2	14	57	17	8	19	21	29
30	9	19	11	13	13	5	15	1	17	13	19	25	30
31	9	23	11	17			15	5			19	30	31

Days	*Jan.* h.	m.	*Febr.* h.	m.	*March* h.	m.	*April* h.	m.	*May* h.	m.	*June* h.	m.	Days
1	19	34	21	42	23	32	1	25	3	18	5	24	1
2	19	38	21	46	23	35	1	28	3	22	5	28	2
3	19	42	21	50	23	39	1	32	3	26	5	32	3
4	19	47	21	54	23	43	1	36	3	30	5	36	4
5	19	51	21	58	23	46	1	39	3	34	5	40	5
6	19	55	22	2	23	50	1	43	3	38	5	44	6
7	20	0	22	5	23	54	1	47	3	42	5	48	7
8	20	4	22	9	23	57	1	50	3	46	5	53	8
9	20	8	22	13	0	0	1	54	3	50	5	57	9
10	20	12	22	17	0	4	1	58	3	54	6	1	10
11	20	17	22	21	0	8	2	1	3	58	6	5	11
12	20	21	22	25	0	12	2	5	4	2	6	9	12
13	20	25	22	28	0	15	2	9	4	6	6	13	13
14	20	29	22	32	0	19	2	13	4	10	6	18	14
15	20	33	22	36	0	23	2	17	4	14	6	22	15
16	20	38	22	40	0	26	2	20	4	18	6	26	16
17	20	42	22	43	0	30	2	24	4	22	6	30	17
18	20	46	22	47	0	33	2	28	4	26	6	34	18
19	20	50	22	51	0	37	2	32	4	30	6	38	19
20	20	54	22	55	0	41	2	36	4	34	6	42	20
21	20	58	22	58	0	44	2	39	4	38	6	47	21
22	21	2	23	2	0	48	2	43	4	42	6	51	22
23	21	6	23	6	0	52	2	47	4	46	6	55	23
24	21	10	23	10	0	55	2	51	4	51	6	59	24
25	21	14	23	13	0	59	2	55	4	55	7	3	25
26	21	18	23	17	1	3	2	59	4	59	7	7	26
27	21	22	23	21	1	6	3	2	5	3	7	11	27
28	21	26	23	24	1	10	3	6	5	7	7	15	28
29	21	30	23	28	1	14	3	10	5	11	7	19	29
30	21	34			1	17	3	14	5	15	7	24	30
31	21	38			1	21			5	19			31

Days	*July* h.	m.	*Auguſt* h.	m.	*Septemb* h.	m.	*October* h.	m.	*Novemb* h.	m.	*Decemb* h.	m.	Days
1	7	28	9	29	11	23	13	12	15	12	17	20	1
2	7	32	9	33	11	27	13	15	15	16	17	25	2
3	7	36	9	37	11	30	13	19	15	20	17	29	3
4	7	40	9	41	11	34	13	23	15	24	17	34	4
5	7	44	9	44	11	37	13	27	15	28	17	38	5
6	7	48	9	48	11	41	13	30	15	33	17	42	6
7	7	52	9	52	11	45	13	34	15	37	17	47	7
8	7	56	9	56	11	48	13	38	15	41	17	51	8
9	8	0	9	59	11	52	13	42	15	45	17	56	9
10	8	4	10	3	11	55	13	45	15	49	18	0	10
11	8	8	10	7	11	59	13	49	15	54	18	5	11
12	8	12	10	10	12	3	13	53	15	58	18	9	12
13	8	16	10	14	12	6	13	57	16	2	18	14	13
14	8	20	10	18	12	10	14	1	16	6	18	18	14
15	8	24	10	21	12	13	14	5	16	11	18	22	15
16	8	28	10	25	12	17	14	8	16	15	18	27	16
17	8	32	10	29	12	21	14	12	16	19	18	31	17
18	8	36	10	32	12	24	14	16	16	23	18	36	18
19	8	40	10	36	12	28	14	20	16	28	18	40	19
20	8	43	10	40	12	31	14	24	16	32	18	45	20
21	8	47	10	43	12	35	14	28	16	36	18	49	21
22	8	51	10	47	12	39	14	32	16	41	18	53	22
23	8	55	10	51	12	42	14	36	16	45	18	58	23
24	8	59	10	54	12	46	14	40	16	49	19	2	24
25	9	3	10	58	12	50	14	44	16	54	19	7	25
26	9	7	11	1	12	53	14	48	16	58	19	11	26
27	9	10	11	5	12	57	14	52	17	3	19	15	27
28	9	14	11	9	13	1	14	56	17	7	19	20	28
29	9	18	11	12	13	4	15	0	17	11	19	24	29
30	9	22	11	16	13	8	15	4	17	16	19	28	30
31	9	26	11	19			15	8			19	33	31

Variations of Longitude.						Variations of Declination.					
	4	8	12	16	20		4	8	12	16	20
M. D.	′	′	′	′	′	M. D.	′	′	′	′	′
Ja. 10	I. 3/4	3. 1/2	5. 1/4	7.	8. 3/4	Ja. 10	S. 2/6	S. 3/4	I.S 1/6	I.S.	I.S 1/6
20	I. 3/4	3. 1/2	5. 1/4	7.	8. 3/4	20	0. 1/2	0. 0/0	I. 2/5	I. 1/6	2. 2/5
30	I. 3/4	3. 1/2	5. 1/4	7.	8. 5/6	30	0. 1/2	I. 1/6	I. 2/3	2. 1/4	2. 1/6
Feb. 9	I. 3/4	3. 1/2	5. 1/3	7.	8. 5/6	Feb. 9	0. 2/3	I. 1/4	I. 5/6	2. 1/2	3. 1/6
19	I. 3/4	3. 1/2	5. 1/3	7.	8.	19	0. 2/3	I. 1/3	2. 0	2. 2/3	3. 2/5
29	I 3/4	3. 1/2	5. 1/3	7. 1/6	9.	29	0. 2/3	I. 1/2	2. 1/6	2. 1/6	3. 1/2
Ma. 10	I. 3/4	3. 1/2	5. 1/3	7. 1/4	9.	Ma. 10	4 2/3	I A 2/5	2 A 1/6	2 A 1/6	3 A
20	I. 5/6	3. 1/2	5. 1/3	7. 1/4	9.	20	0. 2/3	I. 2/3	2. 1/6	2. 5/6	3. 1/2
30	I. 5/6	3. 1/2	5. 1/3	7. 1/4	9.	30	0. 2/3	I. 1/3	2. 0	2. 3/4	3. 2/3
Apr. 9	I. 5/6	3. 1/2	5. 1/3	7. 1/4	9. 1/6	Apr. 9	0. 1/2	I. 1/4	I. 5/6	2. 1/3	3. 1/5
19	I. 5/6	3. 1/2	5. 1/3	7. 1/3	9. 1/6	19	0. 1/2	I. 1/6	I. 3/4	2. 0	2. 5/6
29	I. 5/6	3. 2/3	5. 1/3	7. 1/3	9. 1/4	29	0. 1/2	I 0/0	I. 1/2		2. 1/2
May 9	I. 5/6	3. 2/3	5. 1/3	7. 1/3	9. 1/4	May 9	0. 1/3	I. 4/5	I. 1/2	I. 2/3	2. 0
19	I. 5/6	3. 3/4	5. 1/3	7. 1/3	9. 1/4	19	0. 1/4	I. 1/2	0. 5/6	I. 1/6	I. 2/5
29	I. 5/6	3. 3/4	5. 1/3	7. 1/3	9. 1/3	29	0. 1/6	I. 1/3	0. 1/2	0. 3/5	0. 3/4
Ju. 8	I. 5/6	3. 3/4	5. 1/2	7. 1/2	9. 1/3	Jun. 8	0. 0	I. 0/0	0. 0	0. 1/0	0. 1/6
18	I. 5/6	3. 3/4	5. 1/2	7. 1/2	9. 1/2	18	S. 1/6	S. 1/3	0.S 3/4	0 S 2/5	0.S 3/5
28	I. 5/6	3. 3/4	5. 1/2	7. 1/2	9. 1/3	28	0. 1	I. 1/2	0. 3/4	I. 0/0	I. 1/6
July 8	I. 5/6	3. 3/4	5. 1/2	7. 1/2	9. 1/3	July 8	0. 1/3	I. 3/4	I. 0	I. 2/5	I. 4/5
18	I. 5/6	3. 1/4	5. 1/2	7. 2/3	9. 1/4	18	0. 1/2	I. 0/0	I. 2/5	I. 5/6	2. 1/3
28	I. 1/6	3. 2/3	5. 1/2	7. 1/3	9. 1/4	28	0. 1/2	I. 1/6	I. 2/3	2. 1/3	2. 3/4
Aug. 7	I. 5/6	3. 2/3	5. 1/2	7. 1/2	9. 1/3	Aug. 7	0. 6/5	I. 1/4	I. 5/6	2. 1/2	3. 1/6
17	I. 5/6	3. 2/3	5. 1/2	7. 1/3	9. 1/3	17	0. 2/3	I.	I. 0	2. 2/3	3. 1/3
27	I. 5/6	3. 2/3	5. 1/2	7. 1/2	9. 1/6	27	0. 1/2	I. 2/5	I. 1/6	2. 1/6	3 1/2
Sep. 6	I. 5/6	3. 2/3	5. 1/2	7. 1/4	9.	Sep. 6	0. 3/4	I. 2/5	2. 1/6	2. 1/6	3. 2/5
16	I. 5/6	3. 2/3	5. 1/2	7. 1/4	9.	16	A 3/4	I A 2/5	2 A 1/6	2 A 1/6	3 A 2/5
26	I. 4/5	3. 2/3	5. 1/3	7. 1/4	9.	26	0. 2/3	I. 2/5	2. 1/6	2. 4/5	3. 1/2
Oct. 6	I. 4/5	3. 1/2	5. 1/3	7. 1/6	9.	Octo. 6	0. 3/5	I. 1/3	2. 0	2. 3/5	3. 0/0
16	I. 3/4	3. 1/2	5. 1/3	7. 1/6	9.	16	0. 3/5	I.	I. 4/5	2. 1/6	3. 1/5
26	I. 3/4	3. 1/2	5. 1/3	7. 1/6	9. 5/6	26	0. 1/2	I.	1. 2/3		
No. 5	I. 3/4	3. 1/2	5. 1/3	7. 1/3	8. 5/6	Nov. 5	0 2/5	I. 5/6	I. 1/3	I. 3/4	2. 1/6
15	I. 3/4	3. 1/2	5. 1/3	7.	8. 5/6	15	0. 1/3	I. 2/3	I. 0	I. 1/2	I. 2/5
25	I. 3/4	3. 1/2	5. 1/4	7.	8. 4/5	25	0. 1/5	I. 2/5	0. 3/5	0. 4/5	I. 0
Dec. 5	I. 3/4	3. 1/2	5. 1/4	7.	8. 3/4	Dec. 5	0. 0	I. 1/5	0. 1/5	0. 1/4	0. 1/5
15	I. 3/4	3. 1/2	5. 1/4	7.	3. 3/4	15	S. 1/5	S. 1/5	0.S 1/5	0.S 1/4	0.S 1/5
25	I. 3/4	3. 1/2	5. 1/4	7.	3. 3/4	25	0. 1/4	I. 4/5	0. 3/5	0. 2/5	I. 0
31	I. 3/4	3. 1/2	5. 1/4	7.	3. 1/4	31	0.	I.	0. 1/6	0. 1/5	I. 1/5

A *Catalogue* of ſome eminent *Fixed Stars*, with their *Right Aſcenſions* and *Declinations*; deduced from the *Flamſteedian* OBSERVATIONS, and fitted to the Year 1738.

Stars Names.	Rt. Aſ. *h.*	*m.*	Declin. *d.*	*m.*	N. or S.
The Knot of the String of the Fiſhes —	1	48½	1	29	N
The Bright Star of Aries ——	1	52	22	12	N
The Jaw of the Whale Mandibula —	2	49	3	04	N
The Head of Meduſa ——	2	51	39	55	N
The Bright Side of Perſeus Algenib. —	3	06	48	54	N
The Goat Star Capella ——	4	57	45	43	N
The Bright Foot of Orion ——	5	02	8	32	S
The Left Shoulder of Orion ——	5	11	6	05	N
The Right Shoulder of Orion ——	5	41	7	20	N
The Dog-Star Syrius ——	6	34	16	21	S
The Little Dog-Star Procyon ——	7	26	5	53	N
Caſtor, *or the Head of the Northern Twins*	7	18	32	26	N
The Lyon's Tail ——	11	36	16	02	N
The Virgin's Spike ——	13	12	9	47	S
Arcturus ——	14	04	20	37	N
The Northern Scale of the Balance —	15	03	8	24	S
The Bright Star in the Neck of the Serpent	15	34	16	15	N
The Bright Star in the Northern Crown -	15	24	27	37	N
The Head of the Scorpion Antares	16	13	25	50	S
The Head of Hercules ——	17	03	14	42	N
The Head of Serpentarius ——	17	23	12	46	N
The Right Shoulder of Serpentarius —	17	31	4	42	N
The Bright Star in the Harp ——	18	28	38	33	N
The Bright Star in the Eagle ——	19	38	8	11	N
The Hand of Antinous ——	19	58	1	35	S
The Right Shoulder of Aquarius ——	21	52	1	35	S
The Shoulder of the Flying Horſe Marchab	22	52	13	47	N
In the Production of the Legs of the Flying Horſe Sheat ——	22	52	26	39	N
The Mouth of the Southern Fiſh Fomalhaut	22	43	31	30	S
The Head of Andromeda ——	23	55	27	38	N
The N. moſt of the 4 Stars called the Croſiers	12	17	55	29	S
The Southern Foot of the Croſs ——	12	12	61	29	S
The preceding Foot ——	12	02	57	08	S
The following Foot ——	12	33	58	05	S

SECT. III.

Containing Methods *for determining the Times of the* Riſing *and* Setting *of the* Stars, *their* Amplitude, Azimuths, Hour of the Night, *&c.*

AS the Right Aſcenſion of the Sun ſhews the Degree of the Equinoctial that paſſes over the Meridian with the Sun at Noon; ſo the Right Aſcenſion of the Star ſhews the Point of the Equinoctial that tranſits the Meridian with the Star.

Hence, the Arch of the Equinoctial intercepted between the Points of the Equinoctial, that tranſits over the Meridian with the Sun and any Star; or, *which is the ſame Thing*, the Difference of their Right Aſcenſions meaſures the Space of Time betwixt the Tranſit of the Sun and the ſame Star. Wherefore,

To find the Time of a Star's coming to the Meridian, ſubtract the Right Aſcenſion of the Sun from the Right Aſcenſion of the Star, and the Remainder ſhews the Time of the Star's coming to the Meridian. But if it happen that the Sun and Star are on contrary Sides of the firſt Point *Aries*, *that is*, If the Sun's Right Aſcenſion exceed the Star's, then to the Star's Right Aſcenſion, add 24 Hours.

Example the 1ſt.

If it were required to find at what Time *Arcturus* will come to the Meridian the *1ſt* Day of *April*, 1738.

	h.	*m.*
From the Right Aſcenſion of *Arcturus* —— ——	14	04
Take the Right Aſcenſion of the Sun at the Time propos'd	1	23
There remains the Hour of the Star's Southing ——	12	41

Example the 2d.

If it were required to find at what Time the ſame Star *Arcturus* will tranſit the Meridian on the *25th* of *January*, 1738.

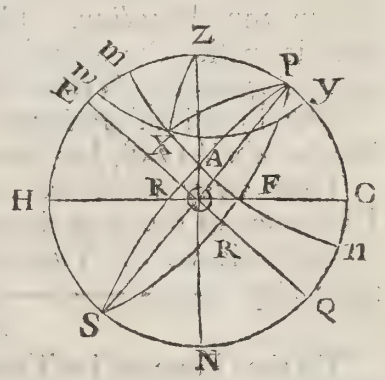
Becauſe the Sun's Right Aſcenſion at that Time is greater than the Star's,

To

	h.	*m.*
To the Right Aſcenſion of *Arcturus*	14	04
Add 24 Hours	24	00
From the Sum	38	04
Take the Sun's Right Aſcenſion at the Time propoſed	21	16
There remains the Time of the Star's coming to the Merid.	16	48

After this Manner may the Culmination of the Moon, or any other Planet be found.

Hence, and from the Latitude of the Place, and the Declination of the Star, may be found the Times of its Riſing and Setting, its Amplitude, the Time when it will appear due Eaſt and Weſt; and from the Star's Height, the Hour of the Night, and its Azimuth at that Time.



For if in the adjacent *Figure*, where P E S Q repreſents the Meridian, P S the Axis, E Q the Equator, Z N the Prime Vertical, H O the Horizon, and *m n* the Parallel of the Star's Diurnal Courſe. If through the Poles P and S, and F the Interſection of the Parallel *m n*, with the Horizon, an Arch of an Hour Circle be drawn, as P F S; in the Triangle C F R are given the Angle at C, the Complement of the Latitude of the Place, ſuppoſe 51° 32′ North, and F R the Star's Declination 20° 37′ in this *Caſe* North: Whence, to find C F its Amplitude, it will be,

S, ∠C : R :: S, R F : S, F C. That is,

As the Co-ſine of the Latitude of the Place 51° 32′	9.7938317
To the Radius	10.0000000
So is the Sine of the Star's Declination 20° 37′	9.5466832
To the Sine of the Amplitude 34° 28′ ½	9.7528515

Which

Which being always the ſame Way with the Declination, is therefore North: And hence, the Star Riſes Eaſt 34° 28′ Northerly, and ſets Weſt 34° 28′ ½ Northerly.

For C R, the *Aſcenſional Difference*, it will be,

t, < C : R :: t, F R : S, C R. That is,

As the Co-tangent of the Latitude of the Place 51° 32′	9.9000865
To the Radius	10.0000000
So is the Tangent of the Declination 20° 37′	9.5754272
To the Sine of the Aſcenſional Difference 28° 16′	9.6753407

Which converted into Time, and added to 6 Hours, becauſe the Latitude of the Place and Declination of the Star are both North, gives the Space of Time the Star requires to paſs from the *Horizon* to the *Meridian*. The Sum taken therefore from the Time that it paſſes the *Meridian*, ſhews the Time of its Riſing; but added, gives the Time of its Setting.

	h.	*m.*	″
The Tranſit of *Arcturus* over the *Meridian* on the 1*ſt* Day of *April* 1738, has been found already to be at	12	41	00
Take away therefore from thence, the Aſcenſional Difference of the Star, increaſed by 6 Hours	7	53	04
And it leaves the Time of the Riſing of *Arcturus* the ſame Day	4	47	56

Again,

	h.	*m.*	″
To the Time of its Tranſit over the Meridian	12	41	02
Add the ſame Sidereal Arch	7	53	04
And it gives the Time of its Setting	20	34	06

If it be required to find its *Height*, and the Time when it will be due Eaſt or Weſt, on the 1*ſt* Day of *April*, 1738.

In the former *Figure*, thro' the Interſection of the Parallel, with the Prime Vertical at A, draw another Hour Circle, as P A R S; then in the Right-angled Spherical Triangle C A R, are given the Angle at C, equal to the Latitude of the Place, and A R the Declination of the Star: whence to find the *Altitude* A C, it will be,

S, < C : R :: S, A R : S, A C. That is,

As

As the Sine of the Latitude of the Place 51° 32′ —— 9.8937452

To the Radius —— 10.0000000
So is the Sine of the Star's Declination 20° 37′ —— 9.5466832

To the Sine of its Height when due Eaſt or Weſt 26° 43′⅓ - 9.6529380

For the *Time*, it will be,

t, ∠C : R : : t, A R : S, R C. That is,

As the Tangent of the Latitude of the Place 51° 32′ - 10.0999135

To the Radius —— 10.0000000
So is the Tangent of the Star's Declination 20° 37′ — 9.5754272

To the Co-ſine of an Arch 72° 36′ ½ —— 9.4755137

Which converted into Time, and ſubtracted from the Time of the Star's coming to the Meridian, gives the Time it will appear due Eaſt; but the ſame Arch added to the Time of its paſſing over the Meridian, ſhews the Time it will be due Weſt.

Wherefore,

	h.	*m.*	″
If from the Time of its coming to the Meridian the 1ſt of *April* ——	12	41	00
Be taken away the Arch laſt found ——	4	50	26
It leaves the Time when it will appear due Eaſt ——	7	50	34

Again,

	h.	*m.*	″
If to the Time of its paſſing the Meridian ——	12	41	00
Be added the ſame Arch ——	4	50	26
It gives the Time that it will be due Weſt ——	17	31	26

If it were required to find the Hour of the Night, and Azimuth, from the Altitude of the Star 42° 00′, its Declination 20° 37′, and the Latitude of the Place 51° 32′. In the former *Figure*, draw *w x y*, parallel to the Horizon, at the Diſtance of 42° 00′: The Star's Altitude, through the Interſection of this Parallel, with the Parallel

m n at ✶, draw the Arch of the Hour Circle P ✶, also the Arch of the Vertical Circle Z ✶; then in the Spherical Triangle Z P ✶, are given Z P the Complement of the Latitude of the Place, Z ✶ the Complement of the Altitude, and P ✶ the Complement of the Declination of the Star in this Case North.

Whence, to find the *Azimuth*,

	°	′
To the Complement of the Latitude of the Place —	38	28
Add the Complement of the Altitude of the Star —	48	00
And the Complement of the Star's Declination —	69	23
The Sum makes —	155	51
From half this Sum —	77	55½
Take the Complement of the Declination —	69	23
The Remainder or Difference is —	8	32½

Then say,

As the Radius —	10.0000000
To the Co-sine of the Altitude of the Star 42° 00′ —	9.8710735
So is the Co-sine of the Latitude of the Place 51° 32′ -	9.7938317
To a 4th Sine —	9.6649052
As the 4th Sine —	9.6649052
To the Sine of the half Sum 77° 55½ —	9.9902832
So is the Sine of the Difference 8° 32½ —	9.1718099
To a 7th Sine —	9.4971879
To which if the Radius be added —	10.0000000

The Sine Complement of 9.7485939, half that Sum, 55° 54′½ doubled, gives 111° 49′, the Azimuth of *Arcturus* from the North.

* Or,

Or,

To the Arithm. Comp. of the Co-ſine of the Alt. 42° 00′ 0.1289265
Add the Arith. Comp. of the Co-ſine of the Lat. 51° 32′ - 0.2061683
Alſo the Sine of the half Sum 77° 55′ ½ —— —— 9.9902832
And the Sine of the Remainder 8° 32′ ½ —— —— —— 9.1718099

Half the Sum of theſe Four Sines —— —— —— 19.4971879

will give —— —— —— —— —— —— —— —— 9.7485939

The Co-ſine of 55° 54′ ½, which being doubled will give 111° 49′ for the Azimuth, the ſame as was before inveſtigated.

For the *Hour of the Night*, it will be,
S, P ϰ : S, < Z : : S, Z ϰ : S, < P. That is,

As the Co-ſine of the Declination 20° 37′ —— 9.9712560

To the Sine of the Azimuth 111° 49′ —— —— 9.9677247
So is the Co-ſine of the Altitude 42° 00′ —— —— 9.8710734

To the Sine of an Arch 47° 29′ —— —— —— 9.8675421

Which converted into Time, and ſubtracted from the Time of the Star's Southing, gives the Hour of the Night, if the Star were on the Eaſt Side of the Meridian; but added to the Time of the Southing, gives the Hour of the Night, when the Star is on the Weſt Side of, or is paſt the Meridian. Hence,

Becauſe on the 1*ſt* of *April*, 1738, *Arcturus* paſſes the Meridian at 12*h* 41′ 00″, if the Star were aſcending; *that is*, ſhort of the Meridian.

	h.	*m.*	″
From the Time of its Southing —— ——	12	41	00
Subtract the Time laſt found —— —— ——	3	9	56
And it gives the Hour of the Night —— —— ——	9	31	04

But if the Star be on the Weſt Side of the Meridian	*h.*	*m.*	″
To the Time of its Southing —— —— ——	12	41	00
Add the Time laſt found —— —— ——	3	9	56
And it gives the Hour of the Night —— —— ——	15	50	56

And indeed the Methods made Uſe of for giving *Solutions* to all the *Problems* ariſing from the *Diurnal Motion of the Sun*, may be applied to the Stars, if, inſtead of the Time the Sun paſſes the Meridian, the Commencement of the *Solar Day*, you make Uſe of the Time the Star paſſes the Meridian, the Commencement of the *Sidereal Day*.

SECT. IV.

Containing Methods *for finding the* Latitudes of Places, *or*, Heights of the Pole, *by the* Meridional Altitudes, *or* Zenith Diſtances *of the* Sun *or* Stars, *and their* Declinations.

THE Latitude of any Place (as hath been already ſhewn) is an Arch of the Meridian, contained between the Pole and the Horizon, and is ever equal to the Arch of the ſame Meridian, intercepted between the Equator and the Zenith.

Fig. the 1ſt.

For in the adjacent *Figure*, where P E S Q repreſents the Meridian, E Q the Equator, P the Northern Pole, H O the Horizon, and Z the Zenith: If from Z O the Diſtance of the Zenith from the Horizon, equal to P E the Diſtance of the Pole from the Equator, be taken away Z P the Diſtance of the Pole from the Zenith, there will remain Z E the Diſtance of the Zenith from the Equator, equal to P O the Diſtance of the Pole from the Horizon, equal to the Latitude of the Place.

As the Declination of the Sun or Star, is their neareſt Diſtance from the Equator, and is equal to the Arch of the Meridian, intercepted between the Sun or Star and the Equator; ſo the Meridional Zenith Diſtance, is the neareſt Approach of the Sun or Star to the Zenith; and is equal to the Arch of the Meridian, contained between the Sun or Star, and the Zenith of any Place, at the Time of their Tranſit over the Meridian.

*

Hence

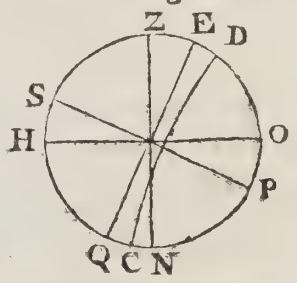
Hence we are taught how, from the Declination of the Sun or Stars, and their Meridional Zenith Diſtances, to find the Latitude of the Place of Obſervation.

1. If the Sun or Star have no Declination, the Meridional Zenith Diſtance is equal to the Latitude of the Place; which if the Sun come to the Meridian South of the Zenith, it is North; but if the Sun come to the Meridian on the North Side of the Zenith, it is South.

Thus, in *Fig. the 1ſt*, where E Q repreſents the Path of the Sun or Star, Z E the Meridional Zenith Diſtance, is equal to P O the Latitude of the Place, or Height of the Northern Pole.

Again,

Fig. the 2d.



In *Fig. the 2d*, Z E the Meridional Zenith Diſtance of the Sun or Star, is equal to S H the Height of the Southern Pole; ſince E, the Place of the Sun or Star, is on the North Side of the Zenith.

2. If the Sun or Star be in the Zenith, the Declination is equal to the Latitude, which is the ſame with the Declination; *that is*, if the Sun or Star have North Declination, the Latitude is North; if the Declination be South, the Latitude is South.

Thus in *Fig the 1ſt*, if Z repreſent the Place of the Sun or Star, then Z E the Declination is equal to PO the Altitude of the Pole, or Latitude of the Place; which is North, as the Declination E Z is North.

Likewiſe in *Fig. the 2d*, if Z repreſent the Place of the Sun or Star, then Z E the Declination South, is equal to S H, the Altitude of the South Pole above the Horizon.

3. If the Declination of the Sun or Star, and their Zenith Diſtances, be unequal, and both the ſame Way; *that is*, if they are either both North, or both South, then their Difference is equal to the Latitude of the Place.

Thus, in *Fig. the 1ſt*, if D C repreſent the Parallel of the Sun or Star's Declination, which is South, as is the Zenith Diſtance, then Z E equal to the Exceſs of Z D the Zenith Diſtance, above D E the Declination, is equal to the Height of the Pole above the Horizon.

Example the 1st.

Being at Sea the *9th* of *January*, 1738, when the Declination of the Sun is 20° 06′ South; I find the Meridional Zenith Distance to be 68° 34′ Southerly.

What is the Latitude of the Place, or Height of the Pole?

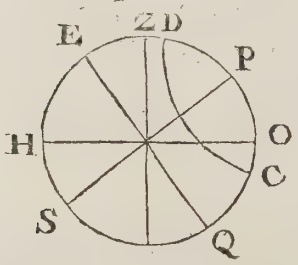
	°	′	
From the Zenith Distance Z D —— —— ——	68	34	South
Take the Declination E D —— —— —— ——	24	06	South
There remains the Latitude —— —— —— ——	48	28	North

Example the 2d.

Admit on the *25th* of *March*, when the Declination of the Sun is 6° 06′ North, I find the Meridional Zenith Distance to be 25° 33′ North.

	°	′	
From Z D, in *Fig. the 2d*, the Zenith Distance -	25	33	North
Take D E the Declination —— —— —— ——	6	06	North
There remains Z E equal to S H the Latitude —	19	27	South

Fig. the 3d.



But if it happen, as in *Fig. the 3d*, that the Declination be more than the Zenith Distance, then the Excess of the Declination above the Zenith Distance, is equal to the Latitude of the Place.

Example.

If the Declination be 22° 20′ North, and the Meridional Zenith Distance 14° 55′ North;

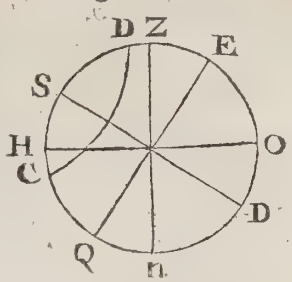
	°	′	
Then from the Declination —— —— —— ——	22	20	N.
Take the Meridional Zenith Distance —— —— ——	14	55	N.
There remains the Latitude of the Place —— —— ——	7	25	N.

In

In like Manner, as in *Fig. the 4th*, if the Declination D E had been 22° 20′ South, and the Meridional Zenith Distance E D had been 14° 55′ South; Then

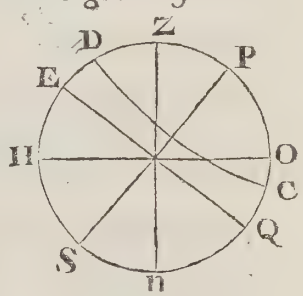
	°	′	
From the Declination ——	22	20	S.
Take the Merid. Zenith Distance -	14	45	S.
There remains the Latitude ——	7	25	S.

Fig. the 4th.



4 If the Declination of the Sun or Stars, and their Zenith Distances be unequal and contrary; *that is*, if one be North and the other South, then their Sum is the Latitude of the Place.

Fig. the 5th.



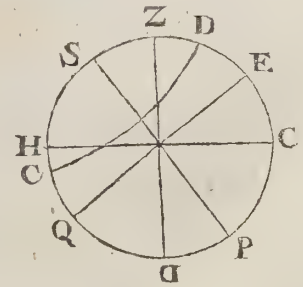
Thus, in *Fig. the 5th*, if to Z D the Zenith Distance, be added D E the Declination, the Sum Z E is equal to the Height of the Pole above the Horizon.

Suppose in *Fig. the 5th*, D E the Declination, to be 15° 27′ North, and Z D the Zenith Distance, to be 32° 20′ South.

To E D the Declination ——	15° 37′	North
Add Z D the Meridional Zenith Distance ——	32 20	South
The Sum is the Latitude ——	47 57	North

Fig. the 6th.

Suppose in *Fig. the 6th*, D E the Declination be 20° 10′ South, and Z D the Zenith Distance, be 27° 20′ North.



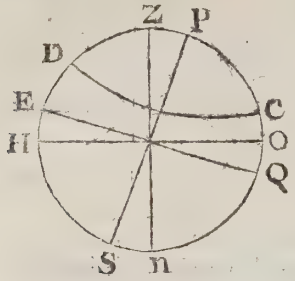
Then

To Z D the Zenith Distance —	27°	20′	N.
Add D E the Declination ——	20	10	S.
The Sum is the Lat. of the Place	47	30	S.

If in either of these last *Examples* it happen, that the Latitude of the Place be greater than the Complement of the Declination, then the Sun or Star will not set, but will have two Meridional Zenith Distances, the one above the Pole, the other below.

If

Fig. the 7th.



If in this *Case* you happen to take the Zenith Distance, when the Sun or Star is below the Pole; to C P the Complement of the Declination, add C O, the Meridional Altitude; the Sum is P O, the Height of the Pole.

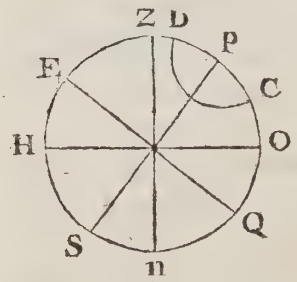
Example.

Suppose the Declination 22° 10′ North, and the Meridional Altitude be 7° 22′.

To the Complement of the Declination ——	67° 50′	North
Add the Meridional Altitude ——————	7 22	North
The Sum is the Latitude of the Place ——	75 12	North

But if the Meridional Altitudes above and below the Pole can be obtained, then half their Sum will be the Latitude of the Place, or Height of the Pole.

Fig. the 8th.



Thus, in *Fig. the 8th*, if C O the least Meridional Altitude of the Star, be 22° 10′ North, and the greatest D O 78° 30′ North.

To DO the greatest Merid. Altitude	78° 30′	
Add CO the least Merid. Altitude-	22 10	
Half that Sum ——————	100 40	
will be the Latitude——————	50 20	N.

If the Zenith Distance be greater than the Complement of the Declination, or, which comes to the same, if the Sun or Star transit the Meridian on contrary Sides of the Zenith, as in *Fig. the 9th*,

Fig. the 9th.

Then

To the least Merid. Altit. CO —	7° 22′	N.
Add the Complement of the other to 180° 00′ DO ——	143 02	
Half that Sum ——————	150 24	
will be the Latitude ——————	75 12	N.

In

In the firſt of theſe two laſt *Examples* (as in *Fig. the 8th.*) where the Sun or Star tranſits the Meridian on the ſame Side of the Zenith;

If from D E the Declination —— —— ——	61°	50′	North
Be ſubtracted Z D, the neareſt Approach of the Sun or Star to the Zenith; or, which is the ſame Thing, the leaſt Zenith Diſtance —— —— ——	11	30	North
There will remain the Latitude of the Place —	50	20	North

Likewiſe in the ſecond *Example* (as in *Fig. the 9th*) where the Sun or Star paſſes the Meridian on contrary Sides of the Zenith;

If to Z D, the leaſt Zenith Diſtance, or neareſt Approach to the Zenith —— —— —— ——	53°	02′	South
Be added D E the Declination —— ——	22	10	North
The Sum is the Latitude of the Place, as before -	75	12	North

Whence it is abundantly manifeſt, that if the Zenith Diſtance and Declination are both North, or both South, their Difference will give the Latitude; but if one be North and the other South, their Sum will give the Latitude: And to find whether it be North or South, take this *General Rule.*

Set off the Zenith Diſtance from the Zenith, if it be North, to the Right Hand of the Zenith, but if it be South, to the Left Hand, and it gives the Place of the Sun; from which Point ſet off the Declination to the Left Hand, if it be North, but to the Right Hand if it be South, and it will give the Interſection of the Equator with the Meridian: From this Interſection, ſet off 90 Degrees, or a Quadrant in the Meridian, towards the Right Hand, and it will give the Place of the North Pole. Now, if the North Pole be above the Horizon, the Latitude is North, but if it fall below the Horizon, then the South Pole will be elevated, and the Latitude will be South.

As this *Rule* is general, and will ſerve in all *Caſes* whatſoever, it ought to be diligently applied to, and throughly underſtood.

In

In all these Operations, the Zenith Distance, and Altitude of the Sun or Star, has been supposed Pure; that is, purged by Refraction: And therefore, before the Altitude or Zenith Distance Observ'd by the Quadrant be applied, according to the former Instructions, they must be all Corrected by Refraction, according to the following TABLE, which shews the Refractions to every Degree of the Quadrant, or how much must be added to the Zenith Distance; and subtracted from the Altitude, Observ'd by the Instrument, to obtain the true Zenith Distance or Altitude.

A Table *of* Refractions, *shewing how much must be added to the* Zenith Distance *Observ'd, or taken from the* Altitude, *in order to obtain the* True Zenith Distance, *or* True Altitude.

Dist. Vert.	Refract.	Dist. Vert.	Refract.	Dist. Vert.	Refract.	Dist. Vert.	Refract.	Dist. Vert.	Refract.
D.	M.	D.	M.	D.	M.	D.	M.	D.	M.
90	33 $\frac{3}{4}$	79	4 $\frac{2}{5}$	68	2 $\frac{1}{6}$	57	1 $\frac{1}{3}$	46	1 0
89	23 $\frac{1}{1}\frac{1}{6}$	78	4 0	67	2 $\frac{1}{12}$	56	1 $\frac{1}{3}$	45	0 $\frac{6}{7}$
88	17 $\frac{1}{8}$	77	3 $\frac{3}{4}$	66	2 0	55	1 $\frac{1}{4}$	40	0 $\frac{3}{4}$
87	13 $\frac{1}{3}$	76	3 $\frac{1}{2}$	65	1 $\frac{6}{7}$	54	1 $\frac{1}{5}$	35	0 $\frac{2}{3}$
86	10 $\frac{4}{5}$	75	3 $\frac{1}{4}$	64	1 $\frac{5}{6}$	53	1 $\frac{1}{6}$	30	0 $\frac{1}{2}$
85	9 ☉	74	3 0	63	1 $\frac{3}{4}$	52	1 $\frac{1}{7}$	25	0 $\frac{2}{5}$
84	7 $\frac{3}{4}$	73	2 $\frac{5}{6}$	62	1 $\frac{2}{3}$	51	1 $\frac{1}{10}$	20	0 $\frac{1}{3}$
83	6 $\frac{3}{4}$	72	2 $\frac{3}{4}$	61	1 $\frac{3}{5}$	50	1 $\frac{1}{11}$	15	0 $\frac{1}{4}$
82	6 0	71	2 $\frac{1}{2}$	60	1 $\frac{1}{2}$	49	1 0	10	0 $\frac{1}{6}$
81	5 $\frac{3}{6}$	70	2 $\frac{2}{5}$	59	1 $\frac{1}{2}$	48	1 0	5	0 $\frac{1}{12}$
80	4 $\frac{2}{5}$	69	2 $\frac{1}{3}$	58	1 $\frac{2}{5}$	47	1 0	0	
79	4 $\frac{2}{5}$	68	2 $\frac{1}{6}$	57	1 $\frac{1}{2}$	46	1 0		

SECT.

SECT. V.

Containing Methods *for finding the* Variation.

AS nothing is of greater Advantage to the Induſtrious Seaman, than a perfect Knowledge of the true Courſe upon which the Ship has, or does Sail; ſo nothing contributes more to it, than a frequent and diligent Obſervation of the *Amplitudes* and *Azimuths* of the *Sun* or *Stars*; ſince by the Help of this he will be enabled, not only to diſcover the Errors or Faults of his *Compaſs*, if any there be, but by comparing the Obſervations with the true Amplitudes or Azimuths, find how much his Needle points to the Eaſtward or Weſtward of the true Meridian, or the Angle that it forms with it.

For,

As the true Amplitude ſhews how much the Sun or Star riſes or ſets to the Northward or Southward of the Eaſt or Weſt Points, number'd in the Arch of the Horizon, and the Azimuth the Angle, the Vertical Circle paſſing through the Sun or Star at any Time, makes with the Meridian, which is meaſured by the Arch of the Horizon, contained between the Meridian and the Vertical Circle: So the Amplitude obſerved by the Compaſs ſhews how much the Sun or Star riſes or ſets to the Northward or Southward of the Eaſt and Weſt Points, according to the Compaſs; and the Azimuth, the Arch of the Horizon, contained between the Meridian pointed out by the Needle and the Vertical Circle; and the Difference between theſe, when compar'd together (if any there be) is what is call'd the *Variation*.

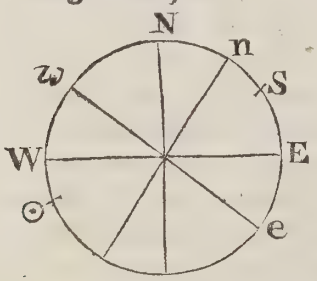
So that the *Variation* is an Arch of the Horizon, contained between the Interſection of the Meridian with the Horizon, or the true North and South Points; and the North or South Points pointed out by the Needle, which is ever equal to the Difference between the true Amplitudes and Azimuths, deduced from Calculation; and the Amplitudes and Azimuths obtained by Obſervation with the Needle.

How the true Amplitude and Azimuth of the Sun or Stars, at any Time, from proper *Data*, may be obtain'd by *Calculation*, has been ſhewn at large in *Sections the 2d and 3d of this Part.* And how the Magnetick Amplitude and Azimuth, or Amplitude and Azimuth obſerv'd by the Compaſs, may be found, is ſo very eaſy in itſelf, and ſo generally known to our Sailors, that it would be ſuperfluous to treat upon it here; and therefore I ſhall haſten to ſhew how, when both are known, to diſcover the true *Variation*; or, how much the Needle points to the Eaſtward or Weſtward of the true North or South Points of the Horizon.

1. If in comparing the true Amplitude or Azimuth, with the Magnetical Amplitude or Azimuth, they be found to agree, then the Needle points exactly true, and there is no *Variation.*

2. If in comparing the Amplitudes together, *viz.* the Magnetical and True, they are found to differ, but are the ſame Way; *that is*, both North or both South, then their Difference is the *Variation*: Which, if it be at Sun-riſing, and the Amplitudes be both North, and the Magnetical exceed the True, it is Eaſt; but if they are both South, and the Magnetical exceed the True, it is Weſt.

Fig. the 1ſt.



Thus, in *Fig. the 1ſt*, if S be the Place of the Sun, S *e* the Magnetical Amplitude, and S E the True; it is manifeſt, that the true Eaſt Point is to the Northward of the Magnetical; and conſequently the North Point pointed out by the Needle, is to the Eaſtward of the true North Point, and conſequently the *Variation* is Eaſterly.

Example.

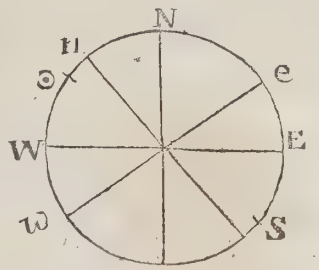
If S *e* the Magnetical Amplitude be 30° 44′ North, and S E the true Amplitude be 22° 15′ North;

From the Magnetical Amplitude —— ——	30° 44′ North
Take the true Amplitude —— —— —— ——	22 15 North
There remains the true Amplitude —— —— ——	8 29 Eaſt.

But

But if, as in *Fig. the 2d*, *e* S be the Magnetic, and E S the true Amplitude Southerly; it is manifeſt, that the Eaſt Point of the Horizon, according to the Compaſs, is to the Northward of the true Eaſt Point of the Horizon; and conſequently the *Variation* is Weſt.

Fig. the 2d.



Example.

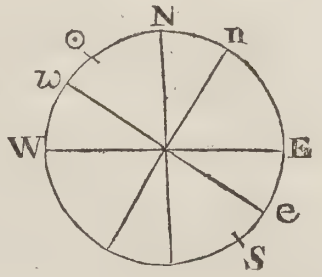
Admit the true Amplitude at Sun-riſe, *viz.* E S were 19° 55′ South, and the Magnetical *e* S were 29° 50′ South.

From the Magnetical Amplitude —— ——	29° 50′ South
Take the true Amplitude —— —— ——	19 55 South
There remains the true *Variation* —— ——	9 55 Weſt

3. If in comparing the Magnetical and true Amplitudes together at Sun-riſing, they are both found to be South, and the true Amplitude exceed the Magnetical, then is the *Variation* Eaſt; but if they are both North, then is the *Variation* Weſt.

Fig. the 3d

Thus, in *Fig. the 3d*, if S E be the true Amplitude Southerly, and S *e* the Magnetical; it is plain, that the true Eaſt Point is to the Northward of the Magnetical; and conſequently the *Variation* is Eaſterly.

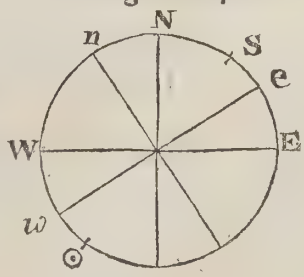


Wherefore,

If from the true Amplitude, ſuppoſe —— ——	24° 15′ South
Be taken the Magnetical, ſuppoſe —— ——	10 00 South
There will remain the *Variation* —— ——	14 15 Eaſt.

Fig. the 4th.

If, as in *Fig. the 4th*, S E be the true Amplitude, and S *e* the Magnetical; it is plain in this *Caſe*, that the *Variation* muſt be Weſt, ſince the Eaſt Point of the Compaſs points to the Northward of the true Eaſt Point.



Wherefore,

If from the true Amplitude, ſuppoſe —— —— 17° 30′ North
Be taken the Magnetical Amplitude, ſuppoſe — 12 10 North

There will remain the true *Variation* —— —— 5 20 Weſt.

4. If in comparing the Amplitudes at Setting, the Magnetical be found to exceed the True, if they are both South, the *Variation* is Eaſt; but if they are both North, the *Variation* is Weſt.

Thus, in *Fig. the 1ſt*, if ⊙ repreſents the Place of the Sun at Setting, ⊙ *w* the Magnetical Amplitude, and ⊙ W the True; it is manifeſt, that in this *Caſe* the *Variation* muſt be Eaſt, ſince the true Weſt Point is to the Southward of the Magnetical.

Wherefore,

If from the Magnetical Amplitude, ſuppoſe — 34° 15′ South
Be taken the true Amplitude, ſuppoſe —— 17 15 South

There will remain the true *Variation* —— 17 00 Eaſt.

If, as in *Fig. the 2d*, the Amplitudes at Setting are both North, it is manifeſt that the *Variation* is Weſt; ſince the true Weſt Point is to the Northward of the Magnetical.

Wherefore,

If from the Magnetical Amplitude, ſuppoſe —— 22° 18′ North
Be taken the true Amplitude, ſuppoſe —— 18 10 North

There will remain the true *Variation* —— —— 4 08 Weſt.

5. If in comparing the Amplitudes together at Sun-ſetting, the True be found to exceed the Magnetical, if they are both South, the *Variation* is Weſt; but if they are both North, the *Variation* is Eaſt.

Thus in *Fig. the 3d*, where ⊙ is the Sun, ⊙ W the true Amplitude, and ⊙ *w* the Magnetical; it is plain, that ⊙ is nearer to the true North Point than to the Magnetical; and conſequently the *Variation* is Eaſt.

Wherefore in this *Caſe*,

If from the true Amplitude, ſuppoſe —— 22° 10′ North
Be taken the Magnetical, ſuppoſe —— —— 14 30 North

There will remain the true *Variation*. 7 40 Eaſt.

If, as in *Fig. the 4th*, where ⊙ W the true Amplitude Southerly, exceeds ⊙ *w* the Magnetical Amplitude; it is manifeſt, that the Magnetic Meridian is to the Weſtward of the True; and conſequently the *Variation* is Weſt.

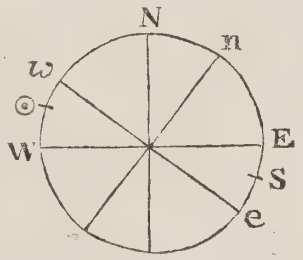
Wherefore in this *Caſe*,

If from the true Amplitude, ſuppoſe ——	17° 30′	South
Be taken the Magnetic Amplitude, ſuppoſe —	12 10	South
There will remain the true *Variation* ——	5 20	Weſt.

6. If in comparing the Amplitudes together, they be found to be contrary; *that is*, If one be North, and the other South, then their Sum is the *Variation*; which, if it be at Sun-riſing, and the Magnetical Amplitude be North, is Eaſt; otherwiſe Weſt.

Thus in *Fig. the 5th*, where the Magnetic Amplitude *S e* is North, and the true Amplitude E S is South; it is plain E *e* equal to N *n*, the Sum of the Amplitudes, is the *Variation* Eaſterly.

Fig. the 5th.



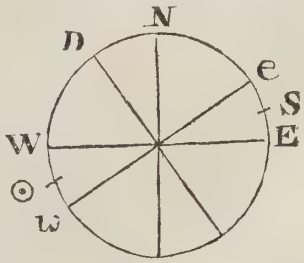
Wherefore,

If the Magnetical Amplitude *e* S be 15° 37′ North, and the true S E, 19° 42′ South;

To the Magnetical Amplitude —— ——	15° 37′	North
Add the True Amplitude —— ——	19 42	South
The Sum is the true *Variation* —— ——	35 19	Eaſt.

If, as in *Fig. the 6th*, the true Amplitude at Sun-riſing S E be North, and the Magnetical *e* S be South; their Sum E *e* is the *Variation* Weſt; ſince the Eaſt Point *e*, pointed out by the Needle, is to the Northward of the true Eaſt Point of the Horizon.

Fig. the 6th.



Wherefore,

If to the Magnetical Amplitude *e* S, ſuppoſe —	22° 30′	South
Be added the true Amplitude E S, ſuppoſe ——	12 10	North
The Sum is the *Variation* —— —— ——	34 40	Weſt.

7. If

7. If in comparing the Amplitudes at Setting together, they be found to be contrary; if the Magnetical be South, the *Variation* is East; if North, the *Variation* is West.

Thus in *Fig. the 5th*, where ☉ *w*, the Magnetic Amplitude is South, and ☉ W the True, is North;

If to ☉ *w* the Magnetic Amplitude, suppose — 15° 37′ South
Be added ☉ W the true Amplitude, suppose — 19 42 North

The Sum W *w* is the true *Variation* —— —— 35 19 East; since the true West Point is to the Southward of the Magnetical.

In *Fig the 6th*, where ☉ *w* the Magnetic Amplitude is North, and the True W ☉ South, it is plain, that their Sum W *w* is equal to N *n*, and the *Variation* West.

Wherefore,

If to ☉ *w* the Magnetical Amplitude, suppose — 22° 30′ North
Be added ☉ W the true Amplitude, suppose — 12 10 South

The Sum is the true *Variation* ———— 34 40 West.

8. If in comparing the Azimuths taken at any Time, *viz.* the Magnetical and True, they are found to disagree, their Difference is the *Variation*; which, if it happens to be in the Forenoon, and the true Azimuth, number'd from the North, exceed the Magnetical counted from the same Point, is East; otherwise West.

Thus in *Fig. the 1st*, *3d*, and *5th*, where S N the true Azimuth, exceeds S *n* the Magnetical; it is manifest, that the true North Point is to the Westward of the Magnetical, and consequently the *Variation* is East.

Wherefore in this *Case*,

If from the true Azimuth, suppose ———— 98° 20′
Be taken the Magnetical, suppose —— —— 76 15

There will remain the true *Variation* —— —— 22 05 East.

In *Fig. the 2d*, *4th* and *6th*, where S *n* the Magnetical Azimuth exceeds S N the True; it is plain, that the true North Point is to the Eastward of the Magnetical, and consequently the *Variation* is West.

Wherefore,

Wherefore,

If from the Magnetical Azimuth, ſuppoſe —— 97° 35′
Be taken the true Azimuth, ſuppoſe —— 92 20

There will remain the true *Variation* —— —— 5 15 Weſt.

9. In comparing together the Azimuths in the Afternoon, number'd from the North, if the Magnetical exceed the True, their Difference is the *Variation* Eaſt; but if the True exceed the Magnetical, their Difference is the *Variation* Weſt.

Thus in *Fig. the 1ſt, 3d* and *5th*, where ⊙ *n* the Magnetical Azimuth, exceeds ⊙ N the True; it is manifeſt, that the true North Point muſt be to the Weſtward of the Magnetic, and conſequently the *Variation* is Eaſt.

Wherefore, in this *Caſe*,

If from the Magnetic Azimuth, ſuppoſe —— 106° 54′
Be taken the true Azimuth, ſuppoſe —— 102 13

There will remain the true *Variation* —— —— 4 41 Eaſt.

If, as in *Fig.* 2, 4 and 6, the true Azimuth from the North in the Afternoon, ⊙ N exceed the Magnetical Azimuth ⊙ *n*; it is abundantly manifeſt, that the true North Point N, is to the Eaſtward of the Magnetical, and conſequently the *Variation* is Weſt.

Wherefore in this *Caſe*,

If from the true Azimuth, ſuppoſe —— 127° 44′
Be taken the Magnetical, ſuppoſe —— 112 30

There will remain the true *Variation* —— 15 14 Weſt.

Whence may be inferr'd this *General Rule.*

If the Amplitudes are both North or both South, their Difference is the *Variation*; but if one be North and the other South, their Sum is the *Variation*, whether it be at Riſing or Setting: And to find whether it be Eaſt or Weſt, ſet off the true Amplitude from the Eaſt, if it be in the Morning, or from the Weſt, if it be in the Afternoon, the ſame Way as the *Caſe* directs; *that is*,

to

to the Northward, if the Amplitude be North; or to the Southward, if the Amplitude be South, and it gives the true Place of the Sun; from whence ſet off the Magnetical Amplitude the contrary Way to its Title; *that is*, towards the South, if it be North, or towards the North, if it be South, and it gives the Magnetic Eaſt or Weſt Point; from whence ſet off 90 Degrees, or a Quadrant upwards, and it gives the Magnetic North Point. Now, if the Magnetic North Point lye to the Eaſtward, or to the Right-Hand of the true North Point, the *Variation* is Eaſt; but if it lye to the Left-Hand, or to the Weſtward of the true North Point, the *Variation* is Weſt.

Likewiſe in Azimuths, if the Magnetic and True are both Eaſt, or both Weſt, their Difference is the *Variation*; but if one be Eaſt, and the other Weſt, their Sum will be the *Variation*: And to find which Way it is, ſet off the true Azimuth from the North to the Eaſtward, if it be Eaſt, but to the Weſtward, if it be Weſt, and it gives the Place of the Sun; from whence ſet off the Magnetical Azimuth the contrary Way to its Title, and it gives the Magnetical North Point; which, if it fall in the Eaſtern Quadrant, the *Variation* is Eaſt; but if it fall in the Weſtern Quadrant, the *Variation* is Weſt.

Theſe two laſt *Rules*, as they are general, ſo they are eaſy, and ought to be well underſtood: for when a Perſon has accuſtomed himſelf to them, he will be able, after he has found the Quantity of the *Variation*, to determine which Way it is, without being at the Trouble of drawing a *Figure*.

A TABLE

A TABLE *of the* Latitudes *of ſeveral* Eminent Towns *and* Sea-Ports, *in the moſt frequented Parts of the World; together with their* Longitudes *from the* Meridian *of the* Royal Obſervatory *at* GREENWICH. *Thoſe Noted with this Mark* (*) *being determined by* Cæleſtial Obſervations; *the reſt corrected by the Help of them.*

Places Names.	Latitude. D.	M.		Diff. of Lon. D.	M.	
Acapulco *in* Mexico	17	30	N	106	15	W
Achin, *the North Point of* Sumatra	05	30	N	94	05	E
Agra, *the Mogul's Court*	26	40	N	76	45	E
* Aleppo *in* Syria	35	40	N	37	15	E
Alexandretta *in* Turkey	36	40	N	37	00	E
* Alexandrina *in* Egypt	31	11	N	30	40	E
* Amiens *in* France	49	54	N	4	20	E
Amoy, *in the* Eaſt Indies	24	35	N	116	55	E
* Amſterdam *in* Holland	52	23	N	5	05	E
Antego, *one of the Leward Iſlands*	17	30	N	60	40	W
Antibes *in* France	43	34	N	7	08	E
* Antwerp *in* Flanders	51	10	N	4	15	E
* Avignon *in* France	43	57	N	4	50	E
Babylon *in* Aſia	34	30	N	48	30	E
* Ballaſore *Road, about* 20 *Miles* ENE *from the Town*	21	20	N	86	07	E
Banjar *in* Borneo	3	00	S	109	50	E
* Bridge Town *in* Barbadoes	12	58	N	58	50	W
Barcelona *in* Spain	41	26	N	2	40	E
* Batavia, *in the Iſle of* Java, *in the* E. Indies	6	15	S	105	45	E
Bahama *Iſland, the North Point*	26	30	N	79	00	W
Bahia de Todos Santos *in* Brazile	13	00	S	40	36	W
Bermudas *in the Weſtern Ocean*	32	30	N	63	45	W
Baſora, *by the River* Euphrates	30	00	N	50	25	E
Bell Iſle, *on the* N. *Point of* Newfoundland	52	10	N	55	10	W
Bencola, *in the Iſle of* Sumatra	3	50	S	101	13	E
Bantam *in the Iſle of* Java	5	47	S	104	54	E
* Bayonne *in* Spain	43	30	N	1	29	W

Places Names.	Latitude. D.	M		Diff. of Lon. D.	M.	
Bay of St. Auguſtine, *near the G. of* Florida	30	10	N	80	45	W
Bengall, *in the* Eaſt Indies	22	27	N	91	50	E
Bergen *in* Norway	61	00	N	8	00	E
Bombay *in* India	18	58	N	72	05	E
* Bononia *in* Italy	44	30	N	11	45	E
* Boſton *in* New England	42	25	N	70	37	W
Borneo *in the* Eaſt Indies	4	20	N	109	50	E
North Point of the Iſland of Borneo	7	20	N	111	30	E
South Point of the Iſland of Borneo	4	14	S	112	52	E
* Bourdeaux *in* France	44	50	N	0	30	W
* Buenos Aires, *on the Eaſt Coaſt of* America	34	35	S	58	00	W
* Breſt *in* France	48	23	N	4	30	W
New Calabar, *on the Coaſt of* Guinea	4	40	N	8	20	E
Cadiz *in* Spain	36	16	N	6	30	W
Calais *in* France	50	57	N	1	45	E
* Camboia *in* India	10	20	N	103	30	E
Cambridge *in* New England	42	45	N	70	37	W
* Grand Canaria	27	40	N	16	10	W
Canea *in* Crete	35	29	N	23	07	E
Candia, *in the Mediterranean Sea*	35	18	N	24	15	E
* Canton *in* China	23	20	N	113	00	E
Campeche *in the* Weſt Indies	19	20	N	89	30	W
Calecut *in* India, *on the Coaſt of* Malabar	11	17	N	75	15	E
Cape Amber, *the N. moſt Point of* Madagaſcar	12	05	S	52	05	E
Cape St. Anthony, *at the Mouth of* Rio da Plata	35	20	S	57	20	W
Cape Agathus, *near the* Cape of Good Hope	35	00	S	17	40	E
Cape St. Anthony, *at the Weſt End of* Cuba	21	30	N	86	00	W
Cape St. Auguſtine, *in* Brazile	8	05	S	35	38	W
Cape Bayador, *on the* Barbary *Coaſt*	26	12	N	16	00	W
* Cape Bon Eſperance	34	15	S	16	30	E
Cape Blanco, *on the Eaſtern Coaſt of* America	46	50	S	71	10	W
Cape Britain *in* Newfoundland	46	10	N	58	50	W
Cape Catine, *on the* Barbary Coaſt	32	16	N	9	00	W
Cape Charles *in* Virginia	37	40	N	73	30	W
Cape Clear *in* Ireland	51	00	N	10	00	W
Cape Cod *in* New England	42	10	N	68	30	W

Places Names.	Latitude D.	M.		Diff of Lon D	M	
* Cape Comerin *in* India	8	00	N	76	45	E
Cape Corientes, *on the West Coast of* America	19	40	N	110	30	W
Cape Finister	43	10	N	9	55	W
Cape Florida, *in* North America	24	48	N	81	55	W
Cape Formosa, *on the* Guinea *Coast*	4	08	N	5	50	E
Cape de Gallo, *the South Point of* Ceylan	6	07	N	80	00	E
Cape Gardafui, *by the Red Sea*	11	50	N	50	40	E
Cape Hateras *in* Carolina	35	10	N	74	25	W
Cape Henry *in* Virginia	37	00	N	74	30	W
Cape Horn, *the S. Point of* Terra del Fuego	57	30	N	79	55	W
Cape de Liampo, *on the Coast of* China	28	30	N	120	30	E
Cape Lopes, *on the Coast of* Africa	1	05	S	10	00	E
Cape Nassaw, *on the East Coast of* America	9	25	N	58	00	W
Cape Negrais, *in the Bay of* Bengall	16	23	N	94	20	E
Cape Negro, *on the Coast of* Africa	16	08	S	13	30	E
North Cape, *on the East Coast of* America	2	05	N	50	00	W
Cape Orange, *on the East Coast of* America	4	05	N	51	30	W
Cape Ortegel, *in the Bay of* Biscay	44	02	N	7	45	W
Cape Palmas, *on the* Guinea *Coast*	4	03	N	7	20	W
Cape Palmiras, *in the Bay of* Bengall	21	00	N	87	40	E
Cape Race *in* Newfoundland	46	30	N	54	08	W
Cape Rasalgat, *near the Gulph of* Persia	22	50	N	60	45	E
Cape Roque *in* Brazile	5	00	S	35	52	W
Cape Sable *in* Nova Scotia	43	45	N	65	15	W
Cape St. Sebastian, *on the East Coast of* Africa	21	40	S	35	43	E
Cape Three Points, *on the* Guinea *Coast*	4	13	N	2	20	W
Cape Siera Leona, *on the* Guinea *Coast*	8	05	N	13	20	W
Cape Spartel, *in the Streights Mouth*	35	56	N	6	00	W
Cape Trio *in* Brazile	22	20	S	42	00	W
Cape Victory, *at the West End of the Streights of* Magellan	52	45	S	83	20	W
* Cape Verde, *at the Isle of* Goree	14	43	N	17	15	W
Cape V. Mary, *near the Streights of* Magellan	52	00	S	75	10	W
Cape St. Vincent	37	06	N	9	30	W
Cape Voltas, *on the Coast of* Africa	29	00	S	15	50	E
* Cartagena *in* America	10	30	N	75	30	W

	Places Names.	Latitude. D.	M.		Diff. of Lon. D.	M.	
*	Cayenne, *in the* Weſt Indies	4	56	N	53	10	W
*	Cayro *in* Egypt	30	04	N	31	26	E
	Charles Town *upon* Aſhley *River*	32	40	N	78	50	W
*	Cheuſan *in* China	30	00	N	120	30	E
	Cubumbo *in* Ceylan	6	35	N	79	10	E
	Cochin *in the* Eaſt Indies	10	00	N	75	45	E
	Conimbra *in* Portugal	40	30	N	9	45	W
*	Conſtantinople *in* Turky	41	07	N	28	55	E
*	Copenhagen *in* Denmark	56	13	N	12	45	E
	Corvo, *one of the Weſtern Iſles*	39	55	N	31	55	W
	Cracow *in* Poland	50	10	N	19	30	E
	Eaſt End of Cuba	20	05	N	74	00	W
	Cuſco *in* Peru	12	25	S	71	35	W
	Darien *in* South America	8	30	N	78	45	W
*	Dantzick *in* Poland	54	04	N	18	57	E
	Derwinda, *in the* Baltic *Sea*	57	15	N	22	01	E
	Dew Point, *on the Coaſt of* Cormandel	16	29	N	81	15	E
*	Diep *in* Normandy	49	56	N	1	50	E
	Saint Domingo *in* Hiſpaniola	18	20	N	71	50	W
*	Dublin *in* Ireland	53	12	N	7	00	W
*	Dunkirk *in* Flanders	51	01	N	2	22	E
	Durazzo *in* Dalmatia	41	58	N	17	20	E
*	Edinburgh *in* Scotland	55	57	N	3	00	W
*	Embden *in* North Holland	53	05	N	7	30	E
	Elſenore, *in the* Baltic *Sea*	56	33	N	12	27	E
	Fero Inſula, *one of the* Canaries	27	35	N	19	15	W
	Fez *in* Morocco	33	10	N	6	00	E
*	Florentia *in* Italy	43	41	N	11	20	E
	Formoſa *North Point*	25	06	N	120	05	E
	Formoſa *South Point*	21	30	N	119	30	E
	Fort St. George *in* India	13	08	N	80	00	E
	Francfort *on the Maine*	50	04	N	8	45	E
	Fyal, *one of the Weſtern Iſlands*	39	18	N	30	05	W
	Geneva *in* Italy	46	22	N	6	30	E
	Genoa *in* Italy	44	27	N	9	45	E
*	Ghent *in* Flanders	51	03	N	3	45	E

Places Names.	Latitude. D.	M.		iff. of Lon D.	M	
Goa *in* India	15	31	N	73	45	E
* Goes *in* Zealand	51	30	N	4	00	E
Gottenberg *in* Swedeland	58	07	N	10	59	E
Greenwich *at the Obſervatory*	51	28½	N	0	00	
* Guadaloupa, *one of the Leward Iſlands*	16	30	N	61	20	W
* Grenoble *in* France	45	16	N	6	00	E
* Hamburgh *in* Denmark	53	41	N	10	30	E
Havana, *in the Iſle of* Cuba	22	40	N	83	00	W
Havre de Grace *in* France	49	30	N	0	07	E
Heidelburg *in* Germany	49	20	N	9	00	E
Hoaignan *in* China	33	35	N	118	00	E
* Jaimaica, Port Royal	17	40	N	76	37½	W
Japon, *South Eaſt End*	35	20	N	41	50	E
Japon, *South Weſt End*	34	30	N	129	30	E
Java, *Eaſt End*	8	50	S	113	37	E
Java, *Weſt End*	6	30	S	103	45	E
Jeruſalem *in* Paleſtine *in* Syria	31	50	N	35	20	E
Ingolſtad *in* Germany	48	40	N	11	37	E
* Inſpruck *in* Germany	47	15	N	11	45	E
Iſland of Aſcenſion, *in the Southern Ocean*	7	40	S	14	50	W
Iſland Trinidada, *in the Southern Ocean*	20	30	S	30	00	W
Iſpahan *in* Perſia	32	25	N	52	50	E
Judda, *in the Red Sea*	21	30	N	40	10	E
* Konningsbergh *in* Pruſſia	54	43	N	21	30	E
Leghorn *in* Italy	43	18	N	13	55	E
* Leipſick *in* Germany	51	19	N	13	15	E
Leige *in* Germany	50	40	N	6	00	E
Lima *in* Peru	12	01	S	78	50	W
* Lintz *in* Auſtriæ	48	16	N	15	00	E
* Lions *in* France	45	45	N	5	00	E
* Lisbon *in* Portugal	38	43½	N	9	07	W
* London *in* England	51	32	N	0	05	W
The Lizard *in* England	50	00	N	5	10	W
Loando de St. Paulo, *on the Coaſt of* Africa	9	20	S	14	15	E
Lubaw *in* Courland	56	32	N	21	41	E
* Lubec *in* Denmark	54	46	N	9	50	E

	Places Names.	Latitude. D.	M.		Diff. of Lon. D.	M.	
*	Macao *in* China	22	13	N	116	00	E
	Madagaſcar Bay *of* Terra dell Gada	19	29	S	44	30	E
	Weſt End of Madera	32	20		16	50	W
	Madrid *in* Spain	40	10		3	15	W
*	Majorca *in the* Mediterranean *Sea*	39	35		3	00	E
*	Malacca *in* India	3	20		100	50	E
*	Malta *in the* Mediterranean Sea	35	54		14	40	E
*	Martinica, *one of the Leward Iſlands*	14	44		60	30	W
	Marſeiles *in* France	43	20		5	27	E
*	Saint Maloes *in* France	48	38		2	10	W
	Saint Mary, *one of the Weſtern Iſles*	36	55		24	50	W
	Meca *in* Arabia Fœlix	23	00		41	20	E
	Meſſina *in* Sicily	38	21		16	30	E
*	Mexico *in* America	20	06		102	10	W
	Muchin *in* Bavaria	48	58		8	45	E
*	Montpelier *in* France	43	36		3	45	E
*	Moſcow *in* Moſcovia	55	34		38	45	E
*	Namur *in* Flanders	50	25		5	00	E
	Nangaſack *in* Japan	32	53		128	45	E
*	Nancy *in* Lorrain	48	39		6	45	E
*	Nants *in* France	47	13		1	36	W
	Naples *in* Italy	40	48		14	22	E
*	Nanquin *in* India	34	55		120	00	E
	Naze *of* Norway	58	10		8	50	E
*	Narbon, *in the* Mediterranean *Sea*	43	15		3	10	E
	Narſinga, *in the* Eaſt Indies	18	15		83	30	E
*	New York Fort *in* America	40	40		74	04	W
	Nerva *in* Livonia	59	47		28	25	E.
*	Nice *in* Provence	43	38		7	15	E
*	Ningpo, *or* Liampo, *in* China	29	58		120	15	E
*	Noremberg *in* Germany	49	29		12	15	E
	Nycopingin *in* Swedeland	58	44		17	01	E
	Oldenborg *in* Germany	55	00		9	59	E
*	Olinda *in* Brazile, *or* Pernambuch	7	48	S	35	00	W
	Oporto *in* Portugal	41	18	N	9	30	W
*	Oxford *in* England	51	44½	N	1	15	W

North Latitude.

	Places Names.	Latitude. D.	M.		Diff. of Lon D	M.	
	Ozaca *in* Japan	35	05	N	136	20	E
*	Padua *in* Italy	45	31	N	11	15	E
	Panama, *on the West Coast of* America	8	56	N	82	18	W
*	Paris *in* France, *at the Observatory*	48	50	N	2	20	E
*	Pekin *in* China, *at the Observatory*	39	55	N	116	55	E
*	Petersburg, *the upper End of the* Baltic *Sea*	60	30	N	30	15	E
	Pico, *one of the Western Islands*	38	45	N	29	56	W
	Pico Teneriff *in the* Western Ocean	28	30	N	18	10	W
	Pernaw, *in the* Baltic *Sea*	58	28	N	25	47	E
	Puli Condor *in the* East-Indies	8	45	N	105	55	E
	Portobello *in* America	9	55	N	80	15	W
	Port St. Julian, *on the East Coast of* America	48	40	S	74	00	W
	Port Royal, *on the East Coast of* America	32	22	N	78	20	W
*	Poudicherri, *on the Coast of* Cormandel	11	54	N	80	15	E
*	Prague *in* Bohemia	50	40	N	14	30	E
	Providence, *one of the* Bahama *Islands*	25	00	N	77	20	W
	Quebeck, New France	46	55	N	69	53	W
*	Ratisbon *in* Germany	48	59	N	12	10	E
	Reggio *in* Italy	42	15	N	13	45	E
	Riga *in* Livonia	57	04	N	25	27	E
	Revel *in* Livonia	59	35	N	24	51	E
*	*The Mouth of the* River Gallego, *near the Streights of* Magellan	52	10	S	75	10	W
	River Amazone, *on the East Coast of* America	0	00	N	50	00	W
*	Rhodes *in* France	44	20	N	2	34	E
*	Rhodes, *in the* Mediterranean *Sea*	36	42	N	28	00	E
*	Rochell *in* France	46	10	N	1	03	W
*	Rome *in* Italy	41	54	N	12	52½	E
*	Roſtock *in* Denmark	54	37	N	11	35	E
*	Rotterdam *in* Holland	51	55	N	4	15	E
*	Roven *in* France	49	27	N	3	45	E
	Salamanca *in* Spain	41	12	N	4	00	W
	Saint Salvador *in* Brazile	12	47	S	40	40	W
	Sal, *one of the* Cape de Verde *Islands*	16	45	N	22	30	W
*	Salonica *in* Greece	40	41	N	23	08	E

Saint

Places Names.	Latitude. D.	M.		Diff. of Lon D.	M.	
Saint Sebastian, *the South End of* Madagascar	25	32	S	47	15	E
* Sevil *in* Spain	37	36	N	5	30	W
Scandroon *in* Turky	36	40	N	37	00	E
* Siam *in* India	14	18	N	100	45	E
* Smirna *in* Ionia, *in the* Lesser Asia	38	28	N	27	19	E
Sphaham *in* Persia	36	14	N	65	00	E
Stockholm *in* Swedeland	59	20	N	18	10	E
Stralsond, *in the* Baltic *Sea*	54	37	N	13	11	E
Streights of Sunday, *in the* East Indies	5	52	S	104	18	E
Surinam, *on the East Coast of* South America	6	30	N	56	56	W
Surrat *in* India	21	10		73	00	E
Syracusa *in* Sicily	37	04		15	15	E
Tangier, *in the Strioghts Mouth*	35	35		5	15	W
Teneriff, *the North Point*	28	25		17	30	W
Toledo *in* Spain	39	46		3	30	W
* Toloun *in* France	43	07		5	45	E
Trinidada, *one of the Charibbee Islands*	10	05		60	18	W
* Tripoly *in* Barbary	32	54		13	00	E
* Tubing *in* Germany	48	34		9	15	E
Valentia *in* Spain	39	30		0	46	E
Venice *in* Italy	45	48		12	30	E
Vera Cruz *in* America	19	12		99	30	W
* Vienna *in* Germany	48	22		17	15	E
Upsal *in* Swedeland	59	00		18	00	E
* Uraniburg *in* Denmark	55	54		12	45	E
* Utrecht, *in the Low Countries*	52	05		5	00	E
Widdah, *on the* Guinea *Coast*	6	35		4	10	E
* Wirtemberg *in* Saxony	51	53		13	00	E
Wolfenbuttle *in* Germany	52	11		11	00	E
Wyborg, *in the* Baltic *Sea*	61	12		28	15	E
* Ylo *in* Peru	17	36		71	13	W
York *in* England	54	00		1	00	W

North Latitude.

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